# CONTEXTS FOR MATHEMATICAL PROBLEM POSING

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Abstract: Mathematical problem posing is a relatively new research topic in mathematics education and activity in the mathematics classroom. The research focus has been on the nature of the mathematical and cognitive elements associated with the problems posed and how they may be analysed. There is less attention on how students construe this activity of posing mathematics problems, what contextual (e.g., situational, psychological, social or cultural) elements are at play in this activity, or what beliefs about mathematics, dispositions toward mathematics, or meanings of "doing" mathematics do pupils or teachers bring to or acquire from this activity. This paper explores the contexts for mathematical problems posing in practice. Illustrative instances are drawn from research literature, personal communications with researchers in this area, and the efforts of a group of pre-service teachers at posing mathematical problems under different conditions or constraints.

## Introduction

Mathematical problem posing, which Silver (1994) defines as the generation of new mathematics problems and the reformulation or transformation of given problems, is a relatively new research topic in mathematics education (e.g., Silver & Cai, 1996, English, 1998) and activity in the mathematics classroom (e.g., Stoyanova, 1999). The research focus has on the nature of the mathematical and cognitive elements associated with the problems posed and how they may be analysed (e.g., Yeap & Kaur, 1999). This paper explores the contextual aspects in mathematical problem posing. It discusses briefly the meaning of *context*, identifies instances from the data collected that illustrate contextual sensitivities in the activity of mathematical posing problems.

What, where and when is the context?

There is more to *context* than just the pre-set environment. Context, it is argued in cultural psychology, is not a given in the environment but constituted by people in interaction (Cole, Ergeström and Vasquez, 1997). Situated cognition theory posits cognition, learning and knowledge to be inseparable from context (e.g., Suchman, 1987; Brown, Collins, & Duguid, 1989; Lave and Wenger, 1991). The work of Carraher, Schliemann and colleagues (e.g., Nunes, Schliemann, & Carraher, 1993), for example, supports this view. They observed performance and conceptual differences in mathematics learnt in two contexts - in the streets and in schools. Cognitive scientists are beginning to accept (Greer, 1989) that human cognition cannot be fully understood without consideration of the affective and contextual aspects in human life (Gardner, 1985). At one extreme, the theoretical position of *contextualism* holds all behaviour as meaningful only within the context in which it occurs (Reber, 1995).

Butterworth (1993) notes that an explicit definition is very often not attempted but a simple definition of *context* is "the setting for thought". The *Collins Cobuild English Dictionary* (1995) defines the word *context* as follows:

> The context of an idea or event is the general situation that relates to it, and which helps it to be understood. …If something is seen in context or if it is put into context, it is considered together with all the factors that relate to it. (p. 353).

It is important to ask what the context or “general situation” in the classroom is that relates to mathematical problem posing, and “which helps it to be understood”, for it seems that a different “general situation” might give someone a different understanding of the activity of posing mathematics problems. What are these factors that relate to problem posing? Potential “candidates” include the obvious such as the physical environment (e.g., inside or outside the classroom), the mathematics topic being taught, the socioeconomic and cultural background of the pupils, and the pupils' motivation to pose a problem. The not so obvious are, for example, the sociomathematical norms (Yackel & Cobb, 1996) and the pupils’ personal epistemologies of mathematics. Deciding what to include as context is a difficult task (Miller, 1996); this point is discussed later.

On a more technical note, the *Penguin Dictionary of Psychology* (Reber, 1995) defines "context" as:

> Generally, those events or processes (physical and mental) that characterize a particular situation and have an impact on an individual’s behavior (overt and covert)…. The specific circumstances within which an action or event takes place. (p. 159)

*Context* is still seen as that something background to the thing of interest, in this case, human behaviour. As Cole, Ergeström and Vasquez (1997) point out, "context" is usually used as an omnibus term for the other factors outside of the variable(s) of concern to the researcher. It is interesting to note that Reber’s (1995) definition above extends the notion of *context* to include the not directly observable mental processes and covert behaviour. In this sense, one may speak of “intra-person" or "internal" context. Hence, a person’s motivation to pose a mathematics problem could be context to the activity, and thereby contributing to the meaning of the activity for that person.

How else has *context* been conceived? Miller’s (1996) uses the term *context* to refer to “the part of the situation (or the field) that is used to determine meaning in general, and in particular is used to resolve potential ambiguities of meaning”. By *contextualisation* is meant “the use of context to determine meaning and resolve potential ambiguities.” (p. 4). "Situation" and "field" are clarified as follows:

> A *stimulus* was a simple energy change that affected a sense organ; a *stimulus object* was a more complicated entity that gave rise to a stimulus; and a *situation* was a very complicated configuration of stimulus objects. A red light is a stimulus; a sentry’s command to halt is a situation. (Miller, 1996, p. 3)
>
> Your field [referring to Kurt Lewin (1936)]… includes just those parts or aspects of the real situation that affect your thought or behaviour at a given moment. A bit of food might be part of the field for a hungry animal, but not for an animal that is satiated. (p. 4)

Miller’s conception of context help explain what it means to say that context is constituted by persons in interaction. Context is more than just the pre-set environment or background to the behaviour of interest. It is that part of the situation (or the field) which affects one's "thought and behaviour *at a given moment*" and that is "used to *determine* meaning or resolve ambiguities of meaning" (my emphasis). However, what is in a person's field (e.g., food) is dependent on a person's mental or physical state (e.g., hungry or satiated animal); food is "not meaningful" to a satiated animal. Hence, *context* is jointly established, or constituted, by and for the person, from elements internal and external to that person. Whether it is the external that initiates the internal, or vice-versa, in constituting context is a moot point.

A group of people in interaction would involve a multitude of fields. At any moment, these fields are being defined, individually by each member and collectively by the group. In principle, different contexts are being constituted at different moments. Given the indeterminable number of situational elements that could be brought into play and the unthinkably large number of ways the interaction could proceed, it is a formidable task to decide what to include as context. It might be appropriate to conceive of cognition, action and context as inextricably bound. This would mean that the unit of analysis in mathematical problem posing should incorporate context (what, where and when) in which the activity is conducted.

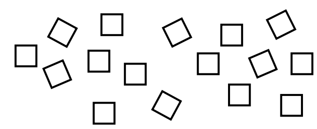
In summary, *context* may be understood in the “static” sense as the given aspects of the pre-set environment for an event or action, and that is called upon to explicate the meaning of that event or action. In the “dynamic” sense, it is the setting being constituted by persons in interaction with the external environment (which may include the other people in the interaction) and that contextualises meaning and behaviour. To understanding human behaviour, it might be necessary, besides just asking what is the context, to ask where is the context and when is the context.

## Illustrative Instances & Discussion

(I) A colleague who is an academic research mathematician was asked in the course of casual conversation to pose a mathematics problem thus: "Suppose you are asked to pose a mathematics problem involving the numbers 3, 5, and 8. What problem would you pose?" Without hesitation, he offered: "What is the sum of three, five and eight?" As a research mathematician, he must have worked on much harder problems. So, if pressed, he might have humoured the request with a more carefully crafted problem. But, the presence of contextualising elements such as talking to a colleague who is teaching the *Teaching of Primary Mathematics* courses, and perhaps his unspoken assumption "surely you can't be asking for a research level problem", seem to have led him to respond as he did. The problem *per se* might be analysed for mathematical elegance or significance. But, inferences about the mathematical ability of the posers must include the context to be meaningful.

(II) Three intact classes of preservice teachers enrolled on the *Teaching of Primary Mathematics* methods courses were asked to pose mathematics problems. The activity was carried out towards the end of a tutorial session and the student teachers were a couple of weeks from completing their teacher education diploma. The student teachers received one of three sets of information for use in posing the problems.

All three sets share the same feature of 16 identical squares randomly placed (see picture on right). They differ as follows: *Set A* contains just the 16 identical squares. In *Set B* the squares were described as coloured 4 red, 4 blue, 4 green, and 4 black squares. In *Set C* the squares were described as numbered 1 to 16.



Not all the participants made use of the embedded cues (colour and numbers) to pose the problems. That is, these items are not *context* for them. A sample of problems involving the cues is:

From *Set A*
1. How many buckets would there be if I put two squares in one bucket?
2. Given that the side is 5 cm
   (a) What is the area of 4 squares?
   (b) What is the perimeter of one square?
3. List two different shapes that can be made up with 16 squares. What are the areas and perimeters for each different shape?

From *Set B*
1. How many squares are left if two black squares and 1 red square are taken away?
2. If one row has one square of each colour, how many rows will there be?
3. (i) Find the total number of reds and blues.
   (ii) Find the total number of reds, blues and greens.

From *Set C*
1. List all the possibilities for the problem below:

$$\square \quad + \quad \square \quad - \quad \square \quad = \quad 15$$

2. 1 , 3 , __ , __ , 9 , __ , __ , 15 [Presumably the intention was to full in the "blanks".]
3. How many pairs can be found that give a sum of 12?

The problems are recognisably school mathematics problems, although there was no instruction to the participants to do so. The general set up (e.g., people, place, and purpose) or context has apparently defined the meaning the word "problem" for the participants. Schoenfeld (1988) found such appropriation of meaning in his study of a geometry class in which the teacher encouraged rote learning. Students interpreted the teacher's instruction to "think" when doing mathematics as memorising procedures.

Similarly, the student teachers have also construed or appropriated the meaning of problem posing as using previously seen mathematical tasks or exercises to produce similar problems under the conditions set by the teacher. Three of many other similar comments support this view.

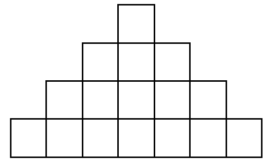
> I thought of familiar maths problems and tried to find a way/problem to suit this number squares into it.
>
> The questions came from my past experiences, like during my school days, teaching practice, etc. They also came from surrounding items that I saw.
>
> I was trying to remember all the assessment books I had read before and recall the sums relevant to this problem. At the same time, I took into account the no. of squares above so that to fit the problems I could remember.

Several student teachers explained that they were concerned that the problems be solvable They formulated or reformulated the problems in order to achieve this end. This is much unlike the problems found in real-world applications of mathematics, much less those faced by academic research mathematicians. Self-report evidence in support of this internal (or "intra-person") context that has defined the activity differently for these participants are

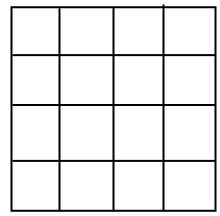
> Think of a mathematical concept… whether the above items can be used to form a question and how… Once decided on the concept and a rough question, see whether it makes sense solvable.
>
> All information given were being taken into consideration to formulate the question… After a question is formed the solution is being sought out. If solution exists, question will be used.
>
> While setting the questions, I try to solve the questions mentally so as to check whether I can solve them myself. At the same time, I am able to find out whether the questions are logical and clear.

(III) A week later, the same student teachers were asked to pose a challenging mathematics problem using the same set of information. There was noticeable effort to make the problems difficult for the intended solver. A change in the goal, from just posing any problem to posing a challenging problem, has constituted a new context for the activity. Had a before-after analysis of the problems posed been carried out, it would be more convincing to suggest that the goal is *context* to be included in the interpretation of the problem posed. A couple of examples are shown below:

Arrange the numbers 1 to 16 such that when you add it (sic) up horizontally and vertically the sum of each line is the same.



Arrange the squares in a sequence (sic) where no squares with the same colour are joined together, either by the side or at the vertices.



(IV) Another three intact classes of pre-service teachers taking the *Teaching of Primary Mathematics* courses worked in groups of not more than four to produce a Maths Trail as part of their coursework. At each location, the student teachers were asked to design mathematics tasks or problems that would be of interest to the people taking part in the trail. The participants were asked to keep their "rough work" and note the thought processes involved.

One group of three students cast the trail as an adventure involving a team of “Weed Busters” tracking down an evil doctor, a *Dr Weed*. A partial set of the problems from this group is:

Objective: To search, find and destroy all traces of Dr Weed’s mutational-causing potion and its formula.

You are now at the basement of the building [Block B], take a lift to the fourth floor and walk up the flights of stairs to the Greenhouse.

The Greenhouse is in a mess. You, the Special team of Weedbusters, while searching the Greenhouse comes across the formula for neutralising and reversing the mutational process.
Your team decides to clear the mess and reset the hydroponics experimenting center to grow plants that would neutralise and reverse the effects of the potion which can turn people into vegetables.
…
Question 2:
After clearing the Greenhouse, you must plant the seedlings in the Greenhouse. Chemicals, X, Y and Z are required so that the plants grown can become antidotes to neutralise the effects of Dr Weed’s potion. In order to work, these 3 chemicals must be mixed in the right proportion with water as shown:

| Water | : X | : Y | : Z |
|---|---|---|---|
| 12 | : 5 | : 1 | : 7 |

If 18 litres of water is required to mixed (sic) with the chemicals, what are the different proportions of chemical X, Y and Z in litres are required to make the antidotes?

Question 3:
Outside the Greenhouse is a hook which can be used for carrying the chemicals from the 4th Floor to the 5th Floor. Estimate the distance between the 2 ends where the hook moves horizontally.

Question 4:

Estimate the height of from the 4$^{th}$ Floor to he 5$^{th}$ Floor and the height of the railing on the 5$^{th}$ Floor. (Hint: use the stairs to estimate).

What are the contextual influences at work in the generation of these problems? The problems had their starting points from aspects of the physical environment that is in the participants' field. Some of the things they noted at the location which are "seeds" for the problems are:

> Greenhouse: In a mess. Pots of discarded plants. A few tanks of shrimps. A rusty, heavy-looking, huge hook. Lots of plants and high trees.
> …
> Ask questions about rate of giving birth of shrimps (?)
> Since plants are dying, how about questions on replanting the plants.
> Can we ask questions about the hook? …

The goal that a group has defined for itself in posing the mathematics problems is another facet of context for the activity. For this group, the goal was changed from the task being "one of the things that a teacher wants us (students) to do and so we had better carry it out" to a highly motivated one of "this exercise is interesting so let's construct really interesting or challenging problems". The motivation to generate "good" problems was captured in their self-reports:

> Our original ideas were amended many, many times….
>
> Upon subsequent meetings, our ideas and locations keep changing, but the Greenhouse in Block B was always at the back of our minds. Then Belle came up with the idea of an adventure, solving a "crime" instead of simply a Maths Trail. The three of us became excited and ideas keep pouring in. (From then on, I began to enjoy (Truly enjoy) discussing about this Maths Trial.)
> The energy level and expressions were highly charged. Both Cat and Belle put forth many suggestions on the plot, problems and solutions to make our Maths Trail Adventure fun and interesting.
> …
> Our proposed questions keep changing.
> Even the area of focus was changed several times.

Another member of the group wrote:

> When we came up with the interesting idea of making it [the maths trail] into an adventure, we somehow like the entire activity more, and we are happily triggered to come up with more interesting ideas. … However, the inclusion of an adventurous plot make the generation of questions at each site more difficult becos we have to think only of questions that can be related to the plot.

By casting the activity of posing problems for a Math Trail as an adventure, this group of student teachers has established a different context and meaning for posing the problem. For one, it made the task of posing problem more demanding for them.

The group or, to borrow a term from Lave and Wenger's (1991) term, "community of practice", is another aspect of context for problem posing. The community defines what counts as a "problem" and decides when the problem is properly posed. Self-reports in support of this facet of context are:

> As we went through the maths trail, we did have a lot of disagreement, especially in setting questions for the Greenhouse. One of the most hot disagreement is the last question for the green house. We were discussing about whether the weight might affect the speed of the hook. Although we agreed that it does not but I felt that this point is quite unnecessary for the P5/P6 pupils to know unless they are gifted pupils, but they [presumably the team-mates] said that if this greenhouse questions are based on out of school mathematics, then it made a difference. Because of this I gave in. Note that problem posing is taken as synonymous to generating questions.

One student in another group wrote:

> I told my friends that I had a problem in hand. The problem was: Find a leaf from plants grown around the vicinity of the National Institute of Education that does not tessellate. Draw it on A4 paper. Modify the leaf until it tessellate.
>
> Although the idea was good, my group mates rejected it as they find it difficult to extend the question.

To further illustrate the influence of the "community of practice" in the classroom on problem posing, consider this problem posed by a pupil: "The heights of three infants are 3 metres, 5 metres, and 8 metres. What is the average height?" It is likely to be rejected by many teachers as unrealistic.

## Conclusion

A variety of contexts is interactionally constituted in the activity of posing mathematics problems. Each contextualises the meaning of mathematical problem posing in particular, and of mathematics and doing mathematics in general. The meaning of problem posing appropriated by pupils (or student teachers) should be more than that of producing problems based on those previously encountered, and that of "problem" should be more than that of just "an exercise to be carried out." It might be ideal that the meaning appropriated resemble those of the practices of a community of mathematicians (applied or academic variety). To this end, Walter and Brown (1990) have hinted at the idea of the classroom as a community of "budding" mathematicians posing, critiquing and solving the problems they generated and deemed significant.

The present inquiry is an interpretative exploration into contexts for mathematical problem posing. The limitations of this approach are recognised. It suggests for rigorous study the notion that thought, behaviour, knowledge and context are inextricably bound. In consequence, it suggests further study of mathematical problem posing in a context interactionally constituted by person (e.g. beliefs), activity (mathematical problem posing), goal (established by person for posing the problems), community (people involved in the activity) and domain (mathematics).

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