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Author(s)	Soon Wan Mei Amanda, Ang Keng Cheng and Teo Kok Ming
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Comparison of Pricing Models with Simulated Demand Data

Soon Wan Mei Amanda
e-mail: wanmei.soon@nie.edu.sg
National Institute of Education
Nanyang Technological University
Singapore

Ang Keng Cheng
e-mail: kengcheng.ang@nie.edu.sg
National Institute of Education
Nanyang Technological University
Singapore

Teo Kok Ming
e-mail: kokming.teo@nie.edu.sg
National Institute of Education
Nanyang Technological University
Singapore

Abstract
Many types of pricing models incorporating different forms of demand functions have emerged in the past years. In an earlier work, a piecewise-defined Complementarity-Constrained Demand Function (CCDF) was discussed to correct certain weaknesses in commonly used demand functions. The authors introduced a Complementarity-Constrained (CC) pricing model incorporating the CCDF in that same work. However, there was a lack of numerical implementations therein. Hence in a separate work, we developed an algorithm using MATLAB to compare a generic pricing model and a CC pricing model. Experiments were performed to compare the revenues from the two models for certain ranges of parameters defining the demand function. In this work, we conduct further numerical testing by simulating the bidding behaviours of different types of customers and using simulated demand data to compare the models. We find that the use of the CC model leads to higher revenues for certain simulated scenarios.

1. Introduction

Demand is a relationship between price and quantity demanded, with all other factors affecting demand being held constant. A demand function (DF) is a mathematical expression that approximates the demand-price relation in the real market. It depends on consumer preferences, relationships between different products (for example, substitutability of products), sensitivities of consumer demands to changes in prices, and so on.

The literature consists of many types of demand functions derived through, for example, utility maximization or data fitting. Soon et al. [6] discussed different types of demand functions used in pricing models prior to their work and the problems with considering those functions. For example, linear functions are commonly used in practice due to their simplicity. However, it is

positive (or zero) on a restricted set of prices, say Ω. Thus prices outside Ω are commonly ignored in pricing models. As for the well-known Cobb-Douglas function, the demand for a product reaches zero only when its price tends to infinity, which is not realistic. Hence in Soon et al. [6], a non-traditional way to construct the demand-price relation was discussed. Firstly, a reasonable function d (linear or non-linear) on a set Ω is considered. Then d is extended to all non-negative prices outside Ω through solving a Complementarity problem. In the process, a Complementarity-Constrained Demand Function (CCDF) is generated.

A huge volume of research on pricing models exists in the literature. See, for example, Bitran and Caldentey [2], and Soon [9] for a review of pricing models. As good pricing strategies are important to retail and manufacturing industries, in revenue management, and so on, it is crucial to use suitable pricing models that incorporate realistic demand functions. Thus Soon et al. [6] developed a model using the piecewise defined CCDF mentioned above, which was termed a Complementarity-Constrained (CC) pricing model.

To make up for the lack of numerical implementations of the CC pricing model in that work, we have done earlier research on the CC model incorporating a suitable function d, in Soon et al. [7,8]. Basically, we developed an algorithm to compare a generic pricing model and a CC pricing model using MATLAB. Our results showed that a CC pricing model generates higher revenues for certain ranges of parameters defining the demand function. This motivated us to extend our work to conduct further investigations of the CC model.

Initially we attempted to use real demand data collected from Amazon and ebay websites to compare the two pricing models. However, due to reasons which will be discussed in a later section, we decided instead to simulate the bidding process for products and use the simulated demand data for our comparisons of the two models. Different types of customer behaviours in the bidding process were simulated and our comparisons showed that the CC model can lead to higher revenues for two different combinations of customer behaviours, and with certain sets of lower bound constraints on prices of products.

In the next section, we will discuss the motivation behind the CCDF in greater detail and illustrate the forms of the CCDF and pricing models. Following that, we examine specific generic and CC pricing models incorporating a particular form of d. A description of difficulties with using real-life data is then provided. We go on to discuss the simulation process and how we use the simulated data to compare two pricing models. Finally some results are shown to justify the use of the CC model.

2. Description of a CCDF and a CC pricing model
For clarity of exposition, we first describe the demand-price relationship for products and explain the motivation behind the use of a CCDF and hence a CC pricing model.

2.1 Motivation behind the use of a CCDF in pricing models
In a simple pricing model, assuming unlimited supply (or the ability to meet all demands), a seller will try to price his products suitably to meet customers' demand and maximize his revenue. Suppose that a seller is considering offering N products. Let $d(p)$ represent the demand for the products at prices represented by a vector $p \in R_+^N := \{ p \in R^N \mid p \geq 0 \}$, and LB represent a given vector of lower bound constraints on prices. A simple generic pricing model can take the form

$$\begin{aligned}\max \quad & p^T d(p)\\ \text{s.t.} \quad & d(p) \geq 0\\ & p \geq LB.\end{aligned}$$

(2.1)

In any pricing model, the choice of the function $d(p)$ is very important, as it should reflect customers' behaviour (for example, tolerance of price increases) and preferences for different products. For simplicity of explanation, suppose that a seller is considering offering two *substitutable* products. If he models customers' demand $d_1(p)$ (for product 1) and $d_2(p)$ (for product 2) using some generic function (for example, a linear function), it is easy to see that since d_i varies negatively with p_i, for a fixed p_j ($j \neq i$), d_i will turn negative at a sufficiently large p_i. Hence $d(p)$ only stays nonnegative on a restricted set of prices. Let such a set be denoted by Ω, that is, $\Omega := \{p \in R_+^N \mid d(p) \geq 0\}$. This means that a seller will usually consider pricing his products at some price $p \in \Omega$ satisfying the lower bound constraints.

However, as argued in Soon et al. [6], why should prices outside Ω be ignored completely? Indeed, it was shown in that work that it is possible to model demand at prices outside Ω. As discussed therein, consider a function $D(p) := d(p)$ for all $p \in \Omega$. This function $D(p)$ can be used to model demand at *all* nonnegative prices. To explain how $D(p)$ looks like for $p \notin \Omega$, suppose that we have a $\bar{p} = (\bar{p}_1, \bar{p}_2)$ on the boundary of Ω, where $d_1(\bar{p}) = 0$ and $d_2(\bar{p}) > 0$. Then since $d_1(\bar{p}) = 0$, for any $p \notin \Omega$ of the form $p = (p_1, \bar{p}_2)$ with $p_1 > \bar{p}_1$, we will have $d_1(p) < 0$. Such a price will be completely ignored usually. However, it is realistic to assume that since there is no demand for product 1 at price $\bar{p}$, the demand for product 1 at a price $p = (p_1, \bar{p}_2)$ with $p_1 > \bar{p}_1$ will still remain at 0. So we can set $D_1(p_1, \bar{p}_2) = d_1(\bar{p}_1, \bar{p}_2)$. And then since at price $\bar{p}$, all demand for product 1 has already shifted to product 2, the demand for product 2 at $p = (p_1, \bar{p}_2)$ should stay the same as that at $\bar{p}$. That is, the price $p = (p_1, \bar{p}_2)$ is "projected" onto the price $\bar{p}$. Thus $D_2(p_1, \bar{p}_2) = d_2(\bar{p}_1, \bar{p}_2)$ as well.

In Soon et al. [6], a map $B(p) = \bar{p}$ (called the projected price of p) was introduced and so $D(p) = d(B(p))$ as a result. This map B was defined via a nonlinear complementarity problem (NCP) as follows.

Definition 2.1 For any $p \in R_+^N$, $B(p)$ is defined as a solution of the $NCP(p)$: find x ($= B(p)$) such that

$$0 \leq d(x) \perp p - x \geq 0,$$

where $0 \leq d(x) \perp p - x \geq 0$ represents $d(x) \geq 0$, $d(x)^T(p - x) = 0$, $p - x \geq 0$.

The authors proceeded to give the following formal definition of the CCDF.

Definition 2.2 The **Complementarity-Constrained Demand Function (CCDF)** $D: R_+^N \rightarrow R_+^N$ is defined by

$$D(p) = d(B(p)), \quad \text{for all } p \in R_+^N,$$

where the map B is as stated in Definition 2.1.

With this possibility of modelling demand at all nonnegative prices, it then makes sense for a seller to consider *all* prices when trying to maximize his revenue. That is, when solving the pricing model (2.1), the function $d(p)$ should be replaced by $D(p)$ instead. Given some fixed LB, we may have a scenario where a $p \notin \Omega$ as described above can satisfy the LB constraints and lead to a higher revenue as compared to the case if all p had to be restricted to Ω, since the feasible set of prices to be considered is now larger. Even though this implies zero sales for one of the products (say product 1), it can still be beneficial for a seller to rely only on the sales of the other product (say product 2) if this leads to overall higher revenue for a seller.

2.2 A specific generic and CC pricing model

For simplicity, we will consider a single seller offering two products in this work. As in Soon et al. [7], we will choose the function

$$d_i(p) = c_i p_i^{a_{ii}} p_j^{a_{ij}} - k_i,$$

(2.2)

for each product i ($i = 1, 2$). Here c_i, k_i, a_{ii}, a_{ij} are some given constants, where $c_i, k_i > 0$ are demand parameters, a_{ii} represents the own-price elasticity of demand for product i, and a_{ij} is the cross-price elasticity of demand for product i with respect to product j. It is clear that $a_{ii} < 0$ holds for normal goods and $a_{ij} > 0$ (or < 0) if products i and j are mutually substitutable (or complementary). Suppose that the products are substitutable. With $k_i > 0$, if p_j is fixed, then the demand d_i goes to 0 before p_i approaches infinity, which seems realistic in the market. That is, the parameter k_i controls the maximum price at which demand d_i is zero. Note that this function is similar to the Cobb-Douglas demand function, but the presence of k_i *corrects* the problem as discussed earlier. Our choice of d is also similar to the functions considered by Chevalier and Goolsbee [3], Ghose and Sundararajan [4], and Ghose et al. [5] to model demand for online products.

Using the function d given in (2.2), the model (2.1) becomes

(2.3)
$$\max \quad p_1\left(c_1 p_1^{a_{11}} p_2^{a_{12}} - k_1\right) + p_2\left(c_2 p_1^{a_{21}} p_2^{a_{22}} - k_2\right)$$
$$\text{s.t.} \quad c_1 p_1^{a_{11}} p_2^{a_{12}} - k_1 \geq 0$$
$$c_2 p_1^{a_{21}} p_2^{a_{22}} - k_2 \geq 0$$
$$p_1 \geq LB_1$$
$$p_2 \geq LB_2.$$

Supposing that we incorporate the CCDF into model (2.1), that is, non-negative prices outside of Ω are allowed, then we have the model

$$\max \quad p^T D(p)$$
$$\text{s.t.} \quad p \geq LB.$$

Using Definitions 2.1, 2.2 and the function in (2.2), this model becomes

$$\max \quad p_1\big(c_1 x_1^{a_{11}} x_2^{a_{12}} - k_1\big) + p_2\big(c_2 x_1^{a_{21}} x_2^{a_{22}} - k_2\big)$$

$$\text{s.t.} \quad 0 \le c_1 x_1^{a_{11}} x_2^{a_{12}} - k_1 \perp p_1 - x_1 \ge 0$$

(2.4)

$$0 \le c_2 x_1^{a_{21}} x_2^{a_{22}} - k_2 \perp p_2 - x_2 \ge 0$$
$$p_1 \ge LB_1$$
$$p_2 \ge LB_2.$$

Our aim is then to compare the revenues obtained using models (2.3) and (2.4), for given parameter values c_i, k_i, a_{ii}, a_{ij}, for $i, j = 1, 2, j \ne i$, and lower bound values LB_1, LB_2. It is easy to see that the model (2.4) is reduced to (2.3) if we set $x = p$. Hence the revenue obtained through model (2.4) is always at least as high as that from model (2.3). We used MATLAB to program an algorithm to solve both models (2.3) and (2.4) in Soon et al. [8]. The reader can refer to therein for details.

Though existence of solution to the CC pricing model has been shown previously, it is not a simple task to prove analytically that the CC model can always ensure higher revenues. This paper focuses on using parameters defining d obtained through simulation studies to compare the two models.

3. Real life demand and simulation of demand
We first discuss some avenues of obtaining real demand data and the difficulties we found associated with these avenues. Then we move on to describe our simulation process and the different types of customer behaviour that we simulated.

3.1 Difficulties with obtaining real life demand data
We attempted to obtain real-life demand data through Amazon and e-bay. However we realised that there were several problems. For example, Chevalier and Goolsbee [3] developed a method to convert sales ranks to sales quantities on Amazon.com and BN.com, and then estimated the price sensitivities of demands for books online. Data was taken over three months in 2001. They observed major price changes by the two sellers. We wanted to use a similar approach but we observed very little changes in prices of products over time. In addition, there may be too much "noise" and censoring of data.

Zhang et al. [10] constructed a mathematical model to extract the demand-price relationship for products using online auction bid data. They used bidders' bid histories on auction websites like e-bay as data. They also estimated the bids that were truncated (unrecorded or censored). Adams and Bivins [1] analysed data from e-bay auctions for three types of telescopes over eight weeks, to estimate the demand for the telescopes. Their results suggested that demand was very elastic (that is, sensitive to changes in prices).

Initially we did collect ebay auction bid data for two cameras (Nikon and Canon) that probably caters to the same pool of customers, as they are often compared in camera reviews. The bidding amounts placed for each camera by different bidders were collected over one month. The bid prices were grouped over several intervals and the number of bids for each interval was used as an indication of the demand for each camera at the median price of the respective interval. However, our analysis of the data collected did not lead to promising results. For certain data, demand showed to be inelastic, and then for some other data, great fluctuations of graphs of

demand versus price were found. Hence we could not use the data to generate a reasonable demand function for our purpose. In addition, we felt that it is hard to know all potential buyers from the data and we have to assume that each person bids his valuation.

Besides attempting to obtain demand data through websites, it is possible to use customer surveys, conduct experiments through discounts offered for limited time and use real transaction data. However, these methods can be costly, time-consuming, difficult to obtain and hard to interpret (for transaction data). Considering all these difficulties with obtaining real demand data, we decided to simulate demand for this work.

3.2 Simulation of bidding process

We simulated different types of customer behaviour in the bidding process for two products using programs written on MATLAB. The two products are assumed to be substitutable for each other and cater to the same pool of customers. For each simulation study, we assumed ten categories of customers corresponding to ten different bid tolerances (indications of the maximum price they are willing to accept for any product). Let r ($1 \leq r \leq 10$) be an index that denotes a customer category, where a larger r value refers to a customer with a larger maximum price tolerated. Note that this direct relationship between r and maximum price is used as the increment in price is given as an increasing function of r (see below). Then the basic steps of the simulation process are:

1. The current price is p.
2. A bidder is randomly chosen from ten types of customers using a uniform distribution and so an ordered pair (max price, r) is obtained.
3. A bid price is calculated based on the bidder chosen: price $= p + dp$, where $dp = p * \frac{r}{200}$.
4. If the bid price is not higher than the bidder's maximum price, the bid is accepted.
5. An accepted bid leads to the new current price to be equal to the bid price.

On top of the above standard procedure in all simulation studies, our programs incorporate additional assumptions on customer behavior for the different studies as follows:

- In study 1, a bidder is assumed equally likely to consider product 1 or product 2.
- In study 2, a bidder goes for the product with the lower price.
- In study 3, there are two types of bidders – one who only considers the product with the lower price, and one who only considers product 1.
- In study 4, there are two types of bidders. The first type considers the product with the lower price. The second type prefers product 1 and his bid is accepted if the price of product 1 is not higher than that of product 2 by more than 20% of his maximum price (and is within his maximum price).
- In study 5, on top of the assumptions made in study 3, there is an extra assumption that should the price of product 1 exceed his maximum price, the bidder who prefers product 1 opts for product 2 whenever its price is within his maximum price.
- In study 6, on top of the two types of bidders as described in study 3, there is a third type who only goes for product 2.
- In study 7, there are three types of bidders. The first type only considers the product with the lower price. The second type prefers product 1 and his bid is accepted if the price of product 1 is not higher than that of product 2 by more than 20% of his maximum price. The last type

of bidders prefers product 2 and his bid is accepted if the price of product 2 is not higher than that of product 1 by more than 20% of his maximum price.
- In study 8, there are three types of bidders. The first type only considers the product with the lower price. The second type prefers product 1 but opts for product 2 if the price of product 1 exceeds his maximum price but the price of product 2 does not. The third type prefers product 2 but opts for product 1 if the price of product 2 exceeds his maximum price but the price of product 1 does not.

For clarity of exposition, we present a flowchart illustrating our simulation study 7 in Figure 1. The MATLAB program written for the same simulation study is also provided in the appendix.

4. Use of simulated demand data to compare pricing models

Our initial thought in this part of the project was to allow a user to input his chosen simulated scenario (out of eight possibilities), the different bid tolerance levels and other parameters; and then provide the user with an output of estimated function d and optimal prices and revenues of the two pricing models. However in our implementation, after collating the bids from a simulated study, we needed to transfer the data to EXCEL to obtain the parameters governing the function d, as we could not do so using MATLAB. That is, there was a "break" in the procedure due to the use of Excel. As a user may not be familiar with the process of data management using MATLAB combined with EXCEL, we did not proceed with our initial aim.

4.1 From simulation to comparison of models

We now describe the procedure of using the bids collected from each simulation study to compare the revenues of pricing models. For each product, the successful bids were first collated over intervals of prices to represent "demand". The median of each interval represented a price level. We then used Excel "data analysis" tool to carry out multivariable nonlinear regression analysis of our bid data, to obtain the parameter values c_i, k_i, a_{ii} and a_{ij} that define the function depicted in (2.2).

Finally, these parameter values were fed into our MATLAB program (see Soon et al. [8]) to compare the revenues of the two pricing models, over different lower bound values. The reader may refer to Figure 2 for a flowchart that outlines the whole process from simulation to the comparison of revenues from the two pricing models.

4.2 Some numerical results

Though we conducted many simulation studies as discussed above, we only present the results of two studies in this paper as they lead to more significant results. Note however, that all studies (with the exception of study 6) do showcase that CC model is better for certain sets of parameter values. For each of the studies presented below, the parameters obtained from certain runs of the corresponding simulation program are provided. The parameters from each run were fed into our MATLAB program to generate the resulting optimal prices and revenues for the two pricing models (for different sets of lower bound values as given below). Note that as discussed in Section 2.2, the revenue obtained from the CC model is always at least as high as that obtained from the generic model.
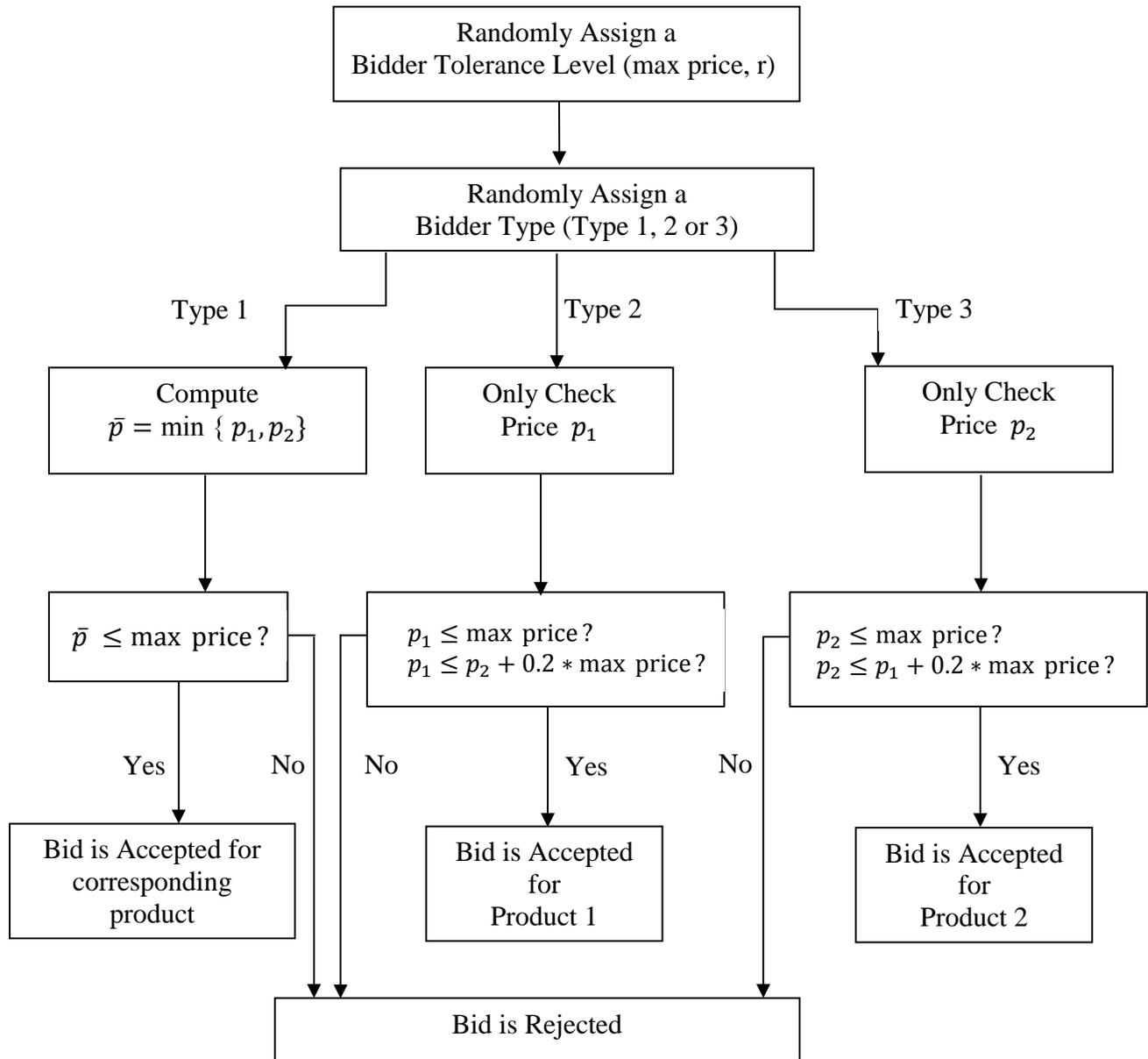
Figure 1: *Flowchart explaining the scenario for simulation study 7*

INPUT
COMPOSITION OF BIDDERS

PROGRAM A
SIMULATION OF SCENARIO

USE OF EXCEL
REGRESSION ANALYSIS

OUTPUT
COLLATION OF BIDS

PARAMETERS
$a_{11}, a_{12}, a_{21}, a_{22}, c_1, c_2, k_1, k_2$

PROGRAM B
COMPARISON OF
PRICING MODELS

OUTPUT
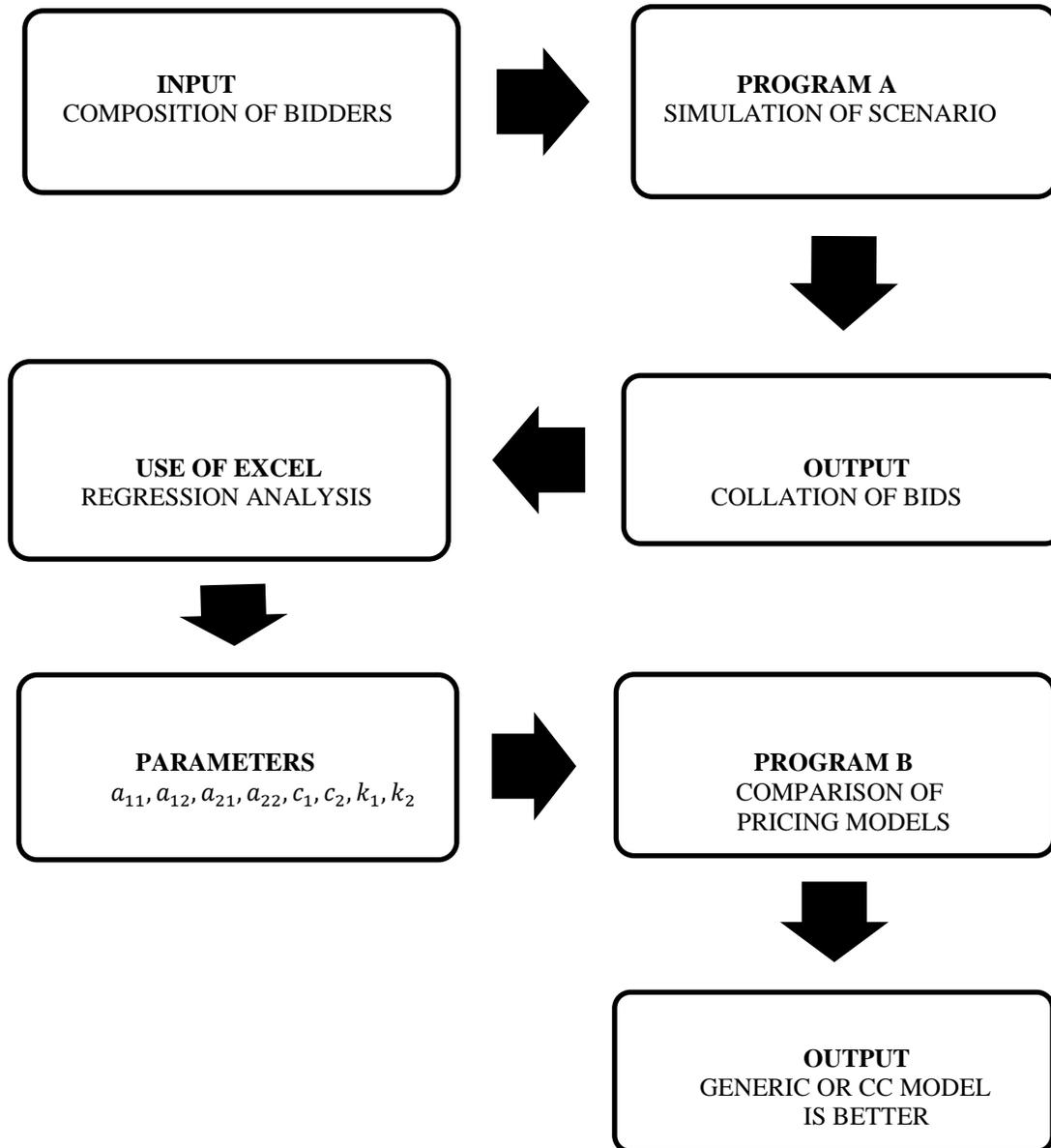GENERIC OR CC MODEL
IS BETTER

Figure 2: *Flowchart illustrating the process from simulation to revenue comparison.*

The results in the tables show significant improvements on optimal revenues at times when CC model is used (as shown by the highlighted values). This shows the superiority of the CC model over a generic one, not just theoretically, but in realistic (simulated) situations.

Study 3 – Summary of results

Run 1's parameters:
$a_{11} = -2.69464, \ a_{12} = 1.67961, \ c_1 = 300842, \ k_1 = 60,$
$a_{21} = 7.06666, \ a_{22} = -8.04625, \ c_2 = 233450, \ k_2 = 60.$

Table 1: *Comparisons between the two models for Run 1's parameters.*

LB_1	LB_2	Generic Model			CC Model		
		P_1	P_2	Revenue	P_1	P_2	Revenue
1800	1000	1800	1045.94	12408414	1800	1000	13982248
1900	1100	1900	1140.72	9848982	1800	1000	10850875
2000	1200	2000	1238.56	7905557	2000	1200	8603216
2100	1300	2100	1339.41	6408965	2100	1300	6943932
2200	1400	2200	1443.19	5241624	2200	1400	5689526
2300	1500	2300	1549.87	4320377	2300	1500	4721645
2400	1600	2400	1659.40	3585468	2400	1600	3961477

Study 7 – Summary of results

Run 1's parameters:
$a_{11} = -2.48558, \ a_{12} = 1.625796, \ c_1 = 156645.2, \ k_1 = 255,$
$a_{21} = 0.286548, \ a_{22} = -1.39228, \ c_2 = 456256.4, \ k_2 = 68.$

Run 2's parameters:
$a_{11} = -1.28132, \ a_{12} = 0.453397, \ c_1 = 127719, \ k_1 = 255,$
$a_{21} = 0.01, \ a_{22} = -1.13877, \ c_2 = 521378.4, \ k_2 = 68.$

Run 3's parameters:
$a_{11} = -3.11225, \ a_{12} = 2.27102, \ c_1 = 140820, \ k_1 = 255,$
$a_{21} = 0.01, \ a_{22} = -1.14067, \ c_2 = 524133, \ k_2 = 68.$

Table 2: *Comparisons between the two models for Run 1's parameters.*

LB_1	LB_2	Generic Model			CC Model		
		P_1	P_2	Revenue	P_1	P_2	Revenue
2300	1700	2300	2756.83	36072	2300	1700	92809
2300	1800	2300	2756.83	36072	2300	1800	83583
2300	1900	2300	2756.83	36072	2300	1900	74514
2300	2000	2300	2756.83	36072	2300	2000	65585
2300	2100	2300	2756.83	36072	2300	2100	56780
2300	2200	2300	2756.83	36072	2300	2200	48088
2300	2300	2300	2756.83	36072	2300	2300	39496

Table 3: *Comparisons between the two models for Run 2's parameters.*

LB_1	LB_2	Generic Model			CC Model		
		P_1	P_2	Revenue	P_1	P_2	Revenue
1900	1800	1900	2754.56	69594	1900	1800	76211
2000	1900	2000	2755.80	36267	2000	1900	67964
2000	2000	2000	2755.80	36267	2000	2000	59801
2000	2100	2000	2755.80	36267	2000	2100	51714
2100	2000	2100	2756.98	3425	2100	2000	59801
2100	2100	2100	2756.98	3425	2100	2100	51714
2100	2200	2100	2756.98	3425	2100	2200	43694

Table 4: *Comparisons between the two models for Run 3's parameters.*

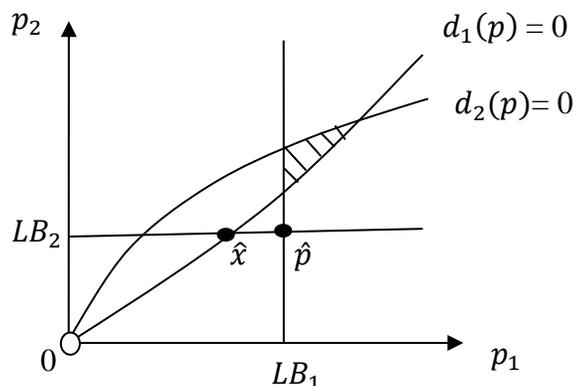| LB_1	LB_2	Generic Model			CC Model		
		P_1	P_2	Revenue	P_1	P_2	Revenue
2400	1600	2400	2736.69	40840	2400	1600	91146
2400	1700	2400	2736.69	40840	2400	1700	82736
2400	1800	2400	2736.69	40840	2400	1800	74430
2400	1900	2400	2736.69	40840	2400	1900	66216
2400	2000	2400	2736.69	40840	2400	2000	58083
2400	2100	2400	2736.69	40840	2400	2100	50025
2400	2200	2400	2736.69	40840	2400	2200	42032

Figure 3: *Optimal $\hat{p}$ outside Ω and $\hat{x}$ on boundary of Ω*

We can verify that in all the comparisons above, the CC model provides better revenues due to the following scenario. Let $\hat{x}$ be the vector where $d_1(\hat{x}) = 0$ and $\hat{x}_2 = LB_2$ (see Figure 3). Then with $LB_1 > \hat{x}_1$, the optimal solution for problem (2.4) is $\hat{p} = (LB_1, LB_2)$ and $\hat{x}$ as defined. The corresponding optimal revenue is then simply $LB_2 * d_2(\hat{x})$. However, the optimal price vector for the generic model (2.3) is restricted to the shaded region in Figure 3 and leads to lower optimal revenue.

Though it would be interesting to provide insight as to why studies 3 and 7 led to significantly better results for the CC model as seen above, we are unable to do so as it is not possible to find a direct relationship between the parameters found and the simulated behaviours. In addition, the possibility of having a situation as depicted in Figure 3 depends on the entire combination of parameters defining the function d and the given lower bound values. Thus the point of the different simulation studies is to obtain parameters based on realistic scenarios, and what we have done is to provide some possible cases where the CC model is better.

The programs written for the above two simulation studies can be downloaded under "Programs and Publications → Simulation of bids" from http://math.nie.edu.sg/wmsoon/price/. Note however, that Studies 3 and 7 above correspond to Studies 1 and 2 respectively in the given website. In addition, the reader may go to the same website under "Programs and Publications → Comparison of two pricing models" to obtain the program used to compare the revenues of the two models.

5. Conclusion

A pricing model can be used to help a seller price the products to be sold to customers. In order to maximize the revenue that can be possibly obtained, a seller needs to be careful with the choice of demand function form that reflects customers' responses to changes in prices of different products. Once the form of demand function is chosen, it is then important to consider the demand and price elasticity parameters that define a demand function. In a bid to compare the generic and CC pricing models, we have considered many different sets of parameters defining the demand functions and lower bound values on prices in earlier work. However, those

chosen parameters did not result from any approximation to real demand. Though we are not able to use real data in this work, our simulations emulate different types of customer behaviours and attempt to provide more reasonable average estimates of demand parameters. This enables us to make better comparisons of the pricing models. Our results show that the use of CC model should be encouraged as it can lead a seller to achieve better revenues as compared to a generic model.

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Appendix (Program for simulation study 7)

```
clc;
global BP;             % Bid Prices
minbid = 100;       % minimum bid
maxbid = 1600;      % maximum bid
N = 300;
CustProp = [1/3 2/3 1.0];
BT=[250 500 750 1000 1250 1500 1750 2000 2250 2500]; % Bidder tolerance
BP=zeros(2,N);
BP(1,1)=minbid;     % set minimum bid for Product 1
BP(2,1)=minbid;     % set minimum bid for Product 2
k1=1;               % bid number for Product 1
k2=1;               % bid number for Product 2
rand('state',sum(100*clock));
randn('state',sum(100*clock));

for i=1:N
  k1=k1+1;
  k2=k2+1;

  % determine tolerance (same for both)
  r=ceil(rand()*10);
  b=BT(r);

  % get current price of product 1
  p=BP(1,k1-1);
  % get current price of product 2
  q=BP(2,k2-1);

  % calculate increments (same only when p and q are equal)
  dp = p*r/200;
  dq = q*r/200;

  % calculate new price of product 1
  p1 = p + dp;
  % calculate new price of product 2
  q1 = q + dq;

  % Generate a random number s
  s=rand();

  % Decide whether customer type is
  % no preference (considers cheaper of the two)
  % or preference for product 1
  % or preference for product 2
  if s < CustProp(1)

    % considers cheaper of the two
    % case 1: p1 and q1 are equal
    % randomly choose product 1 or 2, provided within tolerance
    if (p1 == q1)
      s = rand(); % generate a random number
      if (s < CustProp(1)) % consider product 1
        if (p1 <= b) % price of product 1 within tolerance amount
          BP(1, k1)=p1; % make a successful bid
```

```
        file_1 = fopen('product1.txt','a+');
        fprintf(file_1,' %2d   %6.2f \n',1,p1); % write to file
        fclose(file_1);
        k2=k2-1; % Must remember to decrement other product's counter
      else
        k1=k1-1; % Must remember to decrement
        k2=k2-1; % both counters; no bid is made, go to next customer
      end
    else % consider product 2
      if (q1 <= b) % price of product 2 within tolerance amount
        BP(2, k2)=q1; % make a successful bid
        file_2 = fopen('product2.txt','a+');
        fprintf(file_2,' %2d   %6.2f \n',2,q1); % write to file
        fclose(file_2);
        k1=k1-1; % Must remember to decrement other product's counter
      else
        k1=k1-1; % Must remember to decrement
        k2=k2-1; % both counters; no bid is made, go to next customer
      end
    end
  elseif (p1 < q1) % case 2: p1 < p2
    if (p1 <= b) % price of product 1 within tolerance amount
      BP(1, k1)=p1; % make a successful bid
      file_1 = fopen('product1.txt','a+');
      fprintf(file_1,' %2d   %6.2f \n',1,p1); % write to file
      fclose(file_1);
      k2=k2-1; % Must remember to decrement other product's counter
    else
      k1=k1-1; % Must remember to decrement
      k2=k2-1; % both counters; no bid is made, go to next customer
    end
  else % case 3: p2 < p1
    if (q1 <= b) % price of product 2 within tolerance amount
      BP(2, k2)=q1; % make a successful bid
      file_2 = fopen('product2.txt','a+');
      fprintf(file_2,' %2d   %6.2f \n',2,q1); % write to file
      fclose(file_2);
      k1=k1-1; % Must remember to decrement other product's counter
    else
      k1=k1-1; % Must remember to decrement
      k2=k2-1; % both counters; no bid is made, go to next customer
    end
  end
 elseif r < CustProp(2) % preference for product 1
   if (p1 <= b) & (p1 - q1 <= 0.2*b) % price of product 1 within tolerance
                                       amount and not much higher than product
                                       2
      BP(1, k1)=p1; % make a successful bid
     file_1 = fopen('product1.txt','a+');
     fprintf(file_1,' %2d   %6.2f \n',1,p1); % write to file
     fclose(file_1);
     k2=k2-1; % Must remember to decrement other product's counter
   else
     k1=k1-1; % Must remember to decrement
     k2=k2-1; % both counters; no bid is made, go to next customer
   end
 else % preference for product 2
```

```
        if (q1 <= b) & (q1 - p1 <= 0.2*b) % price of product 2 within tolerance
                                 amount and not much higher than product 1
          BP(2, k2)=q1; % make a successful bid
          file_2 = fopen('product2.txt','a+');
          fprintf(file_2,' %2d   %6.2f \n',2,q1); % write to file
          fclose(file_2);
          k1=k1-1; % Must remember to decrement other product's counter
        else
          k1=k1-1; % Must remember to decrement
          k2=k2-1; % both counters; no bid is made, go to next customer
        end
   end
end

fprintf('Final Price for Product 1 = %6.2f \n',BP(1,k1));
fprintf('Final Price for Product 2 = %6.2f \n',BP(2,k2));
```