PUPILS' UNDERSTANDING OF THE CONCEPT OF REVERSIBILITY IN ELEMENTARY ARITHMETIC

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Primary school pupils are taught how to perform the simple arithmetic operations of addition, subtraction, multiplication and division. The signs +, −, x, ÷ are called the fundamental operators, and pupils are taught how to solve simple open sentences involving them. Such open sentences are of the form, a * b = . . . where * is the fundamental operator and '. . .' is the response blank. Variations of this equation are formed by the different positioning of the response blank (. . . * b = c, a * . . . = c). Problems may also be phrased so that the operators are on the right side of the equal sign (. . . = a * b, c = . . . * b and c = a * . . .)

The solution of these open sentences constitutes the foundation for the solution of simple equations. That is, open sentences such as 4 + . . . = 7 and . . . − 9 = 13 pave the way for the solution of simple equations like $4 + x = 7$ and $x - 9 = 13$. Both types of open sentences and equations shown in the examples require the child to exercise reversibility of thought in problem-solving. For example, in solving the problem, 9 − 3 = . . . , the child asks himself, "nine minus three equals what number?". However, in the problem 9 − . . . = 3, the child asks himself, "nine minus what number equals three?" The child therefore has to think reversibly to solve the problem. Such open sentences requiring reversibility of thought are presented in the primary one curriculum materials of the CDIS Primary Mathematics Project (1982). The examples are well illustrated with coloured pictures, drawings and objects to help the child think reversibly. However, a survey of the examples presented in the pupil workbooks showed that there is a limited number of items requiring reversibility of thought and a predominantly large number of items of the form a * b = . . .

The purpose of this study is to find out the extent to which pupils understand the concept of reversibility in arithmetic given the limited exposure to the solution of this type of problem.

Procedure

The sample came from two classes in a primary school and consisted of 42 boys and 40 girls making a total of 82 pupils.

The test was restricted to items where the operator is on the left of the equal sign. This is because the primary one curriculum materials do not have any items presented with the operator on the right.

Table 1. Distribution of addition and subtraction items

Item Format	Number of Items Addition	Subtraction
. . . * b = c	2	2
a * . . . = c	2	2
a * b = . . .	2	2

All operations are within 10 so that number size is controlled.

The test was administered by the class teachers. No time limit was set and pupils were asked to solve the problems at their own pace. Data analysis involved the classification of common errors following the system devised by Roberts (1968). The errors were classified as follows:

Wrong operation: The problems are solved using the operator other than the one specified in the problem, e.g. substituting the minus (−) sign in 9 − 2 = . . . with a plus sign (+).

Obvious computation error: In this form of error, the pupil uses the correct algorithm but due to carelessness in recalling number facts, the wrong answer is given, e.g. in the problem 3 + . . . = 10, the pupil gives the answer as '6' instead of '7'. In order to differentiate this form of error from the random response type, any answer that deviates by 2 or less from the correct answer falls within the category of obvious computation errors.

Defective algorithm: The pupil uses the wrong algorithm in the problem-solving process, e.g. in the problem . . . − 4 = 3, the pupil gives the answer as '1'. Obviously, by relocating the minus sign he subtracted three from four to get the answer.

Random response: These are errors in which no general pattern is detected.

Results

Since each pupil was given 12 items, the total number of items solved by the sample was 984. Out of this number, 137 or 14% were solved incorrectly. The distribution of the errors classified according to the system devised by Roberts (1968) is as follows:

5 wrong operations
17 obvious computation errors
85 defective algorithms
30 random responses

Total number of wrong answers: 137 (14%)

Wrong operation *(subtraction instead of addition):* All five pupils gave the answer for $6 + 3 = \ldots$ as 3, indicating that the pupil had subtracted 3 from 6 instead of adding 3 to 6.

Obvious computation errors: Answers that differ by one or two units from the correct answer.

Table 2. Obvious computation errors

Problem	Wrong Answer	No. of Cases
Addition		
$4+2=\ldots$	7	2
$\ldots+2=8$	4,7	2
Subtraction		
$8-2=\ldots$	5	1
$9-\ldots=4$	6	5
$7-\ldots=3$	5	4
$\ldots-4=2$	4,8	2
$\ldots-5=7$	7	1

Table 3. Distribution of obvious computation errors according to response blank positions

	Blank Positions	No. of Cases
Addition	a b c	2 0 2
Subtraction	a b c	3 9 1
Total		17

Defective algorithms: The following addition problems were solved incorrectly by adding the given terms:

Problem	Wrong answer	No. of cases
2 + . . . = 5	7	11
5 + . . . = 9	14	7
. . . + 8 = 10	18	6
. . . + 2 = 8	10	7

The following subtraction problems were solved wrongly by subtracting the smaller given term from the larger given term:

Problem	Wrong answer	No. of cases
. . . − 5 = 4	1	28
. . . − 4 = 2	2	19

The following subtraction problems were solved wrongly by adding the given terms:

Problem	Wrong answer	No. of cases
7 − . . . = 3	10	5
9 − . . . = 4	13	2

Table 4. Distribution of defective algorithm errors according to response blank positions

Blank Positions		No. of Cases
Addition	a	13
	b	18
Subtraction	a	47
	b	7
Total		85

Random response errors: As defined, no regular pattern was detected in these errors as answers to the problems were varied and included non-responses. The following are some examples with wrong answers provided by the pupils in brackets.

. . . − 5 = 4 (12, 6, 4)
5 + . . . = 9 (9)
. . . − 4 = 2 (2)
. . . + 8 = 10 (12)
. . . + 2 = 8 (3)
9 − . . . = 4 (2, 9)
6 + 3 = . . . (4)
2 + . . . = 5 (12)

Discussion and Conclusion

The results of the error analyses concurred very well with the findings of Lindvall and Ibarra (1980). Errors such as adding the given terms in addition problems and subtracting the smaller term from the larger in subtraction problems were common. However, one reason why pupils seem to do better with items of the type a − . . . = c over those of the type . . . − b = c is that while pupils make the error of subtracting the smaller given term from the larger in . . . − b = c, the same algorithm used for a − . . . = c will always result in the correct answer. Although Roberts (1968) was of the opinion that pupils used the wrong operation by adding instead of subtracting because of more exposure to addition problems, the present study showed that this error was made the other way round – subtracting instead of adding. The primary one materials have more or less an equal number of addition and subtraction problems and so the question of over exposure does not arise. One conjecture could be that in the present test, two subtraction problems preceded the addition problem, $6 + 3 = \ldots$ leading the pupils to subtract carelessly because the 'minus' operator was still seen in their minds. However, this was not true for the other problems. The seemingly high number of random response type and obvious computation type of errors indicate that pupils do not think carefully in the recall of number facts. Grouws (1974) in his study of solution methods called this rote recall, where the child responded to the items immediately without showing signs of comprehension. In other words, the pupils in this sample tended to be careless.

The study indicates that pupils made errors even with simple problems involving basic skills and the percentage of mistakes made (14%) is considered high given the situation that these operations are within 10 and that most children at the kindergarten level would have grasped these number facts. Their differences in performance over the item types lend credence to the fact that there is an apparent lack of understanding of the concept of reversibility in addition and subtraction operations. The reason could be due to a lesser emphasis in primary one materials on the type of problems in which the response blanks are in positions 'a' or 'b'. It is suggested that teachers can remedy this situation by teaching children at this level to solve more of these problems by the use of counting devices such as sticks and beans. The common algorithmic errors can be pointed out to further reinforce the correct procedures to be adopted.

References

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Appendix A

Test Paper

Name: ______________________________

Sex (Boy/Girl): ______________________

Answer *all* questions

a) $4 + 2 =$ ____

b) $8 - 2 =$ ____

c) $2 +$ ____ $= 5$

d) ____ $+ 8 = 10$

e) $7 -$ ____ $= 3$

f) ____ $- 5 = 4$

g) $5 +$ ____ $= 9$

h) $9 -$ ____ $= 4$

i) ____ $- 4 = 2$

j) $6 + 3 =$ ____

k) ____ $+ 2 = 8$

l) $10 - 6 =$ ____

----------------------- DO NOT WRITE BELOW -----------------------

+	−
a ___ b ___ c ___ Score	a ___ b ___ c ___ Score

Total Score ________