Pre-Service Teachers Engaging in Mathematical Investigation

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Abstract: Personal experience with secondary school mathematics teachers and students in Singapore suggests that they do not know how and what to investigate when given an open investigative task. This agrees with what many literature writers claimed but most of their writings were based on anecdotal evidence. There seems to be a lack of research studies to support the assertion that many teachers and students do not know how to deal with open mathematical investigation. This paper presents the first part of a big research study on the processes of mathematical investigation. It investigates whether pre-service mathematics teachers know what to do with an open investigative task. The study reveals that all of these pre-service teachers had no previous experience in open mathematical investigation when they were students themselves, and that most of them do not know how and what to investigate when given an open investigative task. Since the pre-service mathematics teacher training curriculum in Singapore does not explicitly teach the pre-service teachers how to deal with this type of open investigation, it is unlikely that they will teach their students to do so.

1. INTRODUCTION

The use of open mathematical investigative tasks in the teaching and learning of mathematics is very important in many school mathematics curricula. For example, the Cockcroft Report in the United Kingdom states that “mathematics teaching at all levels should include opportunities for … investigational work” (Cockcroft, 1982, p. 71) and the Australian national curriculum stipulates that “mathematical investigations can help students to develop mathematical concepts and can also provide them with experience of some of the processes through which mathematical ideas are generated and tested” (Australian Education Council, 1991, p. 14). But why do so many school mathematics curricula place such great emphasis on mathematical investigation?

Hawera (2006) claimed that the use of open investigative tasks helps students to focus on "the process of problem solving and the open-endedness of a problem or investigation" (p. 286) because many genuine problems are ill-structured and open in nature. In real life, no one will tell you what the problem is and what the boundaries of the problem are (Simon, 1973). You will have to find the root problem first before you can resolve the issue. Similarly, in mathematical investigation, the task will not tell the students what the problems are. The students will have to think through themselves and pose their own problems to investigate or solve. This helps the students to be more aware of the problem situation and to take charge of the issue at hand. Many educators (e.g., Brenner & Moschkovich, 2002) also favour bringing academic mathematicians' practices into the classroom and this includes letting students engage in a variety of rich mathematical activities which parallel what mathematicians do. And what do academic mathematicians do? They investigate and solve mathematical problems (Civil, 2002). Lampert (1990) believed that such activities encourage students to think mathematically, such as problem posing (Brown & Walter, 2005), conjecturing and generalising (Calder, Brown, Hanley & Darby, 2006).

Although there are many different interpretations of what constitutes a mathematical investigation (e.g. Orton & Frobisher, 1996; Pirie, 1987), this paper will deal only with research using *open* investigative tasks. I will begin with the background of the study, followed by the research methodology and findings, and it will end with a discussion of the data collected and some implications for both research and teacher education.

2. BACKGROUND

The central goal of the Singapore mathematics curriculum is mathematical problem solving (Ministry of Education of Singapore, 1990) and most mathematics teachers in Singapore are familiar with solving mathematical problems. But whenever I mention the term 'mathematical investigation', quite a number of teachers will look at me blankly and ask, "What's that?" Some teachers have this vague idea that mathematical investigation has something to do with guided-discovery learning, but according to Ernest (1991), there are some major differences. Very few teachers actually know what open investigative tasks are, and when faced with such a task, most of them do not know what to do. But this is based on personal communication with pre-service teachers whom I taught and teachers who attended my in-service courses, and there are very few research studies which investigate the effect of giving open investigative tasks to teachers or students who had no prior experience with such investigation.

As part of a bigger research which investigates the nature and development of thinking processes when students with a wide range of mathematical abilities attempt open investigative tasks, there is a need to design a paper-and-pencil test instrument for the bigger study to find out how and what students investigate. But if most of the students do not know how and what to investigate, then not much data will be gathered from such a large-scale study and so it will not be fruitful to conduct such a study in the first place. Therefore, there is a need to do a pilot study to find out how much students can do with open investigative tasks, and if they really do not know what to do, to find out what else can be done to design the test instrument, for example, providing guided investigative tasks at the beginning? However, before conducting a pilot study for willing students who were hard to come by, there availed itself a convenient sample of pre-service teachers which could be utilised to find out how much they know and to finetune the test instrument for the pilot study for students. Thus, this paper describes the first of a series of research studies: do pre-service teachers know how and what to investigate when given an open investigative task? The findings will be used to inform further research.

3. METHODOLOGY

The sample consisted of 21 pre-service (or trainee) secondary school mathematics teachers from an intact class in the National Institute of Education (NIE) in Singapore. They were currently studying for their Postgraduate Diploma in Education (Secondary) or PGDE(Sec). This was a one-year course where the pre-service teachers were required to take two curriculum subjects which they would teach in schools. The trainee teachers in the sample all took the teaching of mathematics as their first curriculum subject and most of them had a university degree in mathematics. Therefore, these pre-service teachers were well grounded in mathematics content.

The paper-and-pencil test instrument consisted of two related tasks given to the trainee teachers separately during a lesson. The first task was as followed:

Task 1

Natural numbers are positive whole numbers: 1, 2, 3, 4, …

Polite numbers are natural numbers that can be expressed as the sum of consecutive natural numbers. For example,

$$\begin{aligned} 9 &= 2 + 3 + 4 \\ 11 &= 5 + 6 \\ 18 &= 3 + 4 + 5 + 6. \end{aligned}$$

Investigate.

Questions: Write down your honest thoughts and feelings when you first see this question. Have you seen this question before? Do you know what to investigate? If you don't know what to investigate, what do you do? If you know what to investigate, start investigating now.

Note: It is ok if you don't know what to do. You will ***not*** get a poorer grade just because you write down you don't know what to do. Just be honest.

The question paper cum answer script was divided into two columns of equal width after the given task: the first column was for working and the second column was for the pre-service teachers to record what they were thinking when they were doing the working. There were questions in the above task to help the trainee teachers write their thought processes if they did not know what to do. The aim of the above note was an attempt to get an honest answer from the pre-service teachers.

The primary purpose of the first task was to find out whether the trainee teachers knew what to do when given an open investigative task with only the word 'Investigate' and no guided questions or problems to investigate or solve. The secondary purpose was to understand their thought processes as they went about to investigate (if they knew how) or what they were thinking when they were stuck and did not know what to do.

After the pre-service teachers had worked on the first task for 15 minutes, their answer scripts were collected back and they were given the second task:

Task 2

Natural numbers are positive whole numbers: 1, 2, 3, 4, …

Polite numbers are natural numbers that can be expressed as the sum of consecutive natural numbers. For example,

$$9 = 2 + 3 + 4$$
$$11 = 5 + 6$$
$$18 = 3 + 4 + 5 + 6.$$

Which numbers are polite?

Questions: Do you know how to start if the task is phrased this way? How do you go about finding the answer? Write down what you are **thinking** when you try to solve this problem. We are interested to find out how you try to solve the problem and why you try to solve it in this way. For example, you can write, "I suddenly think of trying out some examples to see if I can see any pattern…" and then you show your working to try out.

Note: We are also interested in your **draft working** as well. It is ok if you write down some working to solve the problem but then you get stuck halfway. You will ***not*** get a poorer grade just because you write this down. The question will then be, "What are you doing to do next?"

The question paper cum answer script for Task 2 was also divided into two columns of equal width for the same purpose as the question paper cum answer script for Task 1.

The difference between the two related tasks was that the second task contained a specific problem for the trainee teachers to investigate or solve. The primary purpose of the second task was to find out whether the pre-service teachers know what to do if the investigative task was not so open but contained a specific problem for them to investigative or solve. The secondary purpose was the same as that for the first task: to understand their thinking processes as they went about their investigation or when they were stuck. Another difference was that the duration for attempting the second task was 30 minutes, as compared to only 15 minutes for the first task.

4. FINDINGS

After the pre-service teachers had attempted Task 1 for the first five minutes, I observed that many of them were stuck. This was further confirmed by what they wrote about their thinking processes. Some of their comments (all names used in this paper were pseudonyms but maintaining the same gender) were:

Hector:	I have not seen this question before. I have no idea what to investigate.
Kingston:	The question is vague and didn't tell the students what to investigate.
Nora:	Never seen the question before. Not sure on the scope to investigate.
Qamar:	Seriously, I have not seen this question before and do not know what to investigate.
Uma:	I have not seen this question before and I have no idea what to investigate.

But after five minutes, many of them appeared to be investigating! So I thought they finally knew what it meant to investigate this type of open investigative tasks. However, after doing some work for about 10 minutes, most of them did not know what else to do after that. Therefore, 15 minutes after the start of the task, I asked the whole class whether they knew what and how to investigate. To my surprise, most of them said that they still did not know what to do. Then I asked them what they were doing for the past 10 minutes. Some of them mentioned that they just anyhow tried since they were expected to do something. This was best summarized by what Vanessa wrote in her paper:

Vanessa:	My first impression of the question is that it contains a lot of ambiguity. As there are no guided questions to help scaffold the problem, I do not really know how to start. *Since there is no choice but to try* [emphasis mine], my idea of investigating will *perhaps* [emphasis mine] to find out if there is a general formula to represent polite numbers and whether there is an exception to the formula for certain integers *n*.

Hence, for this group of trainee teachers who did not know what to do with this kind of open investigative tasks, they still tried to do something because they were responsible adults. But what about students who do not know how and what to investigate? Will they try to do something or will they give up? Perhaps high-ability students may still attempt the task but what about average and low-ability students, as the larger study involves students of all levels of mathematical abilities?

The quality of their investigation also suggests that the pre-service teachers had not done such investigation before. For Task 1, many of the trainee teachers did either one or more of the following: (a) tried to find a general formula for polite numbers but failed, (b) tried to find out which numbers were polite but failed, (c) listed out some examples and then did not know what to do with them, and (d) tried to find some patterns but failed to observe any non-trivial patterns. Instead, they made trivial conjectures or conclusions, such as, polite numbers can be odd or even, the smallest polite number is 3, and there is no largest polite number.

After the trainee teachers had attempted Task 1 for 15 minutes, their answer scripts were collected back and they were given Task 2 which contained a specific problem, "Which numbers are polite?" Many of the pre-service teachers commented that Task 2 was a lot clearer because they knew what to investigate.

Betty:	Given the question phrased this way, it will be easier to investigate.
Tiffany:	If the task is phrased this way, I'm able to start by looking at what kind of numbers can be written as a sum of consecutive natural numbers.
Uma:	Given the specific question asked in the task, I am clearer about what I'm supposed to do now as compared to in the first task.

But did they know how to investigate? Many of them made a systematic list of natural numbers in ascending order, starting with the smallest natural number, and then tried to express them as the sum of consecutive natural numbers. However, some of them started with the sum of consecutive natural numbers. There were two ways to do this. For those who started with 1, 1 + 2, 1 + 2 + 3, 1 + 2 + 3 + 4, …, 2, 2 + 3, 2 + 3 + 4, 2 + 3 + 4 + 5, …, etc., they found that it was hard going and the patterns were not obvious. But for those who started with the sum of two consecutive natural numbers (i.e., 1 + 2, 2 + 3, 3 + 4, …, etc.), they would be able to observe that all odd numbers, except 1, were polite numbers. Interestingly, only one pre-service teacher continued with investigating the sum of three consecutive natural numbers and the sum of four consecutive natural numbers. Whichever methods that they had used, based on their list of the first 10 or 20 natural numbers, only four of them managed to conjecture that powers of 2 were not polite numbers. But they were not able to prove it, which was to be expected because the proof was not easy.

Hence, the pre-service teachers were not very good in investigating which numbers were polite. They knew how to start by looking at specific examples but most of them were not flexible enough to look at different ways of examining the specific examples, e.g. by starting from the sum of two consecutive natural numbers instead of starting from the natural numbers in ascending order. Most of them were also unable to observe non-trivial patterns but for the few trainee teachers who managed to observe some patterns, they did not jump to hasty generalisations but they understood that these were conjectures to be proven or refuted although they were not able to do so. Thus, some of the trainee teachers did demonstrate, to some extent, the four processes integral in any investigation: specialisation, conjecturing, justification and generalisation (Mason, Burton & Stacey, 1985).

5. CONCLUSION

This research study suggests that most pre-service mathematics teachers may not know how and what to investigate when given an open investigative task with the word ‘Investigate’ because they have not seen or done this type of tasks before, even when they were secondary school students or tertiary students themselves. But they still tried to do something because they were responsible adults. Some of them were able to pose their own problems. However, if the investigative task was rephrased with a specific problem to make the task less open, then the trainee teachers found that the task had become clearer. This seems to help them to start investigating although the quality of the investigation was not very good. But some of them did exhibit to a certain extent some of the core processes in open investigation: problem posing, specialisation, conjecturing, justification and generalisation.

However, these pre-service teachers had a university degree and were studying in NIE to teach secondary school mathematics, meaning that they were rather strong in their mathematics content. So what are the implications if such open investigative tasks are given to average or low-ability secondary school students who may not have such a good mathematics foundation? Will they be able to start investigating or will they give up because they do not know what are the task requirements? Although anecdotal evidence suggests that most students will not be able to do this type of tasks, further research is needed to investigate this. But what if the students are given guided investigative tasks with specific problems? Will this help them to start investigating? Will they know how to investigate? This study with pre-service teachers suggest that giving guided investigative tasks at the start of the written test instrument may help students to understand the task demands and so they can at least start investigating. The quality of their investigation will then be the subject of the larger study. Therefore, there is a need to modify the test instrument to provide more scaffolding to help the students to at least start investigating. Piloting the test instrument with a group of students will further confirm whether this approach will gather enough data for the bigger research study.

Another implication is that if pre-service teachers do not know how and what to investigate when given an open investigative task, and the pre-service mathematics teacher training curriculum in Singapore does not explicitly teach pre-service teachers how to do so, then they will be ill-equipped to teach their students how to deal with open investigative tasks. Since open mathematical investigation is an important aspect in many mathematical curricula, perhaps it is time to expose our pre-service teachers to open investigative tasks.

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