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## **Use of the Geometer's Sketchpad in Secondary Schools**

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**Abstract:** *This paper presents and discusses the results of a survey of Singapore secondary school teachers' use of the Geometer's Sketchpad in their mathematics classrooms. Questionnaires were completed by 41 teachers from ten schools. Responses in the questionnaires were elicited to inform two main questions: Which geometry topics were taught using the software; and, how do teachers prefer to use the software? Findings include that the potential for classrooms to become lab-like environments is not being realized in most classrooms.*

### **The Geometer's Sketchpad**

To Singapore secondary school mathematics teachers, the *Geometer's Sketchpad* (*GSP*) is one of the most familiar computer-software programmes. Frequent references to its use in learning mathematics are found in some commonly-used secondary mathematics textbooks (Teh & Looi, 2001/2002; Sin, 2001/2002).

*GSP* is a member of a family of new generation geometric construction programmes now commonly known as Dynamic Geometry (DG) Software types. Other members of this family include the *Cabri*, *Geometry Inventor* and *Geometric Supersupposer* (Goldenberg & Cuoco, 1998).

As the name suggests, one distinctive feature of DG software is its dynamic visual outputs. So, in *GSP*, objects that are constructed on screen can be manipulated dynamically with the ease of click-and-drag. This is unlike the static diagrams drawn by paper-and pencil. This feature that affords dynamic movements can potentially help students develop visual-spatial tools to manipulate mental objects (Clements & Battista, 1994). In one study, Singapore secondary students who spent a considerable amount of time working with *GSP* in transformation geometry were found to make significant increases in their test scores in the Wheatley Spatial Ability Test (Leong, 2002).

One common didactical use of the *GSP* in geometry education is through the use of its drag-mode. Geometrical objects are constructed in *GSP* with in-built properties so that these properties remain invariant upon dragging. These are known as drag-resistant figures (Hoyles & Noss, 1994). For instance, a drag-resistant square is one that retains all the critical attributes of a square – perpendicularity of adjacent sides, congruence of all sides – while other non-critical attributes such as size and

orientation may be varied upon dragging. Or, in the common cyclic quadrilateral theorem, the sum of measures of opposite angles stays at a constant (at 180 degrees) while the individual measures are dynamically updated upon dragging. This way of helping students abstract geometric properties by viewing virtually infinitely many exemplars is described by Chazan and Yerulshamy (1998) as 'seeing the general in the particular'. Used in this way, students who work with *GSP* can be led to see beyond the graphical aspects of the objects-on-screen to consider the geometry underlying their constructions. This is an exciting prospect for geometry teachers.

Yet the greatest promise of these DG tools is in the way they can revolutionise the teaching of geometry. The modern approach to the teaching of geometry, grounded in constructivist philosophy, favours the engagement of students in actively exploring the objects of focus in diverse pathways. Instead of direct instruction from the teachers, students are to study the tasks given by the teacher, make observations and generalizations, and check their generalizations. The teacher plays the role of a guide and social authority in these experiment-oriented classrooms. To support such a classroom environment, the presence of tools that allow the collaborative pursuit of different paths of thinking is advocated. The *GSP* is looked upon as one such tool. The menu options allow students to perform precise operations on-screen and to check the correctness of their procedures by dragging. Usual editing functions are present to allow quick retracing of steps. The 'measure' menu, in particular, allows students to take accurate 'live' measurement of lengths, angles, areas, and so on, and to perform computations of these measures to aid in the observation for relationships. These inherent features of such software lend themselves easily to students' explorations, as well as to creating and verifying of conjectures (Lampert, 1988; Hoyles, 1993; Laborde, 1995; Olive, 1998) with the possibility of leading to proofs (de Villiers, 1998; Scher, 1999).

### **The Study**

This study sought to investigate how extensive the use is of *GSP* in the secondary schools and how the tool is used by teachers in their geometry lessons.

The method of obtaining the required data was by questionnaire completed by teachers. For the purpose of this report, three main sections of the questionnaire are discussed. The first section elicited brief background information. There were fields for the 'school currently teaching', the 'number of years teaching mathematics' and 'the mathematics grade levels taught in the last three years'. The 'last three years' clause was inserted to coincide with the approximate timing where *GSP* was introduced into secondary schools in Singapore.

The second section of the questionnaire directly addressed the question on how extensive the software was used. The first question in this section was, "have you ever used *GSP* in your teaching of mathematics?" This was followed by a list of topics in geometry (presented in Figure 1) where respondents were invited to tick against the topics where *GSP* was used in their teaching. The topics listed do not follow any particular order and consists of possible overlaps between topics. This delineation was chosen as it matches chapter/topical headings in textbooks that were widely used by schools. It was thus intended to help respondents identify easily with the topics.

- Angle properties of a polygon
- Angle properties of points/lines
- Angle properties relating to circle
- Congruency
- Coordinate Geometry
- Geometrical Constructions
- Locus
- Mensuration
- Properties of special quadrilaterals
- Properties of triangles
- Pythagoras Theorem
- Similarity
- Symmetry
- Transformations
- Trigonometrical 'rules' and 'formulas'
- Trigonometrical graphs
- Vectors
- Others, if any (please indicate topic below)

Figure 1: Geometry topics listed in the questionnaire

As information for the way in which *GSP* was used in their teaching, the third section of the questionnaire presented a list of 'modes of *GSP*-use' (see Figure 2). As there was no known similar prior study on how teachers in Singapore used the *GSP*, the various modes listed were based on the author's knowledge of how the software was used by some teachers through his interactions with them. The numerous choices provided in the list were intended to capture at least one match that would approximately describe the actual way the teacher used *GSP*, though teacher's practice may not have fit exactly with any particular mode described. Respondents were asked to tick against the modes of use and to put a preference-of-use rank against each tick. In addition, there was an item in this section that

prompted the teachers to describe the most preferred mode of use. This item was inserted to seek elaborations on the way the teacher uses the software.

- Draw diagrams for worksheet/test paper
- Let students explore hands-on freely
- Provide templates for students to observe and conjecture properties
- Students observe templates to fill in answers to a given worksheet
- Teacher click-and-drag pre-designed templates to show some geometrical properties
- Teacher demonstrates drawing/construction procedure in class for students to follow
- Teacher shows animation/movement in front of class to aid students' visualization
- Others, if any (please indicate mode below)

*Figure 2. Modes of GSP-use as listed in the questionnaire*

Questionnaires were sent to teacher representatives of 21 randomly selected secondary schools. The invitation for response from the schools included a request for five completed questionnaires from each school. Forty-one completed questionnaires from ten schools were returned.

### Results

Out of the 41 respondents, 33 (more than 80%) indicated that they have used the *GSP* in the teaching of mathematics. The teaching experience of these 33 teachers varies considerably. A breakdown of the years of teaching experience of these teachers is given in table 1 below.

Table 1.

*Distribution of teaching experience of respondents who used GSP*

| Number of years teaching mathematics | Number of teachers |
|--------------------------------------|--------------------|
| 5 years or less                      | 19                 |
| Between 6 to 10 years                | 8                  |
| More than 10 years                   | 6                  |
| Total                                | 33                 |

To have a better picture of the frequency of use, responses in the second section of the questionnaire on the use of the software in particular geometry topics is useful. As the topics listed in Figure 1 include content in both the lower and upper

secondary level syllabuses but, as most teachers specialize in the teaching of only one of the levels, the 33 teachers were further categorized into 'lower secondary (LS)' and 'upper secondary (US)' divisions. The information that the teachers provided in the first section of the questionnaire indeed showed clear specialization in either levels. The task of categorization was therefore straightforward with the exception of two teachers who indicated that they taught at both levels. The data they provided were thus included in both the tables for LS and US. As a result, there were 11 respondents categorized as LS teachers and 24 as US teachers. Topics in Figure 1 were also separated into the levels according to what the respective syllabuses indicate. Where topics can potentially be taught at both levels (as in 'Congruency'), they appear in tables of both levels. Table 2a and Table 2b below show the frequency of *GSP*-inclusion in the coverage of geometry topics in the respective levels.

Table 2a

*LS teachers' use of GSP according to geometry topics in the syllabus (N=11)*

| Geometry topics                      | Number of teachers who use GSP in teaching these topics |
|--------------------------------------|---|
| Angle properties of a polygon        | 8   |
| Angle properties of points/lines     | 7   |
| Congruency                           | 2   |
| Coordinate Geometry                  | 2   |
| Geometrical Constructions            | 4   |
| Mensuration                          | 1   |
| Properties of special quadrilaterals | 4   |
| Properties of triangles              | 6   |
| Pythagoras Theorem                   | 5   |
| Similarity                           | 2   |
| Transformations                      | 6   |
| Symmetry                             | 2   |
| Trigonometric 'rules' and 'formulas' | 2   |

Frequencies peaked with some topics. 'Angle properties relating to circle' and 'Transformations' were favourites with the US teachers for using *GSP* while the corresponding favorites for LS teachers were 'Angle properties of a polygon' and 'Angle properties of points/lines'. 'Mensuration' surprisingly showed low usage of *GSP* despite the powerful 'measure area' function inherent in the software. 'Congruency' and 'Similarity' received low responses for *GSP*-use although 'Transformations' were well-supported for *GSP* use. There appears to be no

Table 2b.

*US teachers use of GSP according to geometry topics in the syllabus (N=24)*

| Geometry topics                      | Number of teachers who use GSP in teaching these topics |
|--------------------------------------|---|
| Angle properties relating to circle  | 21  |
| Congruency                           | 2   |
| Coordinate Geometry                  | 10  |
| Geometrical Constructions            | 7   |
| Locus                                | 12  |
| Pythagoras Theorem                   | 3   |
| Similarity                           | 1   |
| Transformations                      | 19  |
| Trigonometric 'rules' and 'formulas' | 5   |
| Trigonometric graphs                 | 7   |
| Vectors                              | 4   |

relation between these topics in terms of *GSP*-use despite the strong mathematical link by way of congruencies and similarities being compositions of transformations.

The above data seemed to indicate that teachers' use of *GSP* was not 'even' across the geometry domain. Certain common 'pet' topics were favoured over others as appropriate for *GSP* use. It appeared that certain attributes of the software were piece-wise utilized in a fragmented way to fit into bits of geometry, instead of a full integration into the curriculum. Clearly, if such was the case, the full power of the software was not adequately harnessed for students' experience in learning geometry. *GSP* appeared to be still standing as an 'outsider' to the geometry classroom and invited in only once in a while when its dynamic features came in handy for showing some visual effects. This picture of *GSP*-use contrasts drastically from the view described earlier in this paper where its use can potentially transform students into active inquirers through experimentations and verifications.

In the third section on the questionnaire, respondents indicated their modes of *GSP*-use and ranked their preferences of modes. They were also prompted to describe how they conducted a sample lesson with their most preferred mode of *GSP*-use. Most of the teachers indicated a number of modes used. Only the most preferred mode used however shall be presented here for comparison. In questionnaires where respondents did not provide a preference-rank, the description of the sample lesson (where given and discernible) served to indicate the preferred mode of use. Three of the respondents did not indicate preference-rank and did not provide descriptions. Their data were not accounted for in the Table 3 below.

Table 3.

*Distribution of the teachers' most preferred mode of GSP-use*

| Modes of GSP-use |   | Teachers indicating the mode as most preferred |
|------------------|---|--|
| M1               | Draw diagrams for worksheet/test paper  | 7  |
| M2               | Let students explore hands-on freely  | 3  |
| M3               | Provide templates for students to observe and conjecture properties                 | 0  |
| M4               | Students observe templates to fill in answers to a given worksheet                  | 4  |
| M5               | Teacher click-and-drag pre-designed templates to show some geometrical properties   | 6  |
| M6               | Teacher demonstrates drawing/construction procedure in class for students to follow | 4  |
| M7               | Teacher shows animation/movement in front of class to aid students' visualization   | 6  |

Taken together, modes M1 to M7 present many possible inter-overlaps in description of uses of the software and the difference between each mode may appear at first look to be insignificant. By putting on a different 'lens' however, the differences become more conspicuous: M1 indicates a use of the software in an *out-of-class* setting, and therefore not directly related to classroom instruction; M2 and M3 are modes where the use of software in relation to learning geometry is at least partially *student-directed*; M4 to M6 describe modes where the software use is wholly *teacher-controlled*. It may be disputed that M4 is wrongly placed in the teacher-controlled category. However, granted that the phraseology admits the possibility of describing students' direct hands-on with the software, the direction of the learning track – 'fill in answers in a given worksheet' – is nevertheless teacher-controlled. Seen through these three collapsed broader categories, the data shows that most of the teachers (20 out of 30) preferred the use of the software in a teacher-controlled setting. Only three teachers indicated that their first preference was to allow room for students to chart their paths of learning with *GSP*. The other 7 teachers' top preference was to use the software outside of classroom practice.

The teachers' preference-of-use data in the third section of the questionnaire adds support to the earlier conjecture that the full power of the software and the 'promise' of its potential to transform classrooms into lab-like places for students'

inquiries are not realized in most geometry classrooms. Rather, the typical classroom picture looks more like one where conventional instructional activities take place and where the software comes in occasionally as an add-on (or perhaps 'extra') to spice-up the scene. As soon as it serves its visual delectations, teachers direct students back to the main course and work proceeds 'as usual' till there is need again for another occasional visual treat.

### Discussion

Education reformers who wish to see more "integration [of] information technology to enhance the mathematical experience" (UCLES/MOE, 2000) may find the pace of 'integration' as reported in this study slower than hoped for. What could be some reasons for this disparity between policy intent and actual practice in this IT-reform movement?

Manouchehri (1999) conducted a survey to find out how some teachers in the United States respond to the recent mathematics reform movement that emphasized the use of computers in mathematics classroom. She reported that teachers hardly used the computers in teaching. One key reason cited for the lack of appropriate computer usage was the inadequacy in teacher training which led to teachers being ill-equipped to handle technology and the complementary teaching strategies required in the classroom. "Essential ingredients for successful implementation such as modeling of effective pedagogical strategies, questioning techniques conducive to enhanced instruction and assessment, and management issues, most often do not receive adequate attention. The consequence of this practice is obvious . . . the use of technology becomes yet another addition to an already crowded daily schedule" (p.38). This study holds important lessons for Singapore's IT-reform efforts.

One other approach to understanding the difficulties of integrating IT in the classroom stems from seeing teaching as a complex "cultural activity" (Stigler & Hiebert, 1999, p.86). Viewed as such, teaching is a cultural system and – as in all cultural systems – do not change easily. In a system, "the whole is greater than the sum of the parts" (p.97). So conventional reform efforts that twitches one or two features of teaching by changing a textbook here and bringing in some technology there may change merely "surface features . . . [but] the fundamental nature of the instruction does not [change]" (p.98).

There are encouraging signs yet. Thirty-three out of the 41 teachers who responded to the survey indicated that they have tried working with *GSP*. This can be interpreted as indicating teachers' willingness to incorporate new technology into their teaching. Yet, it may mean that the effort to integrate technology into the curriculum takes time.

In France, where there are strong incentives from the French Ministry of Education to promote the integration of information technology in education since 1997, research has begun to develop these programmes of integration. Laborde's (2002) work was in the specialized area of integrating DG software into the mathematics curriculum. She reported that "really integrating technology into teaching takes time for teachers to accept that learning might occur in computer-based situations without reference to paper-and-pencil and to be able to create appropriate learning situations. But it also takes time for them to accept that they may lose part of their control over what students do" (p.32). This concluding note by Laborde provides both the hope for success in Singapore's IT-reform over time and the urge to engage in research to understand the problems of IT-integration in practice.

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