# School Word Problems and Stereotyped Thinking

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## INTRODUCTION

The traditional approach of school instruction on arithmetic has often been to develop a simplified form of word problems to be used as tools for pupils to practise application of the four basic operations. These word problems are formulated with a concise set of data which enables pupils to concentrate on the main task of choosing the mathematical operation that would lead them to the solution. Such standard word problems as they are formulated, induce in the pupils an unwanted rigidity where their responses become very mechanical. When these pupils read a word problem they tend to disregard the actual situation described and instead, they go straight into exploring the possible combinations of numbers to infer directly the needed mathematical operations.

This strategy to find corresponding mathematical operations and to calculate numerical answers is continuously fostered at school. Worksheets and exercises are given in great amount where pupils are encouraged to learn artificial lexical clues and the use of analogy with previously successful and seemingly similar problems. Such short cuts in mastering a skill do not train pupils to solve real world problems but instead induce a stereotyped process of thinking. The result is that pupils begin to view mathematics learning as limited to acquisition of formal algorithmic procedures. They fail to understand the essence of applied arithmetic at school and develop defensive strategies and a kind of phobia which usually end in poor performance. The negative effects of the use of stereotyped word problems are apparent when pupils are asked to write stories for mathematical sentences. For example, given a mathematical sentence such as $0.74 \div 0.21$, some children have been reported to write stories such as:

*Emma had 0.74 sweets.*
*She wanted to share them out so each doll got the same.*
*There was 0.21 dolls. How many did each doll have?*

(Bell et al. 1984, p.139)

These children are unable to connect calculations with appropriate contexts in which they can be embedded. There are also children who attempted to overcome the impasse by converting the problem into a form with which they were familiar. Hence, for $4 \div 24$, these children wrote problem statements such as:

*Mary has 4 lbs apples.*
*She has got to share them between her and her 23 friends.*
*How many apples do they get each? (12 apples to a lb)*

(Bell et al. 1984, p.138)

Standard word problems at school which often do not resemble problems in real situations are constructed by teachers, and these problems are not considered by the pupils themselves to be related to the real world. Many studies have shown that the use of standard word problems in the classroom hinders pupils' reasoning and problem-solving ability as it encourages pupils to exclude real-world knowledge and realistic considerations from the solution process (Davis 1989; Greer 1993; Verschaffel, De Corte & Lasure 1994). Pupils who are used to practising the solving of arithmetic word problems in the style of "school-problems" are not given the opportunity to be engaged in qualitative decision-making through real-world problems. The following is an example of a word problem on rate and proportion found in a common Secondary One textbook used in our schools.

*Given that a typist can type 575 words in 25 minutes, how long will she take to type*

*(a) 3 680 words*
*(b) 8 855 words?*

Teachers and textbook writers may consider such example a real-life problem but there are unrealistic assumptions underlying it. It is assumed that a typist can type 3 680 words or 8 855 words at the same speed as she can type 575 words. Secondary one pupils would have a common-sense knowledge about productivity and have experienced the effects of fatigue on performance but how many of them would transfer the knowledge to the solution of this word problem? How many of them would query the problem statements and the answers given in the textbooks? How many mathematics teachers would actually discuss these problematic modelling assumptions in class?

One of the best documented cases of student failure to apply common-sense knowledge in solving problem is the bus problem in the Third National Assessment of Educational Progress (NAEP). The question reads as follows:

*An army bus holds 36 soldiers.*
*If 1128 soldiers are being bussed to their training site, how many buses are needed?*

Approximately 70% of the sample of pupils tested were able to perform successfully the appropriate arithmetical operation and obtain a quotient of 31 and a remainder of 12. Only 23% of the pupils were able to proceed to deduce that the number of buses required is 32. Most pupils gave an answer (e.g. 31 remainder 12 or 31) that did not make sense when the problem statements were taken seriously. Schoenfeld (1991) described these students as engaging in suspension of sense-making. They failed to see that the solution did not make sense in relation to the given problem statements. Instead, they applied stereotyped thinking and solved the problem with what they thought was the "appropriate" operation rigidly.

If pupils are given opportunities to engage in real-world problems where qualitative analysis of the problem situation requires that they examine alternatives rather than mere calculation with numerical data, then absurd answers to word problems with realistic considerations can be avoided. Using the above mentioned example about the time taken by a typist to type a number of words, the same problem can reformulated in real life, e.g.

*If you have a 2500 word essay to type for your assignment, how long will it take you?*

The presentation of this problem does not involve any given numerical data. The pupils' main task is to find the relevant information for the solution of the problem. In looking for the relevant information, the pupils would have to make some qualitative decisions, for example, decisions concerning when is the best time of the day to do, whether a word-processor is available and when he or she is most productive, etc.

It is the contention of the authors that pupils in most Singapore schools are not provided with classroom activities that connect in-school and out-of-school mathematics to enrich their problem-solving skills. Instead, there is a great reliance on school textbooks which contain multitudes of standard word problems for exercises to reinforce the kind of stereotyped thinking that is detrimental to the development of pupils' true mathematical learning.

These textbook problems do not encourage pupils to use their intuition and mathematical knowledge when they encounter real problem-solving situations in everyday life. Through the textbook problems, pupils are taught the strategies for translating the problem statements to the mathematical representation and the procedural skills for finding the answers. Many pupils terminate their problem-solving process here. When reminded to check their solutions, pupils often limit their checking to the computations of the answers. They seldom apply their common-sense knowledge and make realistic consideration about the problem context.

## DO SINGAPORE PUPILS APPLY COMMON-SENSE KNOWLEDGE IN SOLVING REALISTIC PROBLEMS?

In a recent study by Koay and Foong (1996) of 156 secondary one pupils and 148 secondary two pupils in Singapore, it was found that more than 80% of these subjects had a strong tendency to apply stereo typed thinking in solving "realistic" word problems and consequently, solve them as standard word problems. These pupils came from eight neighbourhood schools and they all have had continuing experience with standard word problems in class. Neither their mathematics teachers nor their mathematics textbooks have addressed the issue of realistic considerations in word problems and stereotyped thinking in the application of arithmetic operations. This finding has important implications for teaching of the revised school mathematics syllabus which places emphasis on developing problem solving and higher-order thinking skills in our pupils.

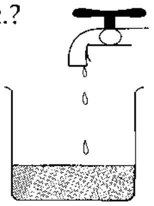
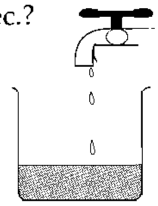
In this study, the pupils were given a test instrument consisting of eight pairs of items (see Table 1), modified from similar research (Greer 1993; Verschaffel et al. 1994). Each pair consisted of a standard word problem and a realistic problem. A standard word problem is one that is often found in school mathematics textbooks. It is usually context-free where the correct application of one or more arithmetic operations on the given numbers will lead to a correct solution. A realistic word problem is one in which pupil needs to take into consideration the realities of the context of the problem statements. Correct application of arithmetic operation(s) will not lead to the correct solution to such context-dependent word problem.

Except for Question 3 in the test, all the other realistic word problems elicited approximately 80% or more responses which indicate that the majority of the pupils in the sample applied stereo typed thinking with straightforward arithmetic operations without considering the realistic constraints of the problem situations. Pupils in this study generally did not activate their real-world knowledge in solving arithmetic word problems. Evidence that pupils are able to recognize the realistic constraints in a problem that depends on the problem situation is shown only in Question 3 about sharing of balloons.

Among the five problem situations, relatively more pupils (59.9%) applied their realistic knowledge in this instance. It can be hypothesized that pupils are very familiar with an object like a balloon which can easily burst when pricked without having to imagine what will happen if one were to "share" it in parts. This finding is consistent with that reported in Greer's study (1993). In his study, Greer reported that most pupils in his sample were able to recognize that it is not appropriate to divide a balloon in half. Similarly, a large majority of his pupils were also not aware of the realistic constraints in the other problem situations.

**Table 1:** Examples of Standard and Realistic Problems

| Problem Context | Standard Form | Realistic Form |
|---|---|---|
| 1. Number of guests | David gave a birthday party. He invited 15 boys and 11 girls. How many children did David invite for his party? | May Ling has 7 friends and May Lee has 9 friends. They gave a party together at the end of the term. They invited all their friends. All friends were present. How many friends were there at the party? |
| 2. Running Time | The average speed of a train is 50 km per hour. How long will the train take to travel 250 km? | Ahmad's best time to run 100 metres is 13 sec. How long will it take for him to run 800 metres? |
| 3. Sharing | If there are 14 bars of Kit-kats for 4 children at a party, how should they be shared out? | If there are 14 balloons for 4 children at a party, how should they be shared out? |
| 4. Distance | Ali went for a nature walk. He walked 7 km before he rested for 15 min. He then continued walking 12 km to reach the end. What was the distance of the trail? | Ali and Ah Meng go to the same school. Ali lives at a distance of 24 km from the school and Ah Meng at 12 km. How far do Ali and Ah Meng live from each other? |

| | | |
|---|---|---|
| 5. Filling flask | This flask is being filled from a tap at a constant rate. If the depth of the water is 2.4 cm after 10 sec, how deep will it be after 30 sec.?  | This flask is being filled from a tap at a constant rate. If the depth of the water is 2.4 cm after 10 sec, how deep will it be after 30 sec.?  |

The followings are excerpts of some the pupils' responses to Question 3 and three of the other problems.

Q3. *If there are 14 balloons for 4 children at a party, how should they be shared out?*

Most pupils (59.9%) recognised that sharing balloons by dividing the remaining two in half was not practical and gave the realistic answer 3. 19.1% of them gave sensible suggestions on how to dispose the remaining two balloons. Their suggestions include bursting the remaining two, giving the remaining two to the youngest or the "best" child, throwing the remaining two away, or keeping the remaining two for future use. 18.4% of pupils did not give any answer, but pointed out that 14 cannot divided by 4, there would be a remainder of 2.

Among those who gave other responses, six pupils appeared to be aware that it would be impossible to give the same amount of balloons to each of the 4 children. They gave the answer as "2 get 4 and 2 get 3". One pupil rounded the quotient of 3.5 to 4 instead of 3. Altogether, there were only 4.1% of the responses that did not indicate the impracticality of cutting a balloon by half for distribution.

Q1. *May Ling has 7 friends and May Lee has 9 friends.*
*They gave a party together at the end of the term.*
*They invited all their friends.*
*All friends were present.*
*How many friends were there at the party?*

Like the pupils in the Belgium study (Verschaffel et al. 1994), none of the pupils in the present study gave a realistic answer for this question. A realistic assumption in this case would be that the two girls are good friends and they have mutual friends. Hence, the realistic response would be "cannot be determined". 84.7% of the pupils in this study gave the expected answer of 16 by finding the sum of the two given numbers.

Some of these pupils commented that the question was "very easy". Others did not write anything in the "answer area" but pointed out that May Lee and May Ling might be friends or one thought they were sisters. About 12% increased the sum of 7 and 9 by two as some argued that the two girls were friends and should be counted as well.

Q2. *Ahmad's best time to run 100 metres is 13 sec. How long will it take for him to run 800 metres?*

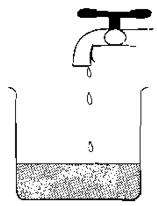
79.0% of the pupils gave the expected answer, 104 seconds or 1 minute 44 seconds. Only 3.8% of the pupils gave the realistic answer of more than 104 seconds. They seemed to be aware that it is impossible to run 800 metres and 100 metres with the same speed. The two comments obtained are as follows:

*By the time he finishes, he would be tired and will slow down.*

*Ahmad's speed might not be constant as he has 800 m to run.*

1.3 % of the pupils did not give any answer or comment while 5.10% did not give any answer but commented either about the inability of a runner to maintain the constant speed for the longer distance or a slower speed would be used because of the lack of stamina. Among the other responses were incorrect number sentences such as the product of 800 and 13.

Q5: *This flask is being filled from a tap at a If the depth of water is 2.4 cm. After10 seconds, how deep will it be after 30 seconds?*



As expected, most of the pupils' responses (86.6%) were in accordance with direct proportionality. The pupils ignored the shape of the flask. It is plausible to assume that most lower secondary pupils are aware that the level of water will not rise at a constant speed in a cone-shaped flask. Yet very few pupils in the study were able to activate this physical knowledge in the context of mathematics word problem. Only one pupil gave a realistic answer of "more than 7.2 cm". However, he did not comment on his response. 1.27% of the pupils did not respond to this problem while 5.73% left the "answer area" blank but gave sensible reasons why there was no solution to the problem. The reasons include:

*...the flask is of irregular shape...,*
*...the flask is conical...,*

*...the side of the flask is not at 90°...,*
*...the flask is not straight...*

## IMPLICATIONS AND SUGGESTIONS

The results of this study on Singapore pupils to investigate their application of common-sense knowledge in solving realistic problems, indicate that somewhere along the way in the teaching of arithmetic word problems in schools, we have failed to recognize the importance of making connections between school mathematics and everyday life. Word problems at school are considered by pupils to be unrelated to reality. Teachers set well-defined word problems where pupils do not have to be engaged in making qualitative and quantitative decisions. To the pupils, what is important is to be able to recognize familiar key words to select the appropriate operation and do some computations from the given data to produce an answer, no matter how unreasonable it can be. This kind of stereotyped thinking hinders the learning of applied arithmetic which is one of the most significant realms of mathematical thinking for future living and which is considered a basic skill for the average citizen.

On the other hand, studies (Carraher et al, 1983) have shown that pupils can often solve quantitative problems of addition and subtraction in everyday life situations but fail to solve similar problems when given word problems at school. As shown in this study, we cannot expect pupils to make these connections spontaneously. Pupils must be shown how school mathematics can be applied to real world situations, and they need to be encouraged to avoid surface processing of information given in problem text. They have to be exposed to different types of word problems in school, in particular, the so-called context problems that invite and sometime force them to use their real-world knowledge and personal experience to analyse and solve the problems. Pupils would be more interested in mathematics if they can see how mathematics is used in their daily live. Many problems found in the textbooks are unreal. These problems would not motivate pupils to solve problems, they only confirm what the pupils' belief that mathematics has nothing to do with the real world. To change this belief, greater attention must be given to establishing the links between school mathematics and the real-world settings.

This call for a modification in the instruction of the arithmetic word problems in school. It is essential that teachers provide classroom activities that connect school mathematics and everyday life to enrich pupils' mathematical experiences. Teachers can also transform textbook exercises into problems that require students to use their mathematical knowledge in ways that reflect how that knowledge is used in everyday situations. For example, when teaching operations with numbers, both standard word problems and realistic problems should be discussed in class. Pupils should be given more open-ended problems formulated as in real life, e.g.

> *How much will it cost each pupil if the school organizes a day-trip to Sentosa?*

The verbal formulation of such open-ended problem does not provide any clues concerning the required mathematical operations and there is no unique answer but a set of options where pupils have to make both qualitative and quantitative decisions. In teaching a topic like percentages, many secondary textbooks contain ideas that involve service charges, GST or profit and discount for purchases and sales contexts. Teachers can use these contexts which are rich with everyday situations to design non-elaborate and yet motivating activities to help pupils connect mathematical ideas from the classroom to out-of-school situations which are related to them. Pupils can be asked to plan a monthly budget for their own families based on the income of their parents. What are the constraints they have to face, how careful they must be with their expenditures and any savings left for a raining day, etc. Such problems should not be meant only for the "better" pupils and are best done in groups where pupils are empowered to apply the basic concepts and skills taught as well as to communicate their own ideas and thinking.

In teaching problem solving, teachers often have a misguided idea that if they can come up with a set of strategies that pupils can follow to solve mathematics problems, then pupils will be well equipped to solve problems. In the textbook context, each arithmetic problem has one and only one solution. The usual problem solving strategy of "read and understand, plan, carry out the plan and check" would help pupils solve these problems. However, it might not help them solve realistic problems which are often unsolvable, have no precise answer or have more than one solution. Hence, teaching problem solving should also include teaching pupils to recognize the assumptions and constraints underlying different problem situations and use reasonable judgement to obtain reasonable solutions. In addition, pupils should be given opportunities to talk about the problems and their solutions.

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