The Mathematics Educator
2006, Vol. 9, No. 2, 48-59

Computer-Based Learning Using *LiveMath* for Secondary Four Students

Joseph Yeo Boon Wooi
Nanyang Technological University, National Institute of Education, Singapore

Abstract: The effect of computer-based learning using *LiveMath*, an interactive computer algebra system, on Singapore Secondary Four students' conceptual and procedural knowledge of exponential and logarithmic curves, was investigated in this study. Sixty-five students from two middle-ability Express classes in an independent school were taught using a guided discovery approach to explore the characteristics of the graphs of exponential and logarithmic functions. The experimental class used *LiveMath* for their exploration whereas the control class used worksheets that contained pre-printed graphs. The findings show that the students in the experimental group performed significantly better than the students in the control group in both the conceptual and the procedural knowledge tests.

Introduction

With the advance of computer technology in the 1960s, there was great interest among mathematics educators in harnessing its potentials for teaching their students (Mariotti, 2002; Nickerson, 1988). The most frequently used mode since the 1970s (Manoucherhri, 1999) is the *tutor* mode where students learn from computers (Taylor, 1980). This usually involves using computer-assisted instruction (CAI) to tutor and drill the students in procedural skills (Jensen & Williams, 1993). But for the 1980s, some academics started to advocate the use of technology in other ways (Taylor, 1980), such as the *tool* mode where the students use the computer as a tool to explore mathematical concepts (Yeo, 2002), or the *tutee* mode where the students teach or program the computer to do mathematics in a microworld environment (Papert, 1980). These approaches were based on the belief that the computer has the ability to make abstract mathematical concepts more concrete for the students (Turkle & Papert, 1990).

In Singapore, the Information Technology (IT) Masterplan in 1997 set the stage for computer revolution in the classrooms (Ministry of Education, 1997). A considerable amount of money was spent on equipping schools with computers, on developing suitable courseware for teaching, and on training teachers how to use the software. But research has shown that many teachers were not entirely convinced of the relevance and benefits of the use of IT in education (Ang, 1999; Cheong, 2001; Ong-Chee, 2000). This uncertainty may be due to an ineffectual use of technology to support the existing curriculum (Shanmugaratnam, 2002), such as showing work on PowerPoint (Yeo, 2004) or putting lecture notes on the World Wide Web which

Roberts and Jones (2000) called the naïve model of online learning. The teachers' doubt of the use of IT in education may also be due to their pedagogical beliefs (Koh, Koh, & Wu, 2004), for example, if a mathematics teacher believes that direct teaching and then drill and practice is the best way to produce results, then there is no need for students to use IT to explore mathematical concepts (Norton & McRobbie, 2000). But the Ministry of Education is still putting in a considerable amount of money for its IT Masterplan 2 (mp2) which was launched in 2002 (Ministry of Education, 2002).

Therefore there is a need to examine whether or not the use of IT really enhances students' learning. Although there may be widespread belief that IT use does enhance students' learning (Barton, 2000; Ho, 1997; Jensen & Williams, 1993; Leong, 2001; Ong, 2002), Oppenheimer (1997) has opened a "can of worms" by claiming that many of these studies are flawed. Therefore, in this paper, I will consider some of these research studies as they relate to IT in secondary mathematics education and attempt to address Oppenheimer's concerns. I will then present a study that I conducted on the use of a computer algebra system in teaching the topic of exponential and logarithmic curves, after taking into account Oppenheimer's comments.

Research Studies on the Use of IT

Singapore-based research studies on the use of IT in mathematics education usually focus on the use of graphing or geometry software to explore graphs or geometry (e.g., Leong, 2001; Ong, 2002). Yet very few of these studies explore the use of a computer algebra system (CAS) (Yeo, 2004). However there are many research studies conducted overseas that focus on the use of CAS or algebraic calculators. But the students in most of these studies used CAS or algebraic calculators as symbolic manipulators to perform algebraic manipulations such as factorisation and differentiation, not as cognitive tools to explore algebraic concepts (Yeo, 2004).

The findings of most of these overseas studies, for example, Barton's (2000) meta-analysis of more than 60 studies, indicated that students in the experimental group usually performed significantly better in their conceptual knowledge test than students in the control group but there was no significant difference in their procedure knowledge test. The Singapore-based studies usually focus on procedural knowledge only and that students in the experimental group usually perform better in their procedural knowledge test than students in the control group (e.g., Ho, 1997; Ong, 2002).

However Oppenheimer (1997) questioned the validity of these studies because they did not control for other critical influences such as differences in pedagogies between the experimental and the control groups. In fact, Barton (2000) defined the control group for the 60 odd studies in her meta-analysis as “the group that was typically taught in a traditional manner” (p. 3). In Singapore, most research studies also compared students who used IT to explore mathematical concepts to students taught using the traditional teacher-directed method (Yeo, 2003). Therefore the better performance of the experimental group may be due to the difference in the pedagogies rather than the use of IT (Oppenheimer, 1997). If this is the case, then why should so much money be spent on computer hardware and software? Teachers might just as well employ a guided-discovery method for students to explore mathematics without the use of IT and anticipate a similar effect!

Therefore a research study was conducted whereby the experimental and the control classes were both taught using the same guided-discovery approach to explore the characteristics of the graphs of exponential and logarithmic functions so that any difference in performance will not be the result of a difference in pedagogies. The difference between the two classes was that the experimental class used a CAS called *LiveMath* for their exploration whereas the control class used worksheets that contained pre-printed graphs.

Research Design

The research study was conducted using a quasi-experimental pretest-posttest control group design (Frankfort-Nachmias & Nachmias, 1996). The research questions for the study were:

(1) What is the effect of computer-based learning on the students’ conceptual understanding of exponential and logarithmic curves?

(2) What is the effect of computer-based learning on the students’ procedural knowledge of exponential and logarithmic curves?

To address these research questions, two null hypotheses were formulated:

(1) There is no significant difference in the mean scores of the Conceptual Knowledge Test (CT) between pupils in the experimental group and the control group after the experiment.

(2) There is no significant difference in the mean scores of the Procedural Knowledge Test (PT) between pupils in the experimental group and the control group after the experiment.

Sample

The sample consisted of 65 Secondary Four students from two middle-ability Express classes in an independent boys’ school in Singapore. Two t-tests were run to show that the two intact classes selected had no significant difference in their academic achievement in mathematics before the experiment (based on the students' Secondary Three Elementary Mathematics and Additional Mathematics Final Examinations results). The classes were then randomly assigned the experimental and the control groups.

Intervention

I taught both classes each for a total of ten 40-minute periods. The experimental class used a CAS called *LiveMath* to explore exponential and logarithmic curves for five 40-minute periods. The remaining lessons were used for teaching the students how to sketch the curves properly and how to solve simultaneous equations involving exponential or logarithmic equations by plotting the graphs.

Like all CAS, *LiveMath* can perform algebraic manipulations. But unlike them, *LiveMath* is interactive: whenever the students change the value of a variable, everything that is linked to the variable will also change automatically. For example, Figure 1 shows a pre-designed *LiveMath* template used by the students in this study to explore the effect of the constant c on the graph of the exponential function of the form $y = e^x + c$. The students just need to change the value of *c*, and the graph and its equation will change instantaneously. This interactive feature of *LiveMath* allows the students to observe the effect immediately, without having to retype the equation and to replot the corresponding graph.

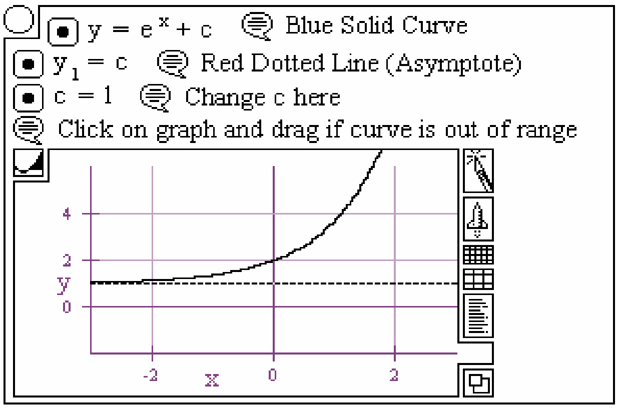

Figure 1. Example of an Interactive *LiveMath* Template

Figure 2 shows another pre-designed *LiveMath* template that allows the students to animate the graph of the exponential function of the form $y = a^x$ where a

increases from 0.5 to 4 in steps of 0.5. Such animation helps the students to visualise the effect of a on the graph of the exponential function.

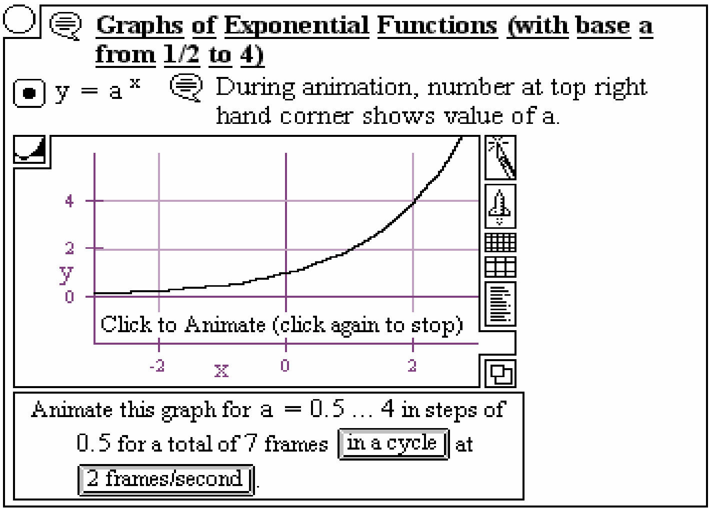

Figure 2. LiveMath Template showing Animation

The students in the experimental class did not need additional lessons to learn the software because the templates were pre-designed in the IT Workbook *Maths Online* (Yeo, 2001). All the students needed to do was follow the instructions in the worksheets, change the values of the variables and observe the effects immediately. Kaput (1992) noted that *LiveMath* (previously known as *Theorist*) was “intriguing because of a unique user interface that allows one to perform ‘natural’ algebraic maneuvers even more ‘naturally’ than one can achieve them on paper” (p. 534).

The students in the control class did not have access to IT. Instead all the graphs that they needed were printed on the worksheets and the students were guided to discover for themselves certain properties of the curves. This is in sharp contrast to most research where the control class was taught using traditional teacher-directed teaching. Similar to the experimental class, the exploratory phase lasted for five 40-minute periods and the remaining five 40-minute periods were used for teaching the students how to sketch the curves properly and how to solve simultaneous equations involving exponential or logarithmic equations by plotting the graphs.

Test Instruments

Two test instruments were designed for the study: the Procedural Knowledge (PK) Test and the Conceptual Knowledge (CK) Test. Three experienced teachers were asked to validate the test instruments; each believed that the CK and the PK Tests were trying to measure students' conceptual and procedural knowledge respectively. The tests were then piloted with a previous batch of Secondary Four students and some questions were modified based on students' verbal feedback during the tests and on their answers. For example, if too many students had problems understanding a particular question, that question was modified to make it clearer. Or, if too many students gave an unexpected answer due to a misinterpretation of the question, that question was also modified.

The purpose of the PK Test was to find out how much students have learnt about certain procedures such as curve sketching, types of transformations from one curve to another, and graphical solutions.

The purpose of the CK Test was to find out how much students have understood about the nature of the curves and their asymptotes using real-life examples. Since it was not easy to design questions that test solely students' conceptual understanding and that such questions were also usually too difficult for the students, the questions were typically structured with many parts to provide guidance to the students. These parts inevitably involved some procedural skills but the aim of the overall question was to test conceptual understanding. For example, the graph in Figure 3 (not drawn to scale) shows the cooling curve of a cup of hot coffee where its temperature θ °C, when it had been cooling for t minutes, was given by the equation $\theta = 30 + 50e^{-kt}$.

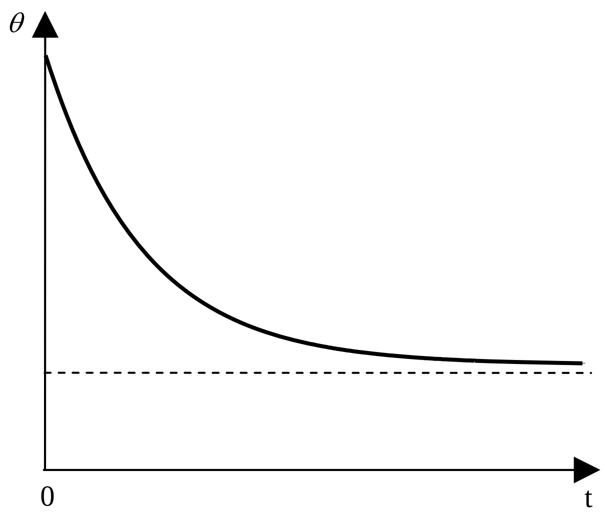

Figure 3. Cooling Curve of Hot Coffee

The students were asked a series of part-questions such as what were the initial temperature of the coffee and the temperature of the room. The students were then asked why the coffee would eventually cool down to room temperature in real life although the nature of the asymptote suggested that the coffee would never reach room temperature. This was not an easy question to answer and so the students were asked to calculate the temperature of the coffee when $t = 1$ hour and hence guided to observe that the difference between the temperature of the coffee and the room temperature was negligible after one hour. Another reason was because the room temperature fluctuated in real life. Although some part-questions inevitably involved calculations which were procedural skills, the entire question attempted to test the students' understanding of the relationship between the exponential equation and its graph, especially the nature of the asymptote.

The reliability of both the CK and the PK Tests (Field, 2000) is shown in Table 1. Since all the values of the Cronbach's alpha were 0.5 and above, the two tests passed the reliability test for achievement tests (Nunnally & Bernstein, 1994).

Table 1
Cronbach's Alpha for Conceptual and Procedural Knowledge Tests

	CK Test	*PK Test*	*Both Tests Combined*
Cronbach's Alpha	0.739	0.501	0.684

Table 2 below shows the mean scores of the CK Test, the PK Test and the overall results for the two classes which took them during the pilot test and for the two classes which took them during the main study. These scores measure the facility level of both tests. Since the mean scores were about 74%, both the CK and the PK tests were of average difficulty.

Table 2
Mean Scores of Conceptual and Procedural Knowledge Tests

	CK Test /25	PK Test /25	Both Tests Combined /50
Mean Score	18.2	18.8	37.0

Data Analysis
Appropriate statistical tests were run to test the two null hypotheses for this research. The Shapiro-Wilk tested whether or not all the different sets of data were

normally distributed (Field, 2000). Some of the CK and the PK Tests results for either one of the two classes were found not to be normally distributed and so parametric tests, such as the t-tests, could not be used to analyse the data. Instead a non-parametric test, the Mann-Whitney test, was used to test whether there was any significant difference in the results so as to accept or reject the other two null hypotheses.

However some researchers believe that a null hypothesis should not be rejected based only on a *statistically* significant test (Burns, 2000). So the effect size was also calculated to see whether it was *practically* significant to reject the null hypothesis. The effect size is "the degree to which the phenomenon is present in the population" (Burns, 2000, p. 167). It is used to determine the strength of the relationship between the independent and the dependent variables and this is independent of sample size. Meta-analysis of findings of quantitative research studies is based on this concept of effect size.

Findings

Significant differences in the scores of both the CK and the PK Tests between students in the experimental and the control classes were found. Tables 3 and 4 summarise the Mann-Whitney Tests for the CK and the PK Tests respectively.

Table 3
Mann-Whitney Test on Conceptual Knowledge Test

Group	n	Mean	Std Dev	Z-value	p-value	Effect Size
Experimental	32	20.7	2.95	–4.238	< 0.001	1.28
Control	33	15.9	4.42			

Table 4
Mann-Whitney Test on Procedural Knowledge Test

Group	n	Mean	Std Dev	Z-value	p-value	Effect Size
Experimental	32	20.2	3.77	–2.532	0.011	0.705
Control	33	17.4	4.23			

The effect size of 0.705 for the PK Test suggests that the experimental approach had a large effect on students' procedural knowledge test performance. But the effect size of 1.28 for the CK Test suggests that the effect was even bigger for the students' conceptual knowledge test performance. These findings are consistent with other Singapore-based research findings that suggest that the use of IT enhances students' procedural knowledge, and are also consistent with overseas research findings that the its use enhances students' conceptual knowledge.

Conclusion

Although many research studies conclude that the use of IT to explore mathematics is more efficient than traditional teacher-directed teaching, Oppenheimer (1997) believed that the differences in results were most likely due to a difference in pedagogies rather than the use of IT. However, this study seems to suggest that there is something inherent in computers that make it easier for students to explore mathematics. In other words, the use of IT to explore mathematics may be a better pedagogical approach than exploring mathematics without access to computers. It may thus be worthwhile to continue funding IT initiatives such as the Singapore-IT Masterplan 2 so that more teachers may be encouraged to make full use of computers "by taking into account new teaching methods that are made possible by technology" (Shanmugaratnam, 2002, p. 5). In fact, these 'new' teaching methods, such as mathematical exploration and guided-discovery approach, are not new (Yeo, 2004) but computer technology has made it easier for educators to use these pedagogies to enhance their students' learning because "the computer stands betwixt and between the world of formal systems and physical things; it has the ability to make the abstract concrete" (Turkle & Papert, 1990, p. 346).

However more research needs to be done to substantiate these findings. A small research study like this has its limitations; for example, a small sample size, the use of only one type of software, and restriction to only the topic of exponential and logarithmic curves. Therefore a direction for future research in this area is a replication of this study with a bigger sample size, the use of other interactive types of software or the use of the same software to explore other topics. However it is essential that both the experimental and the control groups must still use the *same* pedagogy so that any difference in results will not be due to a difference in pedagogies but due more likely to the use of IT.

References

Ang, A. K. H. (1999). *An investigation into the psychological barriers related to teachers' use of computer technology in the school.* Unpublished master's thesis, National Institute of Education, Nanyang Technological University, Singapore.

Barton, S. (2000). What does the research say about achievement of students who use calculator technologies and those who do not? In P. Bogacki & E. D. Fife (Eds.), *Electronic Proceedings of the Thirteenth Annual International Conference on Technology in Collegiate Mathematics.* Atlanta: Addison Wesley.

Burns, R. B. (2000). *Introduction to research methods* (4th ed.). NSW, Australia: Longman.

Cheong, H. Y. (2001). *Staff development in using information technology for teaching: The management perspective.* Unpublished master's thesis, National Institute of Education, Nanyang Technological University, Singapore.

Field, A. (2000). *Discovering statistics using SPSS for Windows.* London: SAGE Publications.

Frankfort-Nachmias, C., & Nachmias, D. (1996). *Research methods in the social sciences* (5th ed.). New York: St. Martin's Press.

Ho, C. S. Y. (1997). *A study of the effects of computer assisted instruction on the teaching and learning of transformation geometry.* Unpublished master's thesis, National Institute of Education, Nanyang Technological University, Singapore.

Jensen, R. J., & Williams, B. S. (1993). Technology: Implications for middle grades mathematics. In D. T. Owens (Ed.), *Research ideas for the classroom: Middle grades mathematics* (pp. 225-243). New York: National Council of Teachers of Mathematics & Macmillan.

Kaput, J. J. (1992). Technology and mathematics education. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 515-556). New York: National Council of Teachers of Mathematics & MacMillan.

Koh, T. S., Koh, I. Y. C., & Wu, W. T. (2004). Integration of information technology in Singapore school mathematics curriculum. In *Proceedings of 9th Asian Technology Conference in Mathematics: Technology in Mathematics: Engaging Learners, Empowering Teachers, Enabling Research* (pp. 17-26). Singapore: ATCM.

Leong, Y. H. (2001). *Effects of Geometer's Sketchpad on spatial ability and achievement in transformation geometry among secondary 2 students in Singapore.* Unpublished master's thesis, National Institute of Education, Nanyang Technological University, Singapore.

Manoucherhri, A. (1999). Computers and school mathematics reform: Implications for mathematics teacher education. *Journal of Computers in Mathematics and Science Teaching, 18*(1), 31-48.

Mariotti, M. A. (2002). The influence of technological advances on students' mathematics learning. In L. D. English (Ed.), *Handbook of international research in mathematics education* (pp. 695-723). Mahwah, NJ: Erlbaum.

Ministry of Education, Singapore. (1997). *Masterplan for IT in education.* Retrieved June 16, 2005 http://www.moe.gov.sg/edumall/mpite

Ministry of Education, Singapore. (2002). *Overview of the Masterplan II for IT in education.* Retrieved June 16, 2005 http://www.moe.gov.sg/edumall/mp2/mp2.htm

Nickerson, R. S. (1988). Technology in education in 2020: Thinking about the non-distant future. In R. S. Nickerson & P. P. Zodhiates (Eds.), *Technology in education: Looking toward 2020* (pp. 1-9). Hillsdale, NJ: Erlbaum.

Norton, S., & McRobbie, C. (2000). A secondary mathematics teacher explains his non-use of computers in teaching. In J. Bana & A. Chapman (Eds.), *Mathematics education beyond 2000* (pp. 481-486). Turramurra: Mathematics Education Research Group of Australasia.

Nunnally, J. C., & Bernstein, I. H. (1994). *Psychometric theory* (3rd ed.). New York: McGraw-Hill.

Ong, M. F. (2002). *Effects of computer-assisted instruction on the learning of angle properties of circles among upper secondary students.* Unpublished master's thesis, National Institute of Education, Nanyang Technological University, Singapore.

Ong-Chee, S. Y. H. (2000). *An inquiry into the impact of IT in education on the teaching and learning in a Singapore school.* Unpublished master's thesis, National Institute of Education, Nanyang Technological University, Singapore.

Oppenheimer, T. (1997). The computer delusion. *The Atlantic Monthly, 280*(1), 45-62.

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas.* New York: Basic Books.

Roberts, T. S., & Jones, D. T. (2000, April). *Four models of online teaching*. Paper presented at the Technological Education and National Development Conference, "Crossroads of the New Millennium", Abu Dhabi, UAE. (ERIC Document Reproduction Service No. ED446280)

Shanmugaratnam, T. (2002, July). *IT in learning: Preparing for a different future.* Speech at IT Conference: IT Opportunities, Innovations and Achievements in Education, Singapore. Retrieved June 16, 2005 http://www.moe.gov.sg/speeches/2002/sp24072002.htm

Taylor, R. (Ed.). (1980). *The computer in the school: Tutor, tool, tutee.* New York: Teachers College Press.

Turkle, S., & Papert, S. (1990). Epistemological pluralism: Styles and voices within the computer culture. *Signs: Journal of Women in Culture and Society, 16*(1), 345-377.

Yeo, J. B. W. (2001). *Maths Online: For additional mathematics.* Singapore: Wellington Publisher Services.

Yeo, J. B. W. (2002, May). *Moving towards interactive exploratory mathematics using computer technology.* Paper presented at the Teachers Network Conference, "The Classroom of the 21st Century – A Paradigm Shift", Singapore.

Yeo, J. B. W. (2003). *The effect of exploratory computer-based instruction on secondary four students' learning of exponential and logarithmic curves.* Unpublished master's thesis, National Institute of Education, Nanyang Technological University, Singapore.

Yeo, J. B. W. (2004). Using *LiveMath* as an interactive computer tool for exploring algebra and calculus. In *Proceedings of 9th Asian Technology Conference in Mathematics: Technology in Mathematics: Engaging Learners, Empowering Teachers, Enabling Research* (pp. 457-464). Singapore: ATCM.

Author:
Joseph Yeo Boon Wooi, Lecturer, Nanyang Technological University, National Institute of Education, Singapore
bwjyeo@nie.edu.sg