Language Proficiency and Rewording of Semantic Structures in P5 Pupils' Mathematical Word Problem Solving[1]

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Abstract: In Singapore the solving of mathematical word problems is a major component both within the instructional programme as well as during formal assessments. Research has indicated that both language and semantic structures play a part in determining pupils' performance in the solving of mathematical word problems. This study aimed to determine if Primary 5 pupils' language proficiency and the rewording of mathematical word problems according to some semantic structures could affect the pupils' problem solving process. The findings of this study revealed that the pupils' English proficiency did not affect performance in solving the word problems. However certain rewording constructs like "chronological order of events" and "repositioning of givens" positively affected the low- and average-ability pupils.

Introduction

In Singapore schools, English is the main medium of instruction for teaching and learning. During mathematics lessons, pupils are thus required to solve mathematical word problems which are presented in English. Over the years, much research have been done locally and internationally to determine if language proficiency and language-related factors affect pupils' performance in solving mathematics word problems (Aiken, 1972; Cuevas, 1984; De Corte & Verschaffel, 1991; Bernardo, 1999; Yeap & Kaur, 2001). Clearly language and its constructs are determinants in the outcomes of word problem solving.

This study was concerned with determining whether pupils' problem-solving ability in mathematical word problems was dependent on a good command of English. This study also addressed whether the rewording of word problems based on certain semantic structures would help pupils perform better.

Cognitive and linguistic factors in understanding mathematics word problems
Earlier studies (1970 – 1995)
In this time period, researchers developed different models with respect to how children understand mathematical word problems. Kintsch and Greeno (1985) and De Corte and Verschaffel (1991) believed that the problem solver has to construct an accurate mental

model of the problem in order to understand the text, which is the basis for choosing the operational processes. Mayer (1983) claimed that good problem solvers possess an array of mental "templates" that organize incoming information into a familiar framework known as a "schema". Mathematical schemas serve to structure the way word problems are viewed and interpreted. Misinterpretation of a problem may be seen as a result of the choice of an inappropriate schema. Nesher (1986) suggested that expert problem solvers use "control systems" that monitor and revise their choice of schema, where such error control helps them to catch their mistakes and revert to the appropriate schema. Novice problem solvers are claimed to have a much smaller set of schema to choose from and inadequate "control systems" to detect inappropriate choices.

Reading comprehension was also a focus of research during this period and cited as a factor contributing to pupils' successful mathematical problem solving achievement. Early studies by Aiken (1971, 1972) showed significant correlations between reading ability and high problem solving ability. Muth and Glynn (1985) investigated how reading and computational skills work together in successful problem solving and concluded that both play important roles in children's successful solution of word problems.

Recent research (1996 to date)
The issue of language contributing towards mathematics learning difficulties has continued in recent years. Fuentes' (1998) asserted that the style of mathematical writing is strikingly different from non-mathematical texts and hence demands that the learner acquire special reading skills to unpack the meaning in the problem text. Similarly, Zevenbergen (2001) contended that the language of mathematics is very specific, and students need to identify correct meanings of words in order to communicate effectively and construct appropriate meanings. Borasi, Siegel, Fonzi, and Smith (1998) found that students lack reading strategies when solving mathematical problems as they were infrequently incorporated into mathematical instruction. More recently, Moreau and Coquin-Viennot (2003) stated that the understanding of word problems leads to the construction of two complementary representation levels - that of the problem model and situational model. Bernardo and Calleja (2005) reported that students performed better in their first and more proficient language as they enabled them to understand the word problems better.

Although relatively few current studies have focused on how rewording of word problems affects pupils' problem-solving performance, findings have been promising. Verschaffel, Greer, and De Corte (2000) found, for example, that rewording of word problems have systematic effects in the problem-solving performance of students when they are faced with ambiguous and abstract language. Rewording of word problems by making the semantic structures more explicit compensates for the less developed semantic schema and facilitates the solving of these problems. Bernardo (1999), from his findings, noted that students showed better

understanding and solution performance when the problems were reworded to state more explicitly the relationship among known and unknown quantities.

Further, as language allows us to describe events in a sequence different to that in which they actually occur, De Corte and Verschaffel (1991) and Teubal and Nesher (1991) argued that when the relationship between the order of mention of numerical data and the order of events in the word problems conflict, they give rise to potential confusion to younger students because of the need to take into account the possibility of having different sequencing of data in terms of real time, description in the text, and in yielding a canonical number sentence.

As well, studies in rewording through personalization (that is, making the context of problems more personalized using familiar words and situations) have also shown positive results. Hart (1996) claimed that through personalizing word problems, pupils' background knowledge could be tapped helping to bridge the gap between existing and new knowledge. Her study showed that both the attitudes and achievement of the students improved as a result. More recently, a study by Heng-Yu and Sullivan (2000) similarly showed performance improvement among low-ability Taiwanese students.

De Corte and Verschaffel (1991) also noted that children's solution processes could be influenced by the sequence of the given numbers presented in the problem text and called for rewording of problems to make the semantic relations more explicit. Casteel (1990) found that chunking sentences into meaningful units of thought helped the low-ability readers more than the high-ability readers in reading comprehension of text.

In summary, it thus appears that there are four types of rewording constructs that could affect pupils' understanding towards better performance in solving word problems - "chronological order of events", "personalization", "chunking", and "repositioning the givens".

Research Questions

This study was thus designed to examine pupils' language proficiency in problem solving in relation to rewording constructs. Specifically, the following research questions were posed:

1. Do good English language performers (GP) have better achievement in mathematical (English) word problem solving than the marginal English language performers (MP)?

2. Is there a difference in the achievement of the pupils in their respective ability groups (Low Ability, Average Ability and High Ability) when word problems have been reworded according to selected semantic structures? If so, what types of semantic structures have an effect on the pupils' achievement?

Methodology

Sample

Eighty-three Primary 5 pupils from two homogeneous, mixed-ability intact classes participated in this study. They were grouped into Good English Performers (GP)[2] and Marginal English Performers (MP) based on their English language performance in the Primary 4 Streaming Examination. The breakdown of the sample participants is shown in Table 1. Pupils with Bands 1 and 4 were not involved in this study as they were in other classes.

Table 1

Number of pupils who achieved Bands 2 and 3 for English Language

Band	Class 1	Class 2	Total
2	19	22	41 (GP)
3	23	19	42 (MP)
Total	42	41	83

Instruments

Two test instruments, the Original Word Problem Test (OWT) and the Reworded Word Problem Test (RWT) were designed for this study. The OWT comprised five word problems modified from samples in mathematics assessment books. The RWT comprised five reworded (reworded from the original and deemed to be parallel to the original) word problems. The OWT and RWT are shown in Appendix A. The problems in the RWT were crafted based on four types of semantic structures, namely, "Chronological Order of Events", "Personalization", "Chunking" and "Repositioning the Givens" as summarized in Figure 1.

Two experienced mathematics teachers (the Head of Department for Mathematics and the Level Head for Mathematics) vetted the instruments to ensure that the questions were parallel. These teachers rated the accuracy of each pair of original and reworded problems against two criteria - the textual content of the reworded problem should retain the meaning of the original problem, and the numerical figures used in the reworded problem should be close to the original problem's figures. The word problems were revised based on the feedback given by the teachers.

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[2]The mark range and descriptors prescribed by the Ministry of Education for each band are as follows:
  Band 1 (85 - 100 marks): Pupil is "very good in the subject"
  Band 2 (70 - 84 marks): Pupil "is good in the subject"
  Band 3 (50 - 69 marks): Pupil "has adequate grasp of the subject"
  Band 4 (below 50 marks): Pupil "has not met minimum requirements for the subject"

Type	Description of Reworded Semantic Structures
1. Chronological order of events	Rewording to make the order of data presentation (order of events) follow real time events.
2. Personalization	Rewording to make the word problem more personalized by the use of personalized items.
3. Chunking	Rewording by breaking up a sentence structure into two separate sentences - also known as "chunking".
4. Repositioning	Rewording by presenting the data through the switching of any two givens to attain a sequential processing order.

Figure 1. A framework using the four types of rewording semantic structures

Procedure

A repeated-measure counterbalanced approach was used. Pupils of one class sat for the OWT while pupils of second class sat for the RWT at the same time. Both the tests lasted an hour, followed by a recess break. After the break, the first class sat for the RWT while second class sat for the OWT.

A generic analytic scoring scale (0 to 3) was used to score the pupils' solutions. This analytic scoring scale was a modification and simplification of an analytic scoring scale by Charles, Randall, Lester, and O'Daffer (1987). Scoring did not take into account computational errors but the flow of thought in the pupils' solutions.

To address the second research question, the pupils were regrouped into Low Ability (LA), Average Ability (AA) and High Ability (HA) based on their problem-solving performance in the OWT. Their scores obtained in the OWT were then compared to scores obtained in the RWT using paired t-tests. A qualitative analysis was also carried out on samples of pupils' solutions.

Results and Discussion

The scores of the pupils obtained from the two mathematical word-problem tests were analyzed to determine if English language proficiency had an effect on mathematical problem solving between the GP and MP groups. The analysis also aimed to determine the effect of rewording according to semantic structures on pupils mathematical problem solving abilities.

For the first research question of whether or not the GP performed better than the MP in both the OWT and RWT, independent groups t-tests carried out to compare the

effects of language proficiency on the pupils' performance in solving mathematical word problems between the two groups revealed no significant difference in both tests, t (81) = 0.496, p > 0.05 for the OWT and t (81) = 0.237, p > 0.05 for the RWT, suggesting that difference in language-proficiency levels does not affect performance in solving mathematical word problems. One explanation for this finding is that as both the GP and MP groups of 11-year old subjects have used English in class for five years, they would not understand word problems too differently as compared to subjects in other research studies who were mainly first or second graders and were at stages where they were still learning the language. Further the distinction between Band 2 and Band 3 pupils would necessarily not be as great as, for example, the distinction between pupils in Band 1 and Band 4.

The purpose of the second research question was to determine if rewording of word problems based on some semantic structures had an effect on pupils' solving of word problems. Each of the constructs is considered below:

Type 1 - Chronological Order of Events

From Table 2, the low-ability (LA) group registered a significant difference, t(19) = -3.387, p<0.05, suggesting that rewording the word problem chronologically had benefited this group of pupils significantly. For the average-ability (AA) group, there is a reasonably large increase in the performance mean for the RWT over the OWT but the increase is not statistically significant, t(28) = -1.995, p > 0.05. The high-ability (HA) group did not show any significant difference.

Table 2
Analysis of LA, AA and HA groups' performance for WP1 and RP1

Group	Problem 1 Type 1	Mean	N	Std. Deviation	Std. Error Mean	t	Sig. (2-tailed)
LA[1]	WP1	1.60	20	1.31	0.29	-3.387	0.003
	RP1	2.40	20	0.88	0.20		
AA	WP1	2.18	34	1.14	0.20	-1.995	0.054
	RP1	2.68	34	0.68	0.12		
HA	WP1	2.86	29	0.35	0.006	0.849	0.403
	RP1	2.72	29	0.80	0.15		

[1]Low-ability (LA);
AA (Average ability);
HA (High ability).

A qualitative analysis of pupils' solutions revealed two similar error patterns. Based on WP1, the opening phrase "After giving 10 clips to Peter", which in real time is not the first event to take place, could have posed a cognitive conflict to some pupils. These pupils in trying to get the correct sequence of events started by subtracting 10, and used the derived number to double Norman's number of clips (as in 40 x 2 = 80), perhaps

having the impression that from the sequence the doubling was consistent with Norman having "twice as many clips as Peter" (Figure 2). The other error pattern is shown in Figure 3, whereby pupils who managed to arrive at the correct solution, for some reason superfluously added 10 as a last step, perhaps thinking that that would be the point in time when Norman had not given any clips to Peter.

Figure 2. Sample solution of WP1

Figure 3. Another sample solution of WP1

The reworded problem, which chronologically ordered the events according to real time, facilitated the pupils' thinking processes to achieve the full and correct solutions as shown in Figure 4.

Figure 4. Sample solution of RP1

Type 2 - Personalization

Based on the paired t-test analysis in Table 3, the LA and AA groups did not show a statistically significance difference in their performance under rewording through personalization. The AA and HA groups showed a decline in their mean performance from 1.06 to 0.82 and 2.93 to 1.41 respectively. The HA group showed a statistical difference in the opposite direction, $t(28) = 5.525$, $p < 0.05$.

Table 3

Analysis of LA, AA and HA groups' performance for WP2 and RP2

Group	Problem 2 Type 2	Mean	N	Std. Deviation	Std. Error Mean	t	Sig. (2-tailed)
LA	WP2	0.30	20	0.80	0.18	-0.418	0.681
	RP2	0.40	20	0.82	0.18		
AA	WP2	1.06	34	1.35	0.23	1.034	0.309
	RP2	0.82	34	1.34	0.23		
HA	WP2	2.93	29	0.37	0.0069	5.525	0.000
	RP2	1.41	29	1.50	0.28		

The results contradicted the positive-impact personalization had in the research by Heng-Yu and Sullivan (2000) and Hart (1996). An evaluation of suspected areas with respect to the poor performance of the HA group in solving the personalized problem pointed to several possible causes. The reworded version RP2 exceeded WP2 by fourteen words which could have made the reading and understanding of RP2 more cumbersome. There was the possibility also that RP2 itself was ambiguous and was not as parallel to the WP2 as intended. The way RP2 was phrased could have led pupils to some misunderstanding. Further analysis was based on examining the pupils' solutions.

In Figure 5, a HA pupil's thought processes is captured as seen in his use of a pictorial model to aid his construction of a representation of the pen-book association. This approach aided his solving of the problem. In Figure 6, however, he assumed each card carried the same number of points, thus he divided 151.2 by 8. Dividing by 8 was the most common mathematical sentence provided by the HA
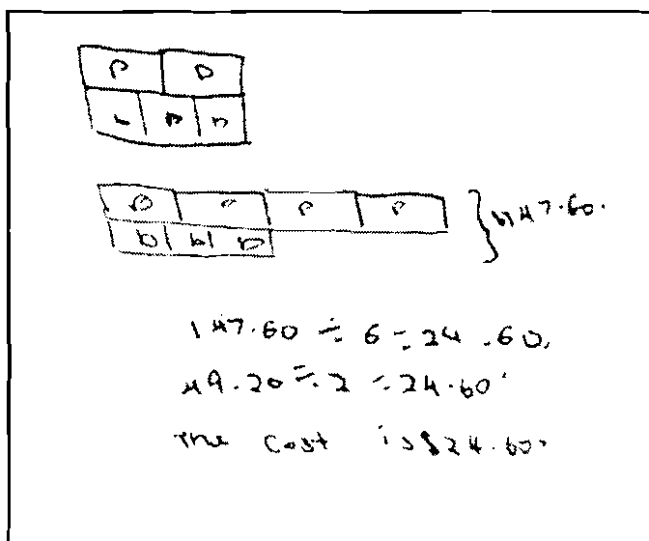

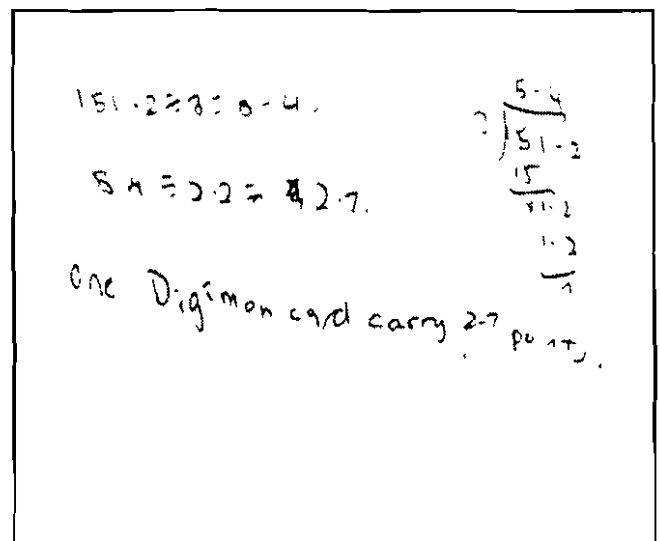*Figure 5.* Sample solution for WP2

Figure 6. Sample solution for RP2

pupils in attempting RP2. For the other pupils who could not solve RP2, their solutions were quite diverse. Evaluating the problem text suggests that the complex algebraic structure could have increased the complexity of the problem. It is also possible that the pupils' "over-developed schemata" with the familiar mathematical term "cost as much as" (frequently used in the school mathematics textbooks and classroom instruction) conflicted with their understanding of the phrase "I can exchange..." thus causing them to read more into the problem than necessary.

Type 3 - Chunking
There were no significant differences for the reworded treatment for all the three groups (Table 4). Analysis of sample solution scripts revealed that chunking had not caused the pupils to process RP3 differently from WP3 as depicted in Figures 7 and 8 respectively. Chunking may be more useful when huge paragraphs of texts are chunked into smaller readable units. In this case, the rewording for RP3 had merely replaced the conjunction "and" from WP3 and provided similar remaining phrases to the first part of both the opening sentences. As these two word problems were not lengthy, chunking perhaps did not have a significant effect in aiding understanding.

Table 4
Analysis of LA, AA and HA groups' performance for WP3 and RP3

Group	Problem 3 Type 3	Mean	N	Std. Deviation	Std. Error Mean	T	Sig. (2-tailed)
LA	WP3	0.85	20	1.18	0.26	-0.507	0.618
	RP3	1.05	20	1.36	0.30		
AA	WP3	2.53	34	1.05	0.18	1.327	0.193
	RP3	2.26	34	1.16	0.20		
HA	WP3	2.97	29	0.19	0.003	1.000	0.326
	RP3	2.93	29	0.37	0.007		

Figure 7. Sample solution of WP3

Figure 8. Sample solution of RP3

Type 4a -- Repositioning the Givens (Algebraic Type)

From Table 5, all three groups showed an increase in their performance means in the comparison suggesting the positive effect of Type 4 rewording. Both the LA and AA groups registered p values less than 0.05 [$t(19) = -2.373$, $p < 0.05$, for LA, and t $(33) = -2.510$, $p < 0.05$, for AA, respectively], indicating significant differences in the pupils' performances.

Table 5

Analysis of LA, AA and HA groups' performance for WP4 and RP4

Group	Problem 4 Type 4	Mean	N	Std. Deviation	Std. Error Mean	T	Sig. (2-tailed)
LA	WP4	1.40	20	1.05	0.23	-2.373	0.028
	RP4	2.20	20	1.15	0.26		
AA	WP4	2.26	34	1.02	0.18	-2.510	0.017
	RP4	2.65	34	0.81	0.14		
HA	WP4	2.45	29	0.91	0.17	-1.361	0.184
	RP4	2.66	29	0.77	0.14		

The finding suggests that when the positions of the givens in the word problem which has an algebraic structure were repositioned, the relationship among the known and unknown quantities became more explicit and mediated the language format, consistent with Bernardo's (1999) view that in most cases when problem-solving performance improved, it was due to better comprehension through rewording to make explicit the known and the unknown quantities.

From sample solutions for WP4, most pupils from the three groups tended to multiply $\frac{3}{5}$ by 75 in the second step as shown in Figure 9. These pupils believed that the resulting "75" was the number of oranges left and therefore were consistent in their

Figure 9. Sample solution of WP4

Figure 10. Sample solution of RP4

interpreting the text "$\frac{3}{5}$ of his oranges left". Likely the position of the givens and the language format led pupils to think that this was a case of finding "the fraction of the remainder". In Figure 10, by switching the positions of the givens, the intention for the known fraction to be operated with the whole first became clearer and this reduced the tendency to interpret the text towards finding "fraction of a remainder".

A similar argument applies for some of the LA and AA pupils who used the "unitary" method in their solutions. In Figure 11, like the above example, "$\frac{3}{5}$ of his oranges left" could have "misled" pupils to equate 5 units with 75, and 2 units to represent the number of rotten apples. The reworded problem in Figure 12 enabled the pupils to understand the problem better and equate 5 units to the superset 60 towards getting the correct solution.

Figure 11. Another sample
solution of WP4

Figure 12. Another sample
solution of RP4

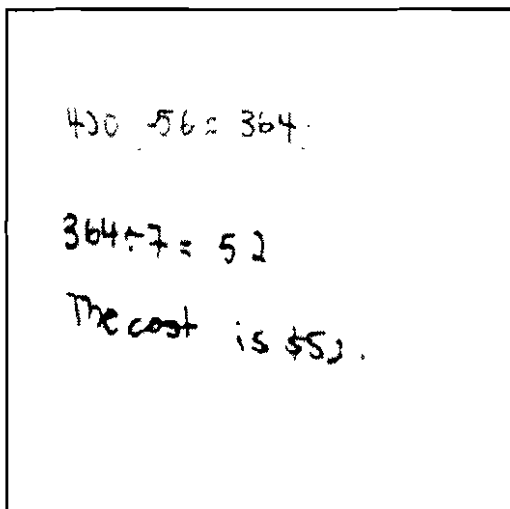## Type 4b - Repositioning the Givens (Mechanical Type)

In an attempt to use the same rewording construct of repositioning the givens with another problem but of a mechanical nature, a problem based on area and perimeter was chosen. The paired samples t-test carried out revealed no significant difference for this construct for all the three groups as shown in Table 6. The insignificance differences in the pupils' performance could be due to the mechanical nature of the problem whereby the knowns and unknowns revolve around a formula pertaining to finding area or perimeter. The positions of the givens therefore did not matter much. However, the improvement in mean performance for the LA group (0.85 for OWT and 1.40 for RWT) showed that this rewording construct had to some extent benefited the weaker pupils.

Table 6

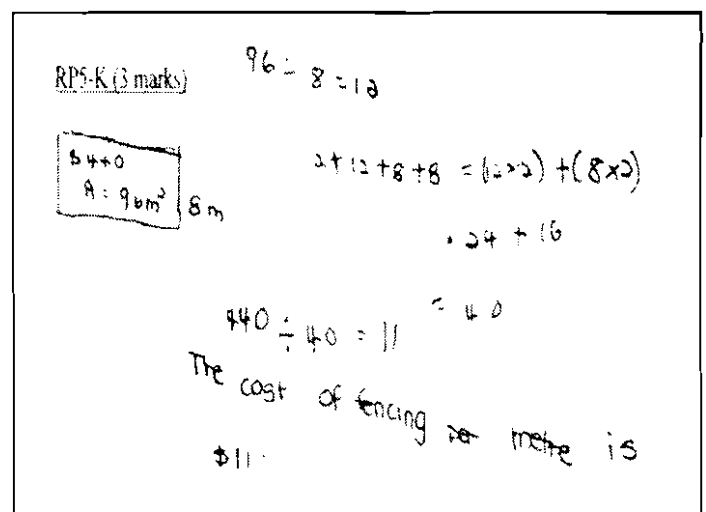*Analysis of LA, AA and HA groups' performance for WP5 and RP5*

Group	Problem 5 Type 4	Mean	N	Std. Deviation	Std. Error Mean	t	Sig. (2-tailed)
LA	WP5	0.85	20	1.31	0.29	-1.927	0.069
	RP5	1.40	20	1.31	0.29		
AA	WP5	2.24	34	1.07	0.18	-0.894	0.378
	RP5	2.44	34	1.11	0.19		
HA	WP5	3.00	29	0.00	0.00	1.000	0.326
	RP5	2.97	29	0.19	0.003		

Figures 13 and 14 show the solutions of one such pupil. Figure 14 shows the improvement made by an LA pupil under this construct.

Figure 13. Sample solution of WP5

Figure 14. Sample solution of RP5

In Figure 13, the pupil used the first given (cost) to operate with the second given (area), and then the third (width) to find the unit cost of fencing. The use of the data sequentially showed a lack of understanding of the problem context. However, for RP5 in Figure 14, where the givens were repositioned such that cost was featured last, the pupil worked on the data sequentially showing appropriate understanding of the concepts involved. The rewording could have aided the pupils' thought processes through the construction of a clearer mental representation of the context. From the analysis, most of the solution scripts pertaining to this "Area and Perimeter" word problem suggest that the LA pupils who could not solve WP5 were affected more by the mechanics of finding the perimeter or area or not able to associate the idea of fencing to perimeter and thus the cost. Repositioning the givens in RP5 could have helped "realign" their thinking towards solving such a formula-based word problem.

Conclusion

Using four types of rewording semantic structures for five word problems in this study implies that the effect on performance is based almost entirely on how the pupils perform in one particular word problem. It might have been more helpful if more word problems pertaining to each type of semantic structures were used. Moreover, the five word problems selected necessarily could not be representative of the many types of word problems in the problem-solving context.

Further, in rewording the original word problems according to the selected semantic structures, the intention of making the reworded problems parallel to the original was considered but perhaps variables other than the structure under study could have affected pupils' performance as well.

Nevertheless, the findings reveal that language proficiency level differences of the pupils (GP and MP) did not have a significant effect in their performance in solving mathematical word problems. This study does support however the positive effect rewording has on pupils' performance in solving word problems, particularly when the rewording is done based on certain semantic structures. Of value are the constructs "chronological order of events" and "repositioning the givens (algebraic type)" where both serve to make the context more explicit. Quite obvious from the findings is that these two types of rewording have aided the pupils to develop a richer and more elaborate construction of their mental representation of the problem and thus achieve success in their problem solving. This finding also suggests that the pupils' initial failure to solve word problems was not due to their lack of arithmetic ability but to their inability to construct an appropriate problem representation due to the way the problem was structured. In this respect, rewording can help overcome some of the difficulties that pupils experience in learning to solve word problems. Teachers therefore can help lower- and average-ability pupils by sequencing problems progressively and aid the pupils' thinking process by rearranging the order of events chronologically or repositioning the givens to provide a clearer mental representation.

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Appendix A

Original and reworded word problems and associated rationales

Type	WP1 (Whole Numbers)	RP1 (Whole Numbers)
1	After giving 10 clips to Peter, Norman had twice as many clips as Peter. If Norman had 50 clips at first, how many clips did they have altogether?	Mei Lin had 80 clips at first. She gave 20 clips to Ali and she found that she then had twice the number of clips as Ali. How may clips did they have altogether?
	Rationale: The order of data presentation in the WP 1 has been reworded hi RP1 to give it a logical real time sequence for explicitness.	

Type	WP2 (Decimals)	RP2 (Decimals)
2	Two similar pens cost as much as three similar books. If 4 such pens and 3 such books cost $147.60, find the cost of a pen.	1 can exchange 3 Pokemon cards for 2 Digimon cards. Each type of card carries some points. I have 6 Pokemon and 2 Digimon cards which carry a total of 151.2 points. How many points does a Digimon card carry?
	Rationale: The mathematical term "cost as much as" in WP2 is replaced in RP2 with "can exchange", and the pronoun "I" is introduced together with familiar cartoon characters to make the reworded problem more personal.	

Type	WP3 (Fractions)	RP3 (Fractions)
3	When Wenhao spent $\frac{1}{4}$ of his money and Amy spent $\frac{1}{3}$ of hers, each of them had $120 left. What was the original sum of money each person had?	After spending $\frac{1}{5}$ of his money, Bala had $160 left. As for Mary, after she had spent $\frac{1}{3}$ of her money, she also had $160 left. How much did each of them have at first?
	Rationale: Breaking up a sentence into two for the reworded problem for easier reading comprehension.	

Type	WP4 (Fractions)	RP4 (Fractions)
4a	A farmer plucked 90 oranges. He gave 15 oranges to his neighbour and he threw some rotten ones away. If he had $\frac{3}{2}$ of his oranges left, how many oranges were rotten?	Mrs Tan had 60 sweets. She gave some sweets to her two children and she had $\frac{2}{5}$ of her sweets left. If one of her children James, got 30 sweets, how many sweets did her daughter Judy get?
	Rationale: Switching the order of data presentation between the subset and the fraction in RP4 sets up a strategy to work with the fraction first in the reworded problem.	

Type	WPS (Area & Perimeter)	RP5 (Area & Perimeter)
4b	It costs $420 to fence a rectangular plot of land which has an area of 56 m^2. If the width of the plot of land is 7m, what is the cost of fencing per metre?	A rectangular garden has an area of 96 m^2 It's width is 8 m long. If it cost $440 to fence up the garden, what is the cost of fencing per metre?
	Rationale: Switching the order of data presentation allows for givens to be used sequentially in RP5.	