12th International Congress on Mathematical Education
Topic Study Group 21
8 July – 15 July, 2012, COEX, Seoul, Korea

STUDIES OF HIGH-QUALITY TEACHING PRACTICE IN MATHEMATICS: A COMPARATIVE AND COLLECTIVE ANALYSIS

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This paper presents comparative and collective analyses of research on classroom practice of high-quality mathematics instruction across three studies of high-quality teachers: one from Japan, one from Singapore, and one from the US. Across the three studies, we focus our analyses on the research methods and tools used in each study; the theoretical assumptions that frame each; and the high-quality practices identified by each study.
Key words: Instructional practices; high-quality teaching; teacher learning; international comparisons.

INTRODUCTION

There is no doubt that images of high-quality mathematics instruction are essential for guiding the education of preservice and inservice teachers towards instructional improvement. Yet little consensus exists around what constitutes high-quality mathematics instruction. In this paper, we analyse three studies conducted in three different countries, by researchers who worked isolated one from the other, using different methods and different data sources. One study examines the outcomes of discussions among teacher educators in Singapore who sought to articulate the features of high-quality mathematics instruction. The second study analysed videotaped lessons of three high-quality mathematics teachers in the US. The third study investigated the conversations between Japanese "cooperating" or mentor teachers and student teachers as the student teachers prepared a lesson plan. Although these studies represent divergent data sources, methods, and national contexts, the cross-study analysis yielded some compelling overlaps regarding conceptualization of high-quality instruction. We consider each study in turn, and then look across the studies for patterns and similarities.

IMAGES OF HIGH-QUALITY TEACHING IN SINGAPORE

The Singapore team of teacher educators started the process of painting an image of high quality mathematics instruction by describing certain features of such practices. This was meant to serve as a starting point in a conversation among Singapore mathematics teacher educators towards a set of commonly shared goals for Secondary mathematics methods courses. With these clearly articulated, teacher educators have a common language and framework to discuss about issues relating to high quality teaching; student teachers are presented with a clear set of criteria to serve as a goal that they can work towards in the process of the mathematics methods courses.

Goal(s)-orientedness

It is standard practice in mathematics methods courses that student teachers are expected to make explicit their instructional goals in written plans for their lessons. The basic rationale for such a practice is that one is better able to realise where one is going in a lesson if there is a clear set of goals to work towards. Moreover, it is hardly possible to evaluate the success of a lesson if there is no targeted end result to compare against.

However, the point made here about goal-orientedness is not merely restricted to the act of stating a set of instructional objectives at the beginning of a lesson. It is about teaching—all of it, and not just before a lesson—that is guided by a focus towards the fulfillment of an *achievable* set of *worthy* instructional goals (including those that emerge during teaching), such that all teaching activities are deliberately geared towards the attainment of these goals.

Trajectory of overarching mathematical concept(s)

The language of "mathematical concept" is in part to present a contrasting image against the unambitious teaching of 'small ideas', better known as teaching for facts and procedures without attempts to connect to the mathematical "Big Ideas" underlying these drills. In using the term "overarching", the focus is on the need to pay attention to the central mathematical concept that acts as an organising frame—tying all the pieces within a unit of instructional practice. Thus, "trajectory of overarching mathematical concept(s)" places a demand on student teachers to consider not just a presentation of a sequence of steps in practice but also the deeper mathematical unifying concept undergirding them. The use of "trajectory" is meant to highlight the need not only to identify the overarching concept but also to carefully thread it through the entire practice; and not necessarily at one-go, but maintained or build-up as the instructional unit unfolds.

Also, more than one overarching concept may be involved in a particular unit of instruction. In this case, the student teacher is required to think of ways to weave these concepts together into a coherent trajectory.

Active student engagement

This point rests upon the principle that students' learning in the classroom cannot take place unless they are vitally connected to the mathematical work that goes on in the milieu. While the choice of whether to engage in the work ultimately rests on the student, an important part of the teaching practice is to set up the instructional environment that motivates students to engage productively in the mathematical work. Many research reports in the literature (especially those that are rooted in social-cultural theories) highlight the importance of active interaction among students—such as small group discussions, students' contributions to whole-class talk, and collaborative investigations—as integral to their mathematical learning. (See, for example, Yackel & Cobb, 1996.) "Engagement" here is also broadened to include students' quiet but deliberate *study* of learning materials that are carefully structured by the teacher. Ways to encourage engagements would include questioning and building upon students responses' in public mathematical reasoning during classroom discourses. It may also involve teachers' consistent enforcement of contemplative quiet work and study as a valued disposition for mathematical thinking.

Powerful representations

Teachers do not just engage students verbally; they also use representations—in various media forms such as drawings on the whiteboard or projections of computer screens—to help students focus visually on the discussion track and sometimes even to provide hints for attack routes to mathematical problems. There are many representational forms commonly used in the mathematical domain: number line, tables, Venn diagrams, matrices, graphs, among others. One aspect to the work of teaching is to harness these representations appropriately to advance the mathematical agenda in the classroom.

Representations that perform this function well are those that help the discussants summarise the salient points readily at a glance and thus helps to forward the discussion in productive ways. In this way, the representation can be said to be "powerful" as it projects the key elements of the discussion in one diagrammatic form to the students. Another conception of "powerful" representation is that its use is not limited only to a one-off or narrow domain; rather, the representation can be adapted for use in other wider mathematical domains or as a tool for solving other problems.

Suitable language

That language plays a prominent role in mathematical learning is indisputable. From the social-cultural perspective, language is the medium for reasoning. In the constructivist paradigm, sense making starts from internal constructions of meanings using informal language which becomes more refined over cycles of reconstruction. Regardless of the theoretical stance one takes on learning, it is clear that mathematical learning cannot be separated from the learning of suitable mathematical language. Thus, in considering high quality instruction, there is a need to include the weaving of appropriate language into classroom practice. This point is not merely about using correct mathematical terms in classroom talk; it is also about highlighting the importance of using mathematical language as an explicit learning goal for students. It is about the teacher using suitable mathematical language as well as helping students develop their use of mathematical language.

Good use of time

Often, student teachers are presented with innovative ways of teaching a particular mathematical topic; but because these approaches require significantly more time than is usually allotted in conventional teaching schedules, they are viewed as not realistic as a regular feature of daily classroom teaching practice—but may remain as 'boutique' examples of practice that are seen as exotic. But the conception of high quality practice here is not restricted to those once-in-a-while situations; rather, it is a practice of quality that is sustainable as regular work of teachers in the classroom on a daily basis. As such, this notion of high quality practice must factor in the realistic and ongoing constraints of classroom practice. One major constraint teachers face regularly is the time pressure. Thus, in considering high quality teaching, we are not merely positing images of high quality teaching *per se*. Rather, we are proposing that we conceive of high quality teaching within the constraints of good use of limited time.

For the first time in the Secondary Mathematics methods course in the current intake of student teachers, these features were spelt out in the course documents as well as embedded into the criteria for assessment of their assignments. There are 84 student teachers participating in this course. As the course is ongoing, data and findings will be reported at a later stage. Nevertheless, the four tutors involved in the course (the third author being one of them) met on a number of occasions to discuss how the re-designed course was progressing. There is consensus on the suitability of these features as criteria for assessing the student teachers' assignment submissions.

IMAGES OF HIGH-QUALITY TEACHING IN THE UNITED STATES

This study analysed six videotaped lessons for each of three high-quality middle school mathematics teachers in the US. To qualify as a "high-quality teacher" for this study, teachers were first identified on the basis of their students' achievement scores on standardized tests, their performance on standardized test items that assessed their mathematical knowledge for teaching or MKT (Hill, Schilling & Ball, 2004), and on the quality their observed instruction using an observational coding instrument (Learning Mathematics for Teaching Project, 2011). Six categories of instructional practice were identified that were common across these high-quality teachers: the use of precise mathematical language; the presentation of mathematical representations that follow student thinking; attention to multiple mathematical proficiencies in single lessons; provision of scaffolding for students' development of mathematical ideas; conduct of teacher-directed discussions that build mathematical ideas; and maintenance of routines that structure mathematical productivity.

High-quality teachers use precise mathematical language

Teachers used accurate and precise mathematical terms throughout their instruction; their turns of talk were invariably dense with the use of mathematical terminology. Teachers frequently revoiced students' ideas using precise mathematical terms, and they revisited mathematical definitions often. The high-quality teachers in this set also seem to generate representations of mathematical problems as the need arises in class. While some of these representations come from the curricular materials teachers are using, many are produced as the lesson unfolds. These teachers have a command of representations and can choose between different representational forms with ease and on the spot. Further, there is "meta-talk" about the advantages of a particular representation in a given problem. This attention to the creation of representations and the meta-discussion about them is distinctive to this set of teachers.

High-quality teachers attend to multiple mathematical proficiencies in a lesson

Teachers traverse a range of "mathematical proficiencies" in their lessons. Across the lessons, we see teachers include a range of mathematical proficiencies, even those that are not specified in the curriculum. The model of "strands of mathematical proficiency" (Kilpatrick, Swafford, & Findell, Eds., 2001) served as an analytic framework to survey the various kinds of mathematical work that the teachers in this sample engaged students in. These "strands of mathematical proficiency" were intended to capture "all aspects of expertise, competence, knowledge, and facility in mathematics" (p. 116). The strands are: conceptual understanding; procedural fluency; strategic competence; adaptive reasoning; and productive disposition. The high-quality teachers observed weave in multiple strands of mathematical proficiency within a single lesson, sometimes supplementing the curriculum materials to do so.

High-quality teachers provide multiple scaffolds for students' mathematical thinking

Teachers in this sample had multiple ways to scaffold students' mathematical thinking in the flow of instruction. Five such scaffolding moves are: referencing relevant earlier work and ideas; checking in for student understanding; recalling simpler versions of a problem or idea to leverage higher or more advanced understandings; anticipating student misconceptions; and attending to individual students with concomitant differentiated instruction one-on-one.

High-quality teachers steer discussions to build mathematical ideas

Teachers in this sample direct whole-class mathematical conversations, and this occupies a good deal of time in their lessons. In these discussions of mathematical ideas, the teachers prompt students for contributions, and then work these student offerings towards a mathematical point the teacher is building. The students' contributions are somewhat circumscribed, the pace is brisk, and the teacher's talk dominates these discussions.

Students' answers from the problems or experiments they do in class are starting points for the teacher's heavily guided discussions, in which teachers unpack student ideas and restate them in precise mathematical language, making them clear for collective reasoning. Generally the string of student and teacher turns ultimately builds to a clear mathematical point articulated by the teacher. The kind of discussion pattern seen is common across the teachers in this set, but the degree of directedness from the teacher, and the amount of student contribution, vary with the content being taught. In problems where students have more variation in solution strategies or ways to express reasoning, the teachers will draw students' thinking out more and be less directive. In either case, though, the teachers remain in firm control of the direction and pace of the conversation. There are few instances of student-generated discussions in the data; most conversational exchanges are “student-teacher;” rarely are they “student-student,” to use Chapin, O’Conner, and Anderson’s characterizations of interactions (2003).

High-quality mathematics teachers structure class time with routines

Teachers in this group were highly organized in the way they ran their lessons. There were a number of routines that teachers used to organize class time which enable a certain kind of mathematical press. For example, teachers had overt ways to check on student assignments, daily assignments were recorded on the white board for the week, and the pace of class was clipped enough to keep up with the teacher's plans.

In lessons observed, all the teachers had routines like this, that were explicit, held individual students accountable, and that gave the teachers information that they used later in their lessons.

IMAGES OF HIGH-QUALITY TEACHING IN JAPAN

In this section we summarize the findings of a study of Japanese middle school mathematics teacher’s conceptions of high-quality instruction (Corey et al., 2010). The findings are a set of six foundational instructional principles that emerged from one-on-one conversations between Japanese cooperating teachers (CTs) and student teachers (STs). Student teachers in Japan teach fewer lessons than their US counterparts. This enables the cooperating teachers to spend a lot of time discussing lesson plans and instructional decisions with the student teacher before the lesson is ever taught. In our study the student teachers and the cooperating teachers spent about three one-hour sessions discussing the lesson plan before the lesson was taught and about an hour after it is taught. We used the conversations, 19 pre-lesson conversations and 8 post-lesson conversations, to explore what the cooperating teachers viewed as important in designing and teaching a good lesson.

Intellectual Engagement Principle: High-quality mathematics instruction intellectually engages students with important mathematics.

This principle appears to be the most central feature of a high-quality mathematics lesson. This was a topic in every single one of the 19 pre-lesson conversations between cooperating teachers and student teachers. Not only was it most frequently discussed across conversations but the other five principles are all closely tied to this single central principle of high-quality mathematics instruction. Although US teachers also emphasize engaging students as important, they tend to emphasize physical engagement rather than intellectual engagement (Wilson, Cooney, & Stinson, 2005; Wang & Cai, 2007). Japanese teachers focus explicitly on intellectual engagement. This was illustrated by a CT to an ST during a pre-lesson conversation. When the CT saw that there was little thinking that would be going on in the lesson by the students he asked the ST, “Are there any places that students use their head?” The ST had to reply, “There is no such a place. Nothing at all.” The CT’s question summarizes this first principle well, because if the answer is no, then there is no hope of it being a good lesson.

Goal Principle: High-quality instruction is guided by an explicit and specific set of goals that consist of student motivation, student performance, and student understanding.

Every Japanese lesson plan begins with a set of goals. The goal statements are very important to Japanese teachers. The goal statements help Japanese teachers balance between mathematical skills and conceptual understanding, something that is often dichotomized in the mathematics education literature. The goals also helped the teachers balance two other issues, to make the mathematics interesting and meaningful to the students while maintaining high mathematical standards.

The goals help to guide teachers in developing a lesson. The cooperating teachers continually referred back to the goal of the lesson to see if the activities suggested by the student teacher were aligned with the goals. They even went beyond checking for alignment but they challenged to student teachers to come up with the best activities that they could that would accomplish all of the goals of the lesson.

Flow Principle: The flow of high-quality instruction begins from a question or a problem that students see as problematic. As students intellectually engage in the problem the students are supported in learning the lesson’s big mathematical idea by building on their previous knowledge.

In our analysis of the data we found frequent references (13 of the 19 conversations) to a concept that the Japanese call the “flow” of a lesson. Flow includes the whole logical structure of the lesson as planned (how it builds on students’ ideas, how the task creates a need for the mathematics, etc.) as well as how the lesson actually plays out (building on specific student comments, transitions, etc.).

The lesson flow answers the natural questions raised by principles one and two. In which problem, questions, or activities will the students intellectually engage (intellectual engagement principle) that will best address the goals of the lesson (goal principle), and in particular, will raise the *hatsumon* or big idea of the lesson. Ideally, the *hatsumon* can be developed largely based on work the students do, but lessons vary on how well the Japanese teachers reach this ideal.

Unit Principle: High-quality instruction is created with close connections to past lessons and to build a basis for future lessons. The lessons have strong connections within a unit as well as connections across grades. The lessons in a unit help students progress to ways of thinking, writing, and representing mathematics evident in the discipline of mathematics.

One interesting result here is that lessons within a unit changed depending on the placement within a unit: beginning, middle, or end. Japanese teachers teach lessons at the beginning of the unit in a very open-ended, exploratory fashion. However, at the end of the unit they lesson are more “focused” and are taught in a more explicit fashion. Although the analogy is not perfect, the beginning lessons look more like proposed “reform” instruction while the ending lessons look more “traditional.” However, all lessons still strive to have students doing intellectual work in a way that naturally builds connections to the new material.

Adaptive Instruction Principle: High-quality instruction adapts so that all students are engaged in mathematical work that appropriately challenges their current understanding.

In contrast to the push in the US that emphasizes differentiation of students (Gregory & Chapman, 2002) Japanese teachers emphasize commonalities among students rather than the differences. They craft lessons based on knowledge common to all students in the class, but challenge all students. Of course the instruction is more effective for some students than others and is more challenging for some than others. Part of the lesson planning process in Japan is to consider how to adapt the lesson to students who are struggling or who are not challenged. The Japanese then adapt instruction by considering two groups of students, those that understand specific content and those that are struggling to understand. For those that understand and are not challenged, they adapt the material to make it more challenging. For the students that are struggling they provide hints or carefully adapt the task so it is still challenging these students, but at their level.

Preparation Principle: High-quality instruction requires a well thought out, detailed lesson plan that addresses the previous five principles and ties them together in a coherent lesson.

We admit that this principle is less about instruction itself and more about what is needed for good instruction to take place. However, it was clear that this was an extremely important principle that cooperating teachers wanted student teachers to learn. It is also clear from the post-lesson conversations that both the cooperating teachers and student teachers thought that many of the disappointments in the lesson could have been solved by better preparation and more “research” on the part of the student teacher.

Formulating instructional principles, rather than just instructional methods, is a tremendous resource for teachers. Instructional principles can be implemented in a variety of ways, with varying methods and instructional strategies. The actual essence of good teaching is more closely tied to principles than to methods or surface level features of instruction. Thus an understanding of principles allows teachers to better evaluate the effectiveness of a lesson by looking past some prominent but secondary features to focus on the deeper foundational aspects that compose high-quality instruction.

DISCUSSION: COLLECTIVE AND COMPARITIVE ANALYSIS ACROSS THE THREE STUDIES

The three studies summarized above were conducted separately and in isolation. And yet the similarities of high-quality practice are striking, particularly the overlaps between the images of high-quality instruction articulated by Singapore teacher educators and the practices of high-quality US teachers. Specifically, the categories of mathematical representation, use of time, and suitable language are nearly isomorphic across those two studies. There are overlaps between the Japanese and Singapore categories of high-quality practice as well. For example, the Japanese principle of "intellectual engagement" shares much with the Singapore notion of "active student engagement." There is some convergence here with the US teachers' practice of including multiple mathematical proficiencies in a single lesson. The Japanese principle of "unit" is related to the Singapore concept of mathematical trajectory. Both take up the issue of goal-directedness across lessons.

How do we interpret these notable findings? One, we see this as a source of validation. Teacher educators in Singapore generated conjectures about the features of high-quality mathematics instruction; a small, empirical, qualitative study of US teachers found some of the same categories of practice that characterized actual high-quality mathematics instruction; and Japanese cooperating teachers give voice to some of the same foundational conceptualizations of high-quality mathematics teaching. Two, these categories of high-quality practice are perhaps not all that surprising. It stands to reason that efficient use of time, careful articulation of mathematical ideas, nimble deployment of mathematical representations, and purposeful goal-oriented lessons are likely to add up to strong instruction. Three, we cannot help but note the degree to which these categories of high-quality instruction transcend national cultures and boundaries, despite vast differences in educational systems on a whole host of dimensions. Four, we notice that none of the findings are in the form of specific teaching methods or particular tasks. All are broader categories of teaching practice.

CONCLUSION

There are striking similarities among the findings of characteristics of high-quality mathematics instruction across these three studies, despite wide divergences in method and context. We believe that further analysis would reveal even more overlaps—that in many cases language and examples mask some underlying similarities not seen on the surface. For example, representation of mathematics does not come up explicitly in the Japanese principles of high-quality instruction, but it may be implied as part of something larger the Japanese pay attention to within the exploration of the mathematics in the lesson.

At the same time, there are clear differences among the findings, and must be explored further. These differences expand and challenge our conceptualization of high-quality teaching practice. A carefully articulated, widely shared, and robust conception of high-quality mathematics teaching is essential in guiding mathematics teacher education at every level, and for appraising mathematics instruction for a multitude of purposes. Ultimately this conception shapes the opportunities for students to learn mathematics.

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