Some Common Misconceptions in Algebra

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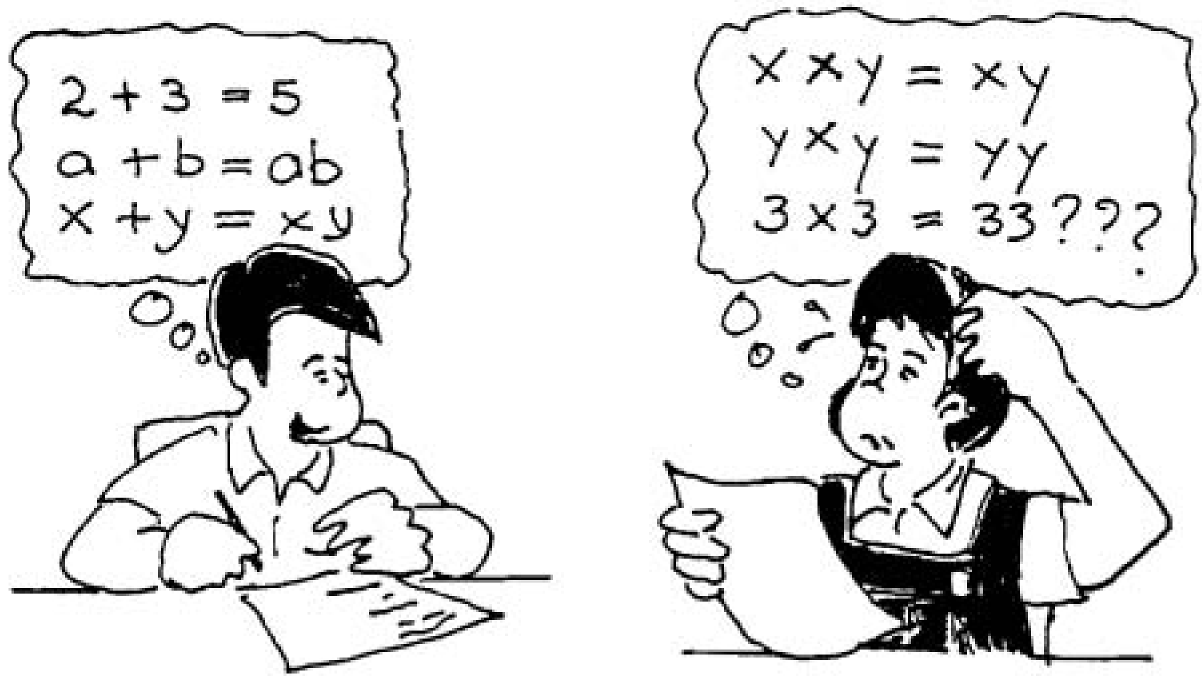


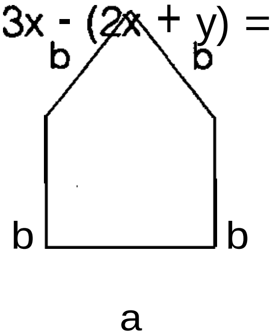
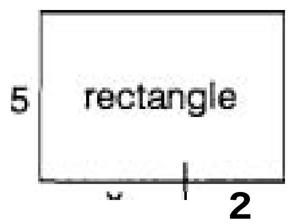
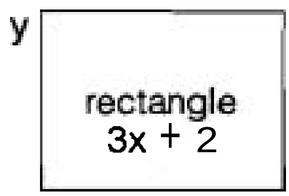
"Algebra has acquired a reputation amongst teachers, pupils and parents alike, as one of the most difficult and troublesome courses in the secondary curriculum."

Kinney, L. B. & C. R. Purdy **Teaching Mathematics In the Secondary School** (New York: Rinehart, 1952 p. 59)

25 Secondary Three pupils in the express stream from a neighbourhood (co-educational) school in Singapore were given an exercise on algebra (see appendix) to do. Sufficient time was given to every individual to complete the exercise.

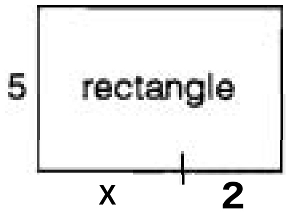
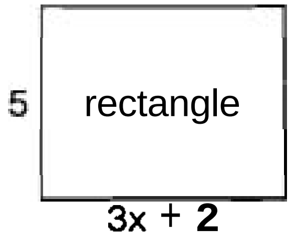
The answer scripts were carefully checked and the incorrect responses of the pupils noted. The analysis of the algebra exercise which centred on a narrow range of algebraic items did shed light on the following areas of difficulty, namely;

1. **Conjoining in algebraic additlon**

Q.	item	"Error" answers	% giving error answer
1(b)	$2x + 3y =$	$5xy$	24%
1(c)	$x + y + 2x =$	$3xy$	12%
1(d)	$2x + 5y - 3x =$	$4xy$	8%
1(e)	$(2x - y) + y =$	$2xy$	12%
1(f)	$3x - 2x + y =$	xy	16%
1(g)	$3x - (2x + y) =$	xy	8%
2(b)	 P = ________	$4ab$	16%
3(b)	 A = ________	$10x$ $7x$	8% **4%**
3(c)	 A = ________	$5xy$	12%

This may have been a result of the desire to obtain an 'answer' (single-term) to algebraic expressions/problems pupils encountered.

2. The non-use of brackets

Q.	item	"Error" answers	% giving error answer
3(b)	 A = ________	x + 2 x 5 5 x x + 2 5x + 2	*8%* **4%** *8%*
3(c)	 A = ________	3x + 2 x y y x 3x + 2	*28%* *20%*

A significant proportion of pupils who did the exercise were either ignorant of the use of brackets or chose to ignore the use of brackets mainly because they considered them unnecessary and took it for granted that operations are performed from left to right.

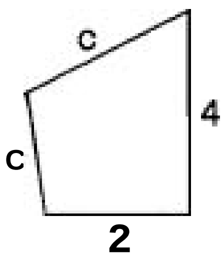
3. The wrong meaning attached to brackets

Q.	item	"Error" answers	% giving error answer
1(e)	$(2x - y) + y$	$2xy - y^2$	12%
1(g)	$3x - (2x + y)$	$-6x + 3xy$	8%
		$-5x - y$	4%
1(h)	$(x+y) + (x-y)$	$x^2 - xy + xy - y^2$	16%
		$x^2 - y^2$	20%
1(i)	$(3x+2y) - (x-2y)$	$2x - 6xy - 2xy - 4y$	8%
		$5xy - 2xy$	4%
		$3x^2 - 2xy - 6xy - 4y^2$	16%

It appears that some pupils have the misconception that 'brackets indicate multiplication'.

4. The meaning attached to letters

While pupils did accept letters representing numbers, at times they handled the numbers as mere entities rather than quantities. This was particularly apparent in answers to question 2c.

Q.	item	"Error" answers	% giving error answer
2(c)		$2c + 12 + 14$	4%
		$1c + 1c \quad 1(2) \quad 1(4)$	
		2c, 14, 12	4%
		2c, 4, 2	8%
	P = ______		

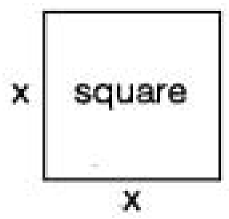
and in more abstract examples such as the simplication of

$$2x + 3y \overset{?}{=} 5xy$$

$$2x + 5y - 3x \overset{?}{=} 4xy$$

where pupils did not interpret the letters at all, but merely 'symbol pushed' using a self-invented rule 'add all the numbers and then write down the letters'.

80% of the pupils gave the following response to question 3(a).



$A =$ ______

$A = xx$

This may have been a consequence of applying the convention that a x b = ab and the ignorance of the use of indices.

Only 2 questions i.e. 1(a) and 2(a) in the whole exercise registered a 100% facility index.

The implication of all of the above would seem to be that we need to devote more attention to the way in which algebraic operations are recorded.

In science lessons the child learns that the addition of oxygen (O) to carbon (C) under certain conditions produces carbon monoxide (CO); the notation C + O -> CO is correct in this context.

When the mathematicians of ancient Rome wanted to write "the number which is three more than five", they wrote the symbols of the two components side by side, as VIII. Today we still make use of this primitive form of coding when we write $6^1/_2$ for the sum of the unlike terms 6 units and 1 half unit.

The travel agent writes 0325 as the departure time of your plane, and you know this is read as 3 hours and 25 minutes. The sum of seven dollars and thirty-five cents is often said "seven thirty five". There is ample justification for the pupil coming to the generalisation xy means x and y.

Teachers in the classroom must stress the fact that the language of generalised arithmetic and algebra is not obvious and intuitive. It is a formal code which has to be learned and practised.

Appendix

Algebra Exercise

Name: ____________________ Class: ______________

School: ____________________

Answer all questions

1. $x + 3x$ can be written more simply as $4x$.

Write the following more simply, where possible:

(a) $2x + 4x =$

(b) $2x + 3y =$

(c) $x + y + 2x =$

(d) $2x + 5y - 3x =$

(e) $(2x - y) + y =$

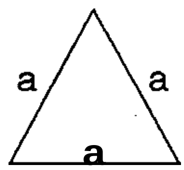
(f) $3x - 2x + y =$

(g) $3x - (2x + y) =$

(h) $(x + y) + (x - y) =$

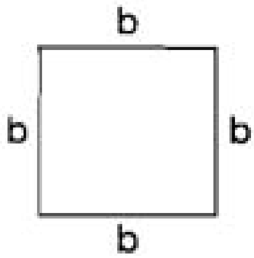
(i) $(3x + 2y) - (x - 2y) =$

2. This triangle has sides of length a, so its perimeter is $p = 3a$.

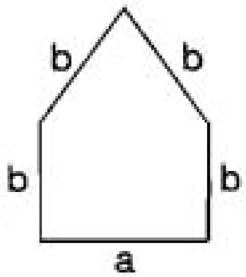


Write down the perimeter of each of the following figures.

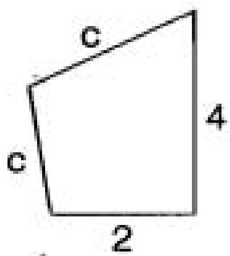
(a)



(b)



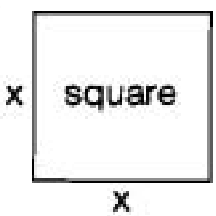
(c)



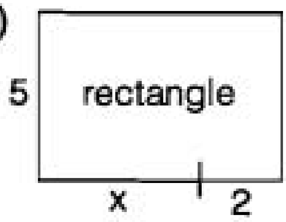
P = ______ P = ______ P = ______

3. Write down the areas of the following figures.

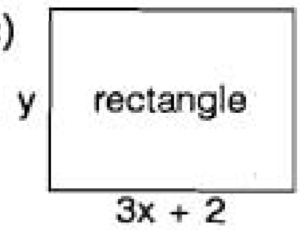
(a)



(b)



(c)



A = ______ A = ______ A = ______