GRAPHS - WHAT DO THEY SAY?

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Abstract

The most widespread use of graphs is to illustrate the variation of some aspect of a real situation, be it in medical sciences, economic forecasts, environmental studies or elsewhere and numerate adults ought to comprehend such representations and draw inferences from the data represented.

Yet many, if not all, approaches to the teaching of graphs in our schools focus mainly on technical issues - for example, algebraic manipulation, point plotting and reading, filling in entries in tables - at the expense of the meaning which is to be conveyed.

160 secondary three girls were each given an exercise to do on the interpretation of graphs of real life situations to assess their ability to communicate the language of graphs. The worksheets used for the study were taken from the teaching package 'The Language of Graphs' produced by the Shell Centre for Mathematical Education at the University of Nottingham, UK.

The findings of the study show that many of the subjects are unable to interpret given graphs of real life situations. The implication that pupils are unable to relate the graphical knowledge that they are taught in school for communication purposes calls for re-examination of the approaches adopted in the teaching of graphs in most if not all schools.

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BACKGROUND

In recent years, graphical representation and interpretation has been a subject of research in many mathematics classrooms (Philips, R., 1985; Bell, A. et al., 1987(a), 1987(b), 1987(c); Kaur, B., 1990). These researchers have found students' unable to interpret graphs of real life situations despite the latters' exposure to the concepts of graphs taught in schools. In a study involving third-year secondary mathematics students in the United Kingdom, Bell, et al (1987(a), 1987(b), 1987(c)) found that the subjects faced difficulties in making the translation between graphs and situations or verbal descriptions. In Singapore, graphical work is a part of the secondary mathematics curriculum. Students basically are taught the techniques of plotting and reading off points and regions from graphs. Although they are fairly competent in the manipulation of these skills, Kaur (1990) noted that students here, are not proficient in the use of graphs to convey meanings.

This study aims to investigate the types of difficulties encountered by Singaporean secondary three pupils , 15 years of age , in the area of graphical representation and interpretation.

METHODOLOGY

Instrumentation :

The instrument (see Appendix) used for this study was a set of five worksheets taken from the teaching package 'The Language of Graphs' (1985) produced by the Shell Centre for Mathematical Education at the University of Nottingham, UK.

Subjects :

The subjects were 160 secondary 3 (15 years of age) female pupils from 4 classes in a girls' school.

Procedure :

Worksheets (1) and (2) were given to the first class while Worksheets (3), (4) and (5) were given to the second, third and fourth classes respectively. Each class was given 40 minutes to complete the exercise. The students were advised not to communicate with or copy from one another or leave out any section of the exercise. They were encouraged to attempt every question to the best of their abilities. The worksheets were collected at the end of the 40 minute period and the responses analysed.

ANALYSIS OF RESPONSES

(A) Interpretation of Points

I. *Interpretation of one point with reference to one variable. [Worksheet 1 : (a) and (b)]*

100% of the students answered correctly for both parts of the question.

II. *Interpretation of two points with reference to one variable. [Worksheet 1 : (c) and (d)]*

95.1% of the students answered correctly for both parts of the question.

These results show that most students do not have any problems working with one variable type of questions.

III. *Interpretation of two points with reference to two variables. [Worksheet 1 : (e), (f), (g); Worksheet 2]*

100% of the students identified the correct bags in (e) and (f) that would give better value for money.

Although everyone could give the correct answer to both (e) and (f), it was very interesting to note the reasons they gave for their answers. From their explanations, it could be observed that they derived the answers intuitively from

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the graph rather than using graphical concepts. However, it was surprising that only 1 student used and related the idea of ratio to the points in the graph.

When asked in (g) to identify the bags that would give the same value for money, only 24.4% of the students could answer bags A and F. 29.3% thought that the answer was B and E, the reason being that B is half as heavy and costly. Another 43.9%, however, chose C and D instead, giving the reason that the cost increases as weight increases.

For part (g), based on the answers given by the students, it was clear that they understood what same value for money meant, that is, a greater weight must incur a greater cost. Here again, they used more of their intuition to identify the two bags that roughly gave the required relationship. None of the students drew a line passing through the two points to see which line passed through the origin. It appears that the students were unable to apply the concept of gradient.

As for worksheet 2, 65.9% of the students gave the correct response that for A, the son is taller and for B, the father is taller. Other responses were:

i.   A; son is shorter, B; father is shorter (2.4%)

ii.   A; sons taller than fathers, B; otherwise (2.4%)

iii. Points A, B are different (14.6%)

iv.   A = 67 inches, B = 69 inches (4.9%)

v.   Father B is taller than father A and son B is shorter than son A (4.9%)

vi.   Sons are shorter than father (4.9%)

The responses to worksheet 2 show that about 34% of the students were not able to interpret the scatter graph.   They just could not relate the two variables given.

(B)   Interpretation of Graphs

I   *Interpretation of actual situations with reference to graphical presentations and vice-versa.   (Worksheet 3)*

The first part of the worksheet required pupils to relate six of the twelve given graphs to six actual situations. The scoring was one mark for each correct match that the student gave.

The scores are summarised in the table below:

Score	0	1	2	3	4	5	6
Percentage	0%	15.4%	48.7%	25.6%	10.3%	0%	0%

The above scores clearly tell us that our students may be very good at graphical methods as required by the examinations, but certainly not at relating real life situations to graphical forms. Perhaps students are not taught the language of langüage of graphs at school.

The second part of the worksheet required the pupils to describe three real situations and relate them to the six remaining graphs. The scoring was again one mark for each correct description that the student gave.

The scores are summarized in the table below:

Score	0	1	2	3
Percentage	*58.8%	38.5%	7.7%	0%

* 20.5% of the students did not attempt this question

Less than 8% of the students could give more than 1 suitable example to match any of the remaining 6 graphs. The above scores reinforce that our students are basically unable to use graphs to describe actual situations.

II   *Translation of graphical representations to daily language. [Worksheet 4]*

A total of 8 marking points were available for this question. The marks are awarded as follows:

i.   C takes the lead   (1 mark)

ii.   C stops running   (1 mark)

iii.   B overtakes A   (1 mark)

iv.   B wins   (1 mark)

v.   Any four of the following: (2 marks)

        -      A and B pass C

- C starts running again

- C runs at a slower pace

- A slows down (or B speeds up)

- A finishes second (or C finishes last)

  (1 mark if any two or three of the above points)

vi.   Lively commentary that mentions hurdles (2 marks)

  (1 mark for lively commentary without mentioning hurdles or a 'report' which mentions hurdles)

The scores of the students are summarised below:

Score	1	2	3	4	5	6
Percentage	10.3%	12.8%	30.8%	28.2%	12.8%	5.1%

From the responses, the following observations may be made:

(a)   Less than 20% of the students scored more than 50% of the total marks available. Many students were unable to translate the mathematical representation into words. This may be due to language difficulties or the inability to interpret the a given mathematical representation.

(b)   Of the 39 students in the class only 1 gave a lively commentary of the race. The rest of the students merely highlighted the essential features of the graph. This indicates that most students do not view graphs as a means of communication. Rather, graphs are viewed as something 'dead'.

(c)   An interesting observation made is that ALL the students were able to identify B as the winner of the race. This showed that students were able to relate the winner as the one that took the least time to complete the race which could be read off the x-axis.

(d)   It was also very interesting to note that 25.6% of the students thought that A and C lost because they took a longer route as compared to runner B. Here, they have associated the graphical lines with distances and hence the longest line was related to the longest distance. It did not occur to them that all three runners took part in a 400 metres hurdles race. This notion of interpreting graphs as pictures is a common mistake made by students (Bell et al; 1987(a), Kaur; 1990)

(e)   There were a few responses that made no reference to the graphical representation at all. For example, one wrote that C lost because he started off too slowly.

III. *Translation of information from pictorials to graphical representations.*
[Worksheet 5: (iv)]

Only 9.8% of the students managed to give the correct sketch of the graph. One student did not even attempt the question. The rest gave partially correct answers.

The responses show that students have difficulties translating information from pictorials to graphical representations. Although students are conversant with the basic concepts involved in graphing (only one was able to associate the turning of a bend to the slowing down of the car which was denoted by a dip in the speed-distance graph) they were just not able to represent the details graphically.

FINDINGS

The responses to worksheets 1 and 2 reveal that students here, in general, do not have any problems interpreting points in a scatter graph provided it is with respect to one variable, for example, weight or cost. However, when two variables are involved, for example, weight and cost, then students begin to face difficulties in comparing any two points in the scatter graph. In such cases, many resort to intuition to give the answers. The students who fail to interpret the points correctly are usually the ones who do not check their answers to see if they are plausible and make sense. Perhaps the students lack knowledge on the use of graphs.

The responses to Worksheets 3, 4 and 5 reveal that the students are also weak in interpreting graphs of real situations. One may expect the matching of graphical representations with the described situations to be within the reach of students who have been taught the concept of graphs. On the contrary, students show that they encounter great difficulties relating situations with graphical representations.

Also many pupils are unable to translate given graphs into verbal form and vice-versa. Hence a very important process skill where graphs are concerned is lacking in our pupils.

It is evident from the analysis that many a time the answer was correct but the explanation and justification given was far from adequate. Perhaps many students feel that in mathematics there is little or no place for writing.

IMPLICATIONS FOR TEACHING

A few conclusions may be drawn from this study:

(a)   The graphical knowledge that our students are taught in schools has little or no application in their daily lives. They lack the training, opportunities and exposure to relate their understanding of graphs with the physical world. To rectify the situation, students should often be made to see the significance and relevance of the content materials.

(b) Apart from the mathematical knowledge that has to be acquired, there must be also emphasis on the acquisition of thinking, writing and communicating skills.

(c) Students should also be taught to appreciate the use of graphs as a means of communication and not just as a set of technical tools to help solve algebraic problems.

A serious implication of pupils being unable to relate graphical knowledge taught in school for communication purposes calls for re-examination of the approaches adopted in the teaching of graphs in most if not all schools.

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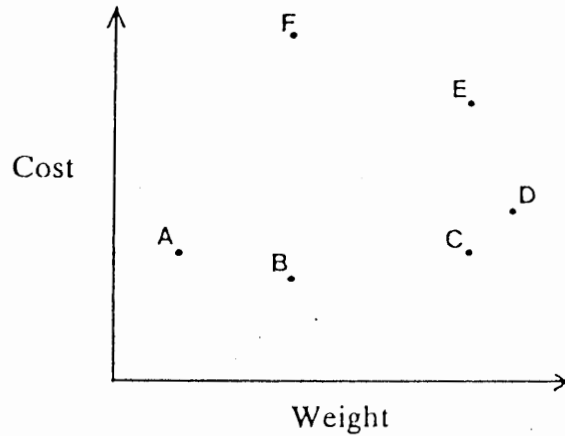
APPENDIX

(A)   Worksheet 1
         - Bags of Sugar

(B)   Worksheet 2
         - Is Height Hereditary

(C)   Worksheet 3
         - Choose the best graph ....

(D)   Worksheet 4
         - The Hurdles Race

(E)   Worksheet 5
         - Going to School

Bags of Sugar

Each point on this graph represents a bag of sugar.
(a) Which bag is the heaviest?
(b) Which bag is the cheapest?
(c) Which bags are the same weight?
(d) Which bags are the same price?
(e) Which of F or C would give better value for money?
    How can you tell?
(f) Which of B or C would give better value for money?
    How can you tell?
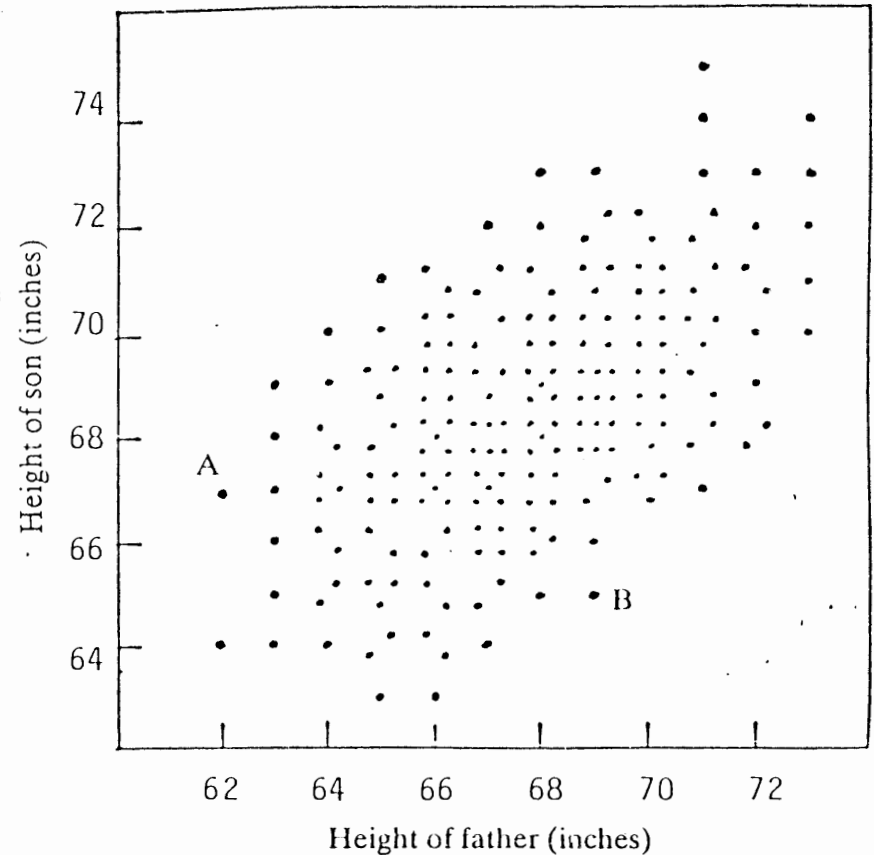(g) Which two bags would give the same value for money?
    How can you tell?

Is Height Hereditary?

In an experiment, 192 fathers and sons were measured.
(The sons were measured when they had attained their full adult height.)
* What can you say about points A and B?
* What conclusions can be drawn from this graph?

Choose the best graph to describe each of the situations listed below. Copy the graph and label the axes clearly with the variables shown in brackets. If you cannot find the graph you want, then draw your own version and explain it fully.

1) The weightlifter held the bar over his head for a few unsteady seconds, and then with a violent crash he dropped it. (height of bar/time)

2) When I started to learn the guitar, I initially made very rapid progress. But I have found that the better you get, the more difficult it is to improve still further. (proficiency/amount of practice)

3) If schoolwork is too easy, you don't learn anything from doing it. On the other hand, if it is so difficult that you cannot understand it, again you don't learn. That is why it is so important to pitch work at the right level of difficulty. (educational value/difficulty of work)

4) When jogging, I try to start off slowly, build up to a comfortable speed and then slow down gradually as I near the end of a session. (distance/time)

5) "In general, larger animals live longer than smaller animals and their hearts beat slower. With twenty-five million heartbeats per life as a rule of thumb, we find that the rat lives for only three years, the rabbit seven and the elephant and whale even longer. As respiration is coupled with heartbeat—usually one breath is taken every four heartbeats—the rate of breathing also decreases with increasing size. (heart rate/life span)

6) As for 5, except the variables are (heart rate/breathing rate)
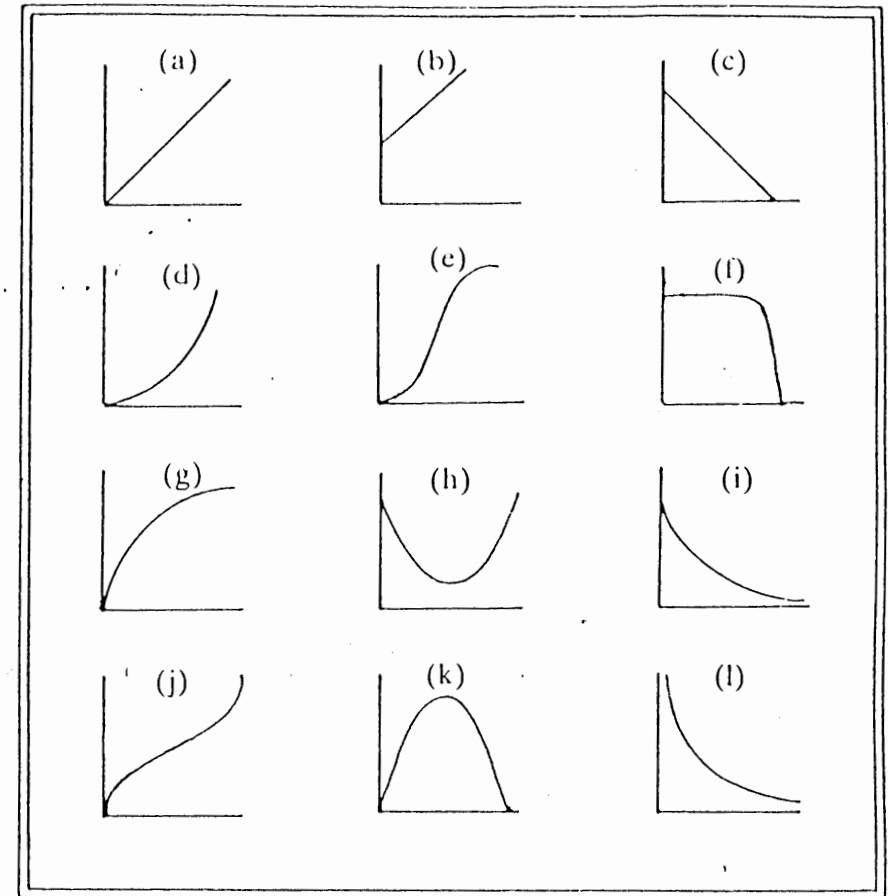
Now make up three stories of your own to accompany three of the remaining graphs. Pass your stories to your neighbour. Can they choose the correct graphs to go with the stories?

THE HURDLES RACE

400 —

Distance
(metres)

———— A
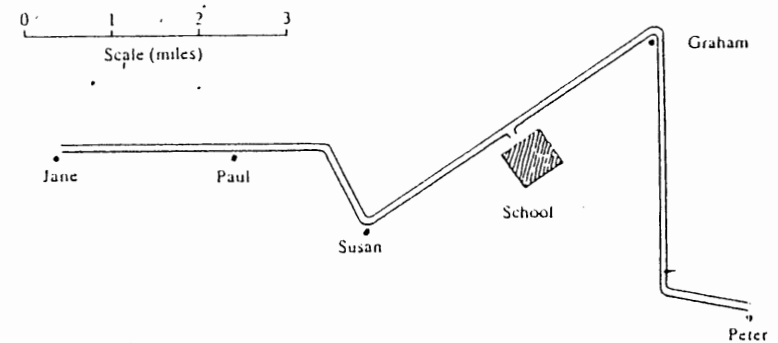—·—·— B
------- C

0

60

Time (seconds)
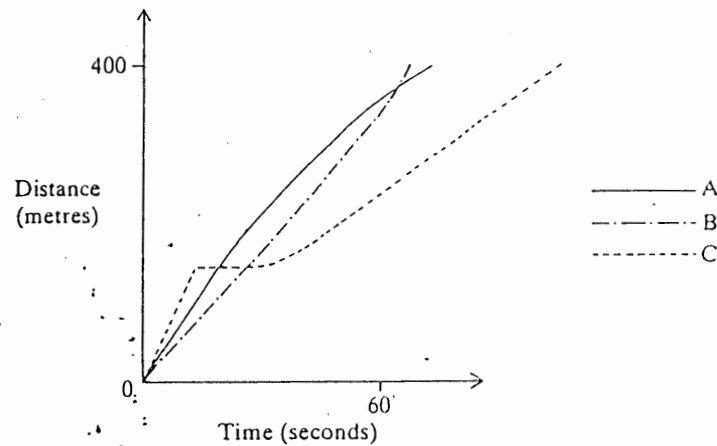
The rough sketch graph shown above describes what happens when 3 athletes A, B and C enter a 400 metres hurdles race.

Imagine that you are the race commentator. Describe what is happening as carefully as you can. You do not need to measure anything accurately.

GOING TO SCHOOL

0   1   2   3

Scale (miles)

Graham

Jane   Paul

School

Susan

Peter

Jane, Graham, Susan, Paul and Peter all travel to school along the same country road every morning. Peter goes in his dad's car, Jane cycles and Susan walks. The other two children vary how they travel from day to day. The map above shows where each person lives.

Peter's father is able to drive at 30 mph on the straight sections of the road, but he has to slow down for the corners. *Sketch* a graph on the axes below to show how the car's speed varies along the route.

Peter's journey to School

30 —

Car's
Speed   20 —
(mph)

10 —

0   1   2   3   4   5   6

Distance from Peter's home (miles)