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An Assessment Of Mental Mathematics Programs For Young Children

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Abstract

In this article, we present criteria to assess the usefulness of mental mathematics programs and using these criteria, we compare the abacus and a finger-based speed mathematics system.

Introduction

Ministries of Education throughout the world, including Singapore, Malaysia and the United States, have recently embarked on ambitious programs to improve mathematics education for younger learners.

- Singapore’s Ministry of Education recently introduced the abacus in Primary Schools on a pilot basis (Chong, Yeap & Fong, 1996) and plans to introduce abacus classes in all Primary Three pupils by 1998 (The Straits Times, 1996a).
- In Malaysia, Standard Five pupils will be required to use the abacus and the Education Ministry is investing M$1 million on abacus training for teachers and M$800,000 on equipment (The Straits Times, 1996b).

So far these programs appear to be yielding some benefits. For example Dr. Fong Chan Onn, Malaysia’s Deputy Education Minister noted that Standard Four and Standard Five pupils showed improvements in mathematical skill after learning the abacus (Kaur, 1996).

Despite these pronouncements, the introduction of these programs has nevertheless led parents, principals, teachers and even students to ask:

- What do we want to achieve with these mathematics programs?
• How are programs such as these (e.g. the abacus courses) achieving these objectives?
• Is there a better way to achieve these objectives?
• How do we choose from the plethora of alternative programs being offered?

Is Mental Mathematics only about Speed and Accuracy?

Without a doubt, the needs of our fast changing society are driving changes in our educational systems. Graduates today are expected to know more than was expected of graduates one or two decades ago. Two decades ago when employers talked about basic skills they were referring to basic fluency in language and numbers. Nowadays basic skills include not only proficiency in mathematics and language but also the ability to use the latest software, make a succinct presentation, write persuasively, organize information and draw conclusions from it (Avishai, 1996). This requires a degree of creative thinking skills on the part of the graduate.

As a consequence educational systems worldwide are being asked to not only teach more in less time, but they are also being pressed to incorporate some new components to develop creative thinking skills in students.

Guidelines for an Ideal Mental Mathematics Program

Keeping this in mind let us now lay out our criteria for a good mental mathematics program and what such a program should achieve.

Help the learner to do well in mathematics

Clearly a good mental mathematics program should help to improve a student’s calculative ability in speed and accuracy. However we also note that to do well in mathematics requires other attributes and skills (Chong, Yeap & Fong, 1996) or as Professor Lee Peng Yee, head of Nanyang Technological University’s mathematics division aptly put it, “in school, speed is not the issue, understanding is” (The Straits Times, 1996a). Therefore in order for a mental mathematics program to strengthen a learner’s ability in mathematics, it needs to develop both
calculative ability and proficiency in skills such as forming or grasping mathematical concepts, developing number sense, analyzing and solving problems. Furthermore, this must be done at a level the learner can follow and absorb. That is to say the program must be suitable for young children.

What should we look for in determining whether a program is suitable for teaching young children? We can consider whether the program:

- has the in-built capability to facilitate the teaching of mathematics
- has the in-built capabilities to help students learn and comprehend mathematical concepts more productively
- was designed, from the start, with modern pedagogical requirements for young learners in mind
- is simple enough for young children to learn, apply and comprehend (i.e. it is logical, consistent and straightforward in its implementation).

But if we drill down to a more fundamental level, we see that the underlying issue is whether the program can achieve successful engagement of the pupil's learning faculties.

Build creative thinking skills

If we expand the scope of our examination, we find that educationists, industrialists and national leaders nowadays do not want our schools to produce students who are merely walking calculators or libraries. They want graduates who are well rounded and imaginative.

But in order for creativity to flourish in a young student this child must first embark on the development of his thinking skills. In addition the student must also develop the confidence needed to tread new ground or try something new -- as well as the courage not to be derailed but to learn from the failure which is inevitable in most creative endeavors. As Alvin Toffler (1990) noted, "Innovation requires experimental failure to achieve success."

Traditionally, creative development has never fallen within the purview of mathematics instruction. But this does not mean that creative development could not be successfully combined with substantive, uncompromised mathematical instruction for younger learners. In fact, if we were able to accomplish the above educational objective, i.e. combining creativity with a
mathematics program, it would be a significant breakthrough. Such a program would certainly merit serious investigation.

Mathematical validity

We must be careful to ensure that a program to teach mathematics should itself be mathematically sound. This means that the program should be able to withstand rigorous mathematical validation and offer some logical consistency in its approach.

Teach efficiently

As we noted earlier, teachers have more to teach in less time. Therefore efficiency of instruction is desirable. This efficiency can be obtained from one of two ways -- or both:

- by helping the student learn more productively and/or
- by helping the instructor teach more productively.

However this efficiency must not come at the cost of compromised educational integrity.

Other considerations

An ideal contemporary mental mathematics program should offer more than pedagogical value and instructional efficiency. It should also:

- be enjoyable for young learners
- help develop learners’ confidence in and comfort with numbers and mathematics

But is it unreasonable for us to expect that a mental mathematics program, besides teaching mathematics efficiently and effectively can also offer further benefits such as building confidence and continue extending further benefits to the development of creative thinking skills?
That is the real question this paper seeks to address.

What makes a Mental Mathematics Program Effective and Suitable for Young Learners?

Educationists generally agree that a good mathematics program should have, as an integral pedagogical element the capability to teach and equip students not only with the necessary skills and knowledge but also the conceptual understanding to enable a young learner to advance to higher levels of learning.

Engaging the pupil in the learning process

How well such a program teaches and equips a student is one measure of its effectiveness. But in order to provide this academic development, the program needs to first engage the student in the learning process at not just one but the three domains that may affect or contribute to the learning and teaching processes:

- The Cognitive domain

- The Affective domain – e.g.:
  - Does the student find the class lively?
  - Does the student find the program stressful to learn?
  - Does the student find the program fun and game like?
  - Does the program require the student to perform a lot of mechanical work and/or rote learning?
  - Does the program build confidence in the student?

- The Psychomotor domain

If a program does not properly engage the student at all three aforementioned domains, its instructional effectiveness can be significantly reduced.

Cognitive concerns

A suitable program must obviously account for a young learner’s state of cognitive development. We note that cognitive structures are progressively organized into stages from rudimentary to complex (Arends, 1994). Therefore if a
program requires a significant degree of *pre knowledge* in mathematical vocabulary or concepts as well as a high degree of proficiency in language the applicability of such a program may be restricted -- especially for young learners.

Affective concerns

It is safe to assume that young children possess relatively more fragile psychological structures than those of older children. Young children can be easily frustrated by a course that they find:

- tedious,
- stressful,
- difficult to apply or
- confidence draining.

This frustration can later degenerate into a lack of confidence in mathematics causing students to develop anxieties around mathematics. As a result, students become demotivated, demoralized or in extreme cases, *mathophobic* (Chong, Yeap & Fong, 1996).

How can these factors influence a student?

1. **Fun or tedious?** When young children, because of either from their own prior experience or from the experience of others, expect a classroom environment to be a *leaden* rather than a *lively* learning experience, this already is *demotivating*. (We should make careful note that an educational program can be fun and game like. In fact well designed games can offer significant pedagogical value (Koay, 1996)).

2. **Stressful or fun?** A program that requires the learner to memorize a voluminous number of formulae and/or procedures, or *subjects* the learner to spend *endless hours* in repetitive mechanical practice can induce undue *anxiety*. Moreover programs imposing highly rigid conditioning have long been recognized by educationists and psychologists to be a *potential impediment to developing creative thinking skills* (Koh & Yong, 1996).
3. **Difficult or easy to apply?** A program involving considerable procedural or psychomotor complexity can also inordinately raise anxiety levels among young learners.

4. **The confidence factor:** Though confidence may be an intangible quality, its long term benefits cannot be ignored. A child may be proficient in mathematics yet because of a lack of confidence may appear to be mediocre. Furthermore a student who is not confident in his or her ability in mathematics may develop a phobia around numbers. Such a student could become educationally handicapped in the long term.

A program able to build confidence as well as impart a love for numbers and mathematics can provide the learner with an important psychological edge that will propel him or her to progress further to new, higher levels of learning with a zest and zeal that less confident students will lack. Furthermore, as noted earlier, for creativity to flourish the underlying confidence must first exist.

**Psychomotor concerns**

When young children are involved, it is vital to see that psychomotor considerations are also addressed, particularly if an external device such as an abacus is involved.

1. **Acceptance or rejection:** As part of the growing process, young children spend endless hours learning how to manage and build their muscles as well as their muscular coordination. When a child is additionally asked to master a new, external device, the child must not only struggle to overcome his or her own psychomotor limitations but at the same time master the new contraption.

2. **Ergonomic incompatibility can further exacerbate this process.** That is if an external device is additionally uncomfortable for the student to use, the learning process becomes even more of a strain. This can lead the child to become frustrated which in turn can lead him or her to:

   - not react well to the device,
   - develop a phobia for the device or
   - simply reject it outright.
3. **Stimulation of the brain:** There is also the degree of psychomotor application to consider as this directly impacts the stimulation of the brain. The significant importance and implications of this shall be discussed next.

**Creative Stimulation**

While the development of creative thinking skills is beyond the scope of this paper we nevertheless can take note of a few scientific developments that have led us to a better understanding of how our bodies, in particular the biological infrastructures for thinking and creativity, function.

- In 1981 Dr. Roger Sperry won a Nobel Prize for his work that identified the right side of the brain as the creative hemisphere (Zdenek, 1983).

- Scientists further showed that the left side of the body is controlled by the right brain and that input from the faculties of the left side of the body feeds directly to the right brain and vice versa.

- In 1988 Chiye Aoki and Philip Siekevitz demonstrated that the brain responds to stimulation from the senses and the faculties by physically growing. This growth is proportional to the amount of stimulation received (Harvey, 1994). Thus stimulation of the right brain (that comes from the left side of the body) results in growth in the right brain.

A program can help to build creativity in a student by helping to promote the development of the right brain. Such a program can also help by developing confidence in the young pupil.

**Mathematical Validity**

Earlier we remarked that a program to teach mathematics should itself be mathematically sound; that is, the program should be able to successfully withstand rigorous mathematical validation and offer some logical consistency in its approach. While the need and benefits of this should be apparent, relatively few mathematics programs offer this strict substantiation. Thus when a system has been proven to withstand strict mathematical scrutiny, for this reason alone it
merits serious attention. One such system in use today, the Speed Maths System, has been validated by rigorous mathematical testing (Lai, 1996).

Good management of time and energy

Given the time pressures on teachers today -- i.e. they must teach more in less time -- and the fact that both a learner and teacher have a finite amount of time and energy to devote to the process of learning, it behooves us to identify programs that provide maximum educational returns for the required investment in time and energy of both the learner and teacher. But how does any educational program manage both this limited time and limited energy well?

As noted before, educational efficiency can be obtained:

- by helping the student learn more productively and/or
- by helping the instructor teach more productively.

This can be achieved through:

1. the elimination of unnecessary tasks
2. application of a logical, straightforward methodology or approach and
3. energizing students in the process of learning.

Eliminate unnecessary tasks

Numerous business management textbooks cite process improvements as ways to increasing organizational efficiency. Process improvement exercises improve efficiency through the elimination of unnecessary steps or procedures.

Let us consider for the moment two mental mathematics programs that produce similar results, the major difference being that one program requires the user to master an external device (after spending considerable time and effort) while the other does not (and consequently saves substantial instruction time). Of these two which would be preferable?

Obviously if a program does not require the use of an external device:
it avoids all the problems of managing an external apparatus (e.g. bringing it along, not forgetting it, not breaking it, etc.) -- which require energy on the part of the learner and the teacher and

it does not require the user to spend time and effort mastering a foreign device.

There is another reason why we should avoid the necessity of learning an external device -- particularly when young children are involved. Science has shown that a young child experiences prodigious biological growth that peaks, then drops off precipitously at a young age. This growth explains why young children are capable of astonishing feats of learning (Harvey, 1994). If we want to take advantage of this early biological development we must start introducing educational content at a young age. By imposing the need to master a device we not only delay the introduction of this instruction, thereby losing the opportunity to capitalize on this biological development, but we also divert student’s precious learning and development energies and time to activities that may seem unnecessary.

We should take note that the need to master an external device is not necessarily undesirable, for example in learning the piano the need to master an external device is inevitable. Rather it is our contention that for a mental mathematics program if an alternative to an external device exists, the alternative should be considered.

Enhance efficiency through a logical approach

As noted previously, a good program should be logical, consistent and straightforward in its procedural approach. It should incorporate good mathematical subject content and should engage the learner in the learning process. There are very sound pedagogical reasons why procedural streamlining is valuable.

A logical, straightforward approach yields several important benefits in addition to increasing instructional efficiency. A logical approach is:

1. more readily comprehended by the learner and less likely to lead to confusion
2. less stressful for the learner to apply and
3. *less prone* to error unlike a system that requires the user to execute numerous and highly involved procedural steps or involves an over-usage of formulae.

Furthermore a logical system is *easier for the instructor to use and apply*. The last point is especially noteworthy.

A system that involves many procedural steps is inherently more prone to error than one that accomplishes the same results with fewer procedures. Procedural complexity can *cause a learner to err not because of the learner's lack of mathematical proficiency but because of a higher probability that a procedural mistake or omission may occur*. This will not only cause unnecessary stress on the part of the learner but may incur the *added undesirable effect of lowering the learner's confidence levels in mathematics for reasons that are, in fact, unrelated to mathematics.* It should now be obvious that a mathematics program, particularly for young learners, should be procedurally straightforward, consistent and mathematically sound yet there are cases where this is not true.

*A Chinese Paradox?*

Out of academic interest, let us examine one such example. There appears to be a method originating from China called the *Shi* technique that uses a finger representation scheme on the left hand for *addition only*. It curiously *deviates* from this finger-based scheme for other operations (e.g. subtraction, multiplication and division). Rather, for these other operations, considerable paper and pencil work as well as memory work is involved.

For subtraction the *Shi* technique employs transformation of subtrahends to combined numbers and utilizes mathematics that is currently not in vogue. Although it is a well known fact that subtraction is the logical inverse of addition, the *Shi* technique does not exploit this fact to handle subtraction.

In the case of multiplication the user is subjected to the task of memorizing a completely new set of tables that are clearly different from the current common multiplication tables taught in schools. This causes great concern to both students and educators. Furthermore, to complete the multiplication task, the user must apply a dizzying array of number manipulations; conversions and
procedures far too involved for many older learners to begin to fathom; for younger children this is a nearly impossible feat.

So far, the Shi technique has not succeeded in producing anything tangible for division.

Finally no mathematical validation or any reference to such mathematical validation can be found in his documentation (Shi, 1991).

Energizing pupils

Returning to the subject of efficiency, we note that a program that can motivate and energize students in the process of learning also increases efficiency of instruction.

Children that want to learn and who are galvanized to learn require less energy from the teacher and are far preferable to lifeless individuals resisting instruction. How can we motivate students to want to learn? We noted earlier that program which is game-like can bring about significant positive effects on a child’s affective domain. Such programs also can strongly motivate students to learn (Koay, 1996). Furthermore it has also been shown that the body can be energized by external stimuli (Ostrander & Schroeder, 1994). If a program is able to tap into these faculties and motivate students to learn, it would increase instructional efficiency manyfold.

Clearly a program that applies a child’s energies fully in the learning process to maximize productivity and invigorate students is far more desirable than one that dissipates learners’ energies with unnecessary activity which, in turn, has the potential of enervating and demotivating these pupils.

Design Suitability

This brings us to the question of how to assess whether a program is suitably designed for teaching mathematics to young learners.
The Abacus

Let us consider the history, design and capabilities of the abacus for a moment.

A short history of the abacus

Today’s abacus traces its ancestral roots to the Roman hand abacus which itself derived from the Counting Board. The counting board was the preserve of traders, merchants, tax collectors and other local officials (Flegg, 1983) and there is no historical evidence to show that the abacus was originally designed for anything other than adult users.

The abacus was designed to be a calculating aid primarily to handle addition and subtraction. Over the course of time, several variants of the abacus emerged, notably the Russian abacus, the Chinese abacus and the Japanese Sorobon (Flegg, 1983; Kojima, 1954) -- the last of which was the most significant change in the physical design of the device though this design change came only after the abacus had been around for several centuries (Kojima, 1954).

Addressing modern pedagogical needs

More importantly, given that the abacus originated centuries ago, it is extremely improbable that the abacus could have been designed with any contemporary teaching objectives in mind. As noted earlier, today we want education programs that are efficient and effective which means they should positively engage the student at the psychomotor and affective domains. We also recognize that a good program today must help to not only develop proficiency in a particular subject but also lead to the nurturing and development of creative thinking skills as well.

Let us now objectively assess how the abacus measures up in these categories.
1. **Efficiency**

At the time the abacus was designed, efficiency was not of paramount concern. Over time the abacus remained virtually unchanged. Apart from the previously noted introduction of the Sorobon, the physical design of the Chinese abacus itself has not been updated to improve efficiency of use. While numerous new schemes involving the abacus have emerged (Flegg, 1983; Kojima, 1954; Li, 1988), ostensibly designed to make the abacus faster and easier to use, the abacus user still must contend with numerous formulae as well as new formulae that need to be committed to heavy memory work.

2. **Psychomotor Concerns**

Besides efficiency considerations, the abacus has inherent ergonomic limitations which in turn, have implications for psychomotor compatibility. Human hands come in all different shapes and sizes. Clearly an abacus optimized for an adult to use may be highly uncomfortable and thus unsuitable for a child and vice versa. Thus with the abacus there is a very real issue of physical ergonomic compatibility with the hand which we noted earlier can make it even more challenging for the student to master and may lead to potential problems of rejection.

3. **Affective Concerns**

So far there is no documentation to conclusively demonstrate that when the abacus was first invented affective considerations were ever taken into account. Students are saddled with the task of learning a large number of formulae and lengthy procedures that a learner had to work on and still must commit to memory.

4. **Creative Stimulation**

Most importantly, if we step beyond the scope of mathematics, it is easy to deduce that the abacus was never intended to help develop the biological infrastructure that leads to independent, creative thinking skills.
Traditionally the abacus has applied the right hand, specifically one, two or three fingers on the right hand to manipulate the beads within a limited range of motion. As a result the abacus:

- has limitations in its psychomotor benefit
- more significantly, is limited in the amount of stimulation it can provide the brain and
- stimulates the left brain but not the right brain – the creative hemisphere.

Management, Supply and Acquisition

Finally with the abacus there are also the problems of management (care, guarding against breakage, repair, etc.), acquisition and configuration – plus the user constantly needs to remember to bring the abacus along. These are not trivial concerns. At the moment Malaysia is struggling to supply an adequate number of abaci to its schools. The country also has problems with the type of abacus being used. Last year it was training students on the two by five abacus, now the Malaysian Education Ministry is training teachers on the use of the one by four abacus (Ragavan, 1996).

The Speed Maths System

The Speed Maths System (refer to References) was designed as a finger-based system to teach young students mathematics, including mental mathematics, and is presently used to teach mathematics involving high speed addition, subtraction and multiplication and much more. The system initially applies the left hand, then later both hands in the learning and teaching process.

Efficiency and pedagogical value

Recent studies have shown that finger-based systems have bona fide pedagogical value in teaching mathematics (Gardner, 1977; Hunting, 1986; Cobb, 1986). This is not surprising because finger mathematics schemes allow for mathematical data to be input directly to the brain without requiring the learner to divert his attention and energies to manipulate an intermediate, external device.
The *Speed Maths System* actively employs the hand to productively convey a vast amount of mathematical information – numbers, concepts and operations, in a direct biological path to the brain. By using the full hand and all fingers directly to perform the operations rather than manipulate an external device, no energy is lost in the manipulation of an object foreign to the body. Thus more energy can be usefully channeled to the learning process thereby increasing instructional efficiency.

The *Speed Maths System* is straightforward to apply, mathematically logical in its approach. For example it uses the hand consistently to perform addition, subtraction and multiplication.

**Ergonomic concerns – Engagement of the learning faculties**

Because no external device is involved, the *Speed Maths System* suffers none of the limitations inherent in systems that depend on an external apparatus. There is no external device to acquire, lose, manage, break or forget. Furthermore, ergonomic compatibility is simply not an issue with the *Speed Maths System*.

To further improve the efficiency of instruction, the system has in-built capabilities to help students learn and comprehend mathematical concepts more productively.

Of most importance, the *Speed Maths System* simultaneously applies and engages the body’s learning faculties – visual, aural and tactile – in the learning experience. It uses the full hand and the arm as well as visual and aural faculties thereby adding considerable value to the overall learning and teaching process.

**Affective Considerations**

In addition, the designers of the *Speed Maths System* not only designed the system to allow students to perform a variety of calculations quickly and accurately but also incorporated value added features benefiting students. For example, the designers took into account affective considerations and eliminated many potential sources of stress by:
- minimizing homework
- minimizing dependence on rote learning
- enlivening classroom sessions
- designing the system to be highly game-like
- as well as painstakingly instituting straightforward procedures in the solution of problems.

Some of these features had the added bonus of motivating and energizing students to want to learn. Furthermore, in reducing students' anxiety levels, the system had the added benefit of reducing teachers' stress levels.

By streamlining its procedures, eliminating the need for heavy rote learning, students in the Speed Maths System are not bogged down with monotonously repetitive exercises, stress-inducing memory work and have more energy to devote to learning. As an added bonus, the system is highly malleable and can be applied to teach a wide range of mathematical topics and concepts.

Creative Stimulation

The Speed Maths System's use of a full range of a young child's faculties is one of its most salient features. This not only allows for highly efficient instruction but also provides the user's brain with considerable stimulation that, as we saw earlier, promotes growth. This growth is needed to build the body's thinking infrastructure that leads to improved thinking skills and ultimately, creativity.

Conclusion

In finding a suitable program to teach 'mental mathematics' to young learners, we should look for a program that:

1. is designed with contemporary needs and realities in mind
2. is mathematically sound
3. addresses the affective, cognitive and psychomotor aspects of the learning and teaching process
4. is simple to use
5. improves not only calculative skills but understanding of mathematical concepts and
6. is efficient and non-stressful to apply.

Most importantly, any program that comes with the added feature of stimulating creative thinking is a bonus.

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The Speed Maths System. International Educational Laboratory and Research Team, Singapore, Pasir Ris Drive 6, #B1-458, Block 472, Singapore 510472, Republic of Singapore.


