Fine-Tuning in a Design Experiment

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Quek, Tay, Toh, Leong, and Dindyal (2011) proposed that a design-theory-practice troika should always be considered for a designed package to be acceptable to the research users who, in this case, are teachers and schools. This paper describes the fine-tuning to the MProSE problem-solving design made by the teachers in the school after first round of teaching. This process involved teacher input from their experience, and detailed time-consuming discussions and learning between the researcher-designers and the teacher-implementers.

A mainstream school participated in our research project *Mathematical Problem Solving for Everyon*e (MProSE) (Toh, Quek, Leong, Dindyal & Tay, 2011). As part of the initialization phase of MProSE, the research team conducted a ten-hour professional development course for the mathematics teachers to use the MProSE design to teach mathematical problem solving. The Head of Department (HOD) of Mathematics, with the endorsement of the principal, coordinated and oversaw the implementation of the problem-solving module for four classes of Year 7 students (130 in total) over Term 1 and early Term 2 in 2012. The ten one-hour lessons were taught within the allocated curriculum time by four teachers, one to each class. These four teachers had different years of teaching experience. The HOD and these four teachers worked closely with the researchers.

The design experiment approach used in MProSE posits that a design-theory-practice troika should always be considered for a designed package to be acceptable to the final research users, who in this case are the teachers and the school (Quek, Tay, Toh, Leong, & Dindyal, 2011). The design could be fine-tuned for improvement based on the feedback by the implementers. However, the modifications due to the fine-tuning should stay within the parameters specified by the theory for the design. In the sections following we describe two rounds of fine-tuning. In the first round of fine-tuning, which took place while the module was being taught, the teacher-implementers were most concerned with the types of problems that were most suitable for their students. As such, the focus of the fine-tuning efforts was centered around discussion of problems—changing the problems as well as having deeper understanding of existing problems. In the second round of fine-tuning, which is in anticipation of the next run of the module, we also looked at other aspects of implementation, which include the overall scheme of work, the structure of lesson, the activities to develop positive attitudes in the students, and scaffolding measures.

First Round of Fine-tuning

Good and appropriate mathematical task design is a crucial component in teaching of the MProSE problem-solving lesson. A well-designed task is essential to developing students' abilities and confidence in handling unfamiliar problem situations. In designing the tasks for the first round of implementation of the MProSE design, the research team

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proposed a set of 17 problems. These problems were crafted based on students' mathematical competency (as reported by their teachers). The 17 problem-solving tasks were to be used in the ten lessons. A problem could be used as the Problem of the Day (POD), which is to be attempted by students in class, or as a Homework (HW) problem. For each task, the designer-researchers provided the problem-solving heuristics and scaffolding hints, which they derived from Polya's and Schoenfeld's work. The students attempted the task by using the *Practical Worksheet*. This worksheet, which is a key feature of the MProSE design, is described in Toh, Quek, Leong, Dindyal and Tay (pp. 21-24, 2011). The 17 problems, though prepared in advance by the designer-researchers, were not meant to be "cast-in-stone" but could be fine-tuned to fit the implementers' needs and constraints.

There were three modes of fine-tuning (a) discussion by teacher-implementers by themselves before each lesson was conducted, (b) discussion immediately after the MProSE lesson with designer-researchers, and (c) discussion via emails between HOD and designer-researchers. The examples that follow illustrate tweaks that result from each of these modes respectively.

The MProSE teacher-implementers held a pre-lesson discussion, which were not attended by the designer-researchers, to talk about how to solve the POD and to fine-tune the problem tasks and teaching approaches. The teachers reported that these pre-lesson discussions helped them to clarify doubts, build confidence, fine-tune the teaching strategies proposed by the designer-researchers and feel better prepared to facilitate the MProSE problem-solving lessons. For example, the teachers knew that their students had not learnt factorization by Term 1 to attempt the proposed POD for Lesson 2:

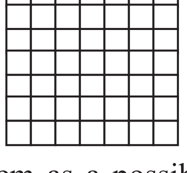
Simplify $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+...+\frac{1}{100\times 101}$, leaving your answer in the simplest form in terms of n. Justify your answer.

As a result of their discussion, this POD was replaced by the problem: Find the last digit of 13^{77}.

The designer-researchers would sit in different classes to observe the lesson and take field notes. A post-lesson discussion with the designer-researchers was held after each lesson to exchange perspectives and suggest further fine-tuning. The teachers would give views on their own teaching (e.g., hard for them to provide adequate scaffolding) and surface any observations they made (e.g., the students reported not liking to fill up the *Worksheet*).

We report here about an example of fine-tuning that resulted from an extended discussion. Lesson 5 focused on the Check and Expand stage. The teachers were not aware that the expansion to a problem could be linked to the structure of the solution to the original problem which in this case is:

HW5: The figure is a 7 × 7 array where each cell is a square. Find the number of squares contained in this 7 × 7 array.



Even though it was suggested to them as a possible extension—Find the number of rectangles in an $m \times n$ rectangular array—the teachers did not consider it because they could not see the link in the solution. The designer-researchers walked them through the solution by referencing the original solution to the number of squares, through step-by-step questioning. The following is a recall of the incident.

(Designer-researcher) DR : How did you solve the number-of-squares problem?
(Teacher-implementer) TI : By considering cases.
DR : Would you like to do the same for the number-of-rectangles problem?
TI : I don't see how it could be done…there are too many cases to consider.
DR : Just try…start with some cases …
TI : Let me see…is 1 by 1 a rectangle? I think so. There are 49. What's next … 1 by 2 … there are 7x6 rectangles … (starts recording on a piece of paper)… Now 1 by 3 … 7x 5 rectangles … Oh, I see a pattern now!! … (finally completes the solution, looking overjoyed and beaming with confidence).
If we had known this, we would have guided the students along similarly!

One teacher-implementer was candid enough to admit a lack of mathematical ability and knowledge. The post-lesson discussion seemed important for the teachers to understand the solution and the problem at a deeper level.

Typically a pre- and post lesson discussion among teachers would take an hour each. Thus sustaining these weekly discussions required dedication from the teacher-implementers and support from the school-leaders (Principal and HOD) in terms of time for discussion (as a professional learning community), curricular adaptations, and words of encouragement.

The last mode of discussion for fine-tuning the design was via emails. The HOD would email his concerns anytime to the research team. For example, in one email, the HOD sought advice from the designer-researchers on the choice of a HW question (see Figure 1 below).

> 2. We are also concern with the homework problem. Can we replace it with the average speed problem that we have proposed please ? (See homework slide).
> We hope that you would be able to advise us. We will be conducting the first run on Wednesday 0745-0845 with 1A and the second run on Thu 1415 - 1515.
> Your advice, please.
> Thank you.
> with regards,
> Name removed

Figure 1. An excerpt of email from the HOD to seek advice

The original HW8 problem was on the "Intersection of Two Squares" (for details of problem, refer to Schoenfeld, 1985). After discussion, the problem was changed to:

> Jeremy and James wondered which car has the highest average speed. Each of the cars, Ferrari and Maserati were to make two runs and the average speed will be measured.
>
> The Ferrari ran two lengths of 1000 m in 10 sec and 8.5 sec respectively.
>
> The Maserati completed two runs of 10 sec, covering 1000 m and 1175m respectively.
>
> Jeremy went on to calculate the speed for each run and took the average of the two speeds for each car. He found that the Ferrari was the faster car but James thinks that Jeremy made some mistakes in his calculations. Was Jeremy's calculation correct? When will average speed be the average of two speeds? Why?

As a result of these three modes of discussion, three proposed problems were changed and one not used.

Fine-Tuning for the Second Round

After the first run of the module, the teachers were generally positive about the outcome and had decided to carry it out again for the next cohort of Year 7 students. In between the two implementations, we intend to enter into intense discussions on the second round of fine-tuning. At the time of writing, two sessions had been conducted. Each session took about two hours. The discussions were in such great detail that Teacher S remarked that "the fine-tuning process in determining whether a problem is suitable for the students or not is not that clear-cut and [is] time-consuming". He added that "thinking how to help students in class and explain clearly to the students is rather challenging and the different types of problems we need to discuss benefit me". We acknowledge that the fine-tuning process is resource-intensive. In fact, we think it needs to be necessarily so in order to bring about deep understanding of changes through the teacher development process.

We state briefly here the changes that were made for the next round. First, they evaluated the suitability of having the MProSE lessons conducted in the "Supplementary lessons" slot—what was done in the first run, after usual lesson hours in the afternoon. The school had since decided that MProSE is such a vital piece in their mathematics instructional programme that they would carry it out during normal lesson times. Next, to better equip the Year 7 students with more cognitive resources for problem solving, the module will be taught in Term 2 instead of Term 1 and end in middle of Term 3. This is to allow students to have opportunities to accumulate relevant content knowledge in the first term. Teachers found that this is beneficial also from the perspective of having more time for preparations. A new suggestion put forward by the researchers was the use of colour scaffold stickers to differentiate the different levels of cognitive scaffold that a teacher may need to give to the students. They agreed to try out this innovation. [In fact, the scaffolding cards were tested in another MProSE school and the preliminary results are reported in another paper in this symposium]. There were also more discussions about suitability of problems. For example, the "Phoney Russian Roulette" problem was deemed inappropriate due to students' lack of pre-requisite knowledge of probability, and was replaced by the problem of "finding the sum of all the digits from 1 to 99".

Conclusion

In this paper, we report the support given to teachers as we fine-tune various aspects of the MProSE design. It is resource-intensive, and no less than the "hard and unglamorous work" that Schoenfeld (1985) envisioned. But from the learning and changes that we observed, we are encouraged to know that it is also necessary and far-sighted work. In order to cater to the specific needs of students to improve their problem-solving ability and to empower the teacher-implementers, the fine-tuning of the MProSE design is integral to a sustainable and effective implementation of the problem solving module. The iterative nature of the design experiment methodology has made the participating teacher-implementers aware that it takes time and effort to understand the learning needs of the students.

References

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