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Source	*The Mathematics Educator, 17*(1), 79-103
Published by	Association of Mathematics Educators, Singapore

The Mathematics Educator
2017, Vol. 17, No.1, 79 - 103

Uneven Teacher Learning in Lesson Study: Towards a Theory of Teacher Learning

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Abstract: In this article, we confront the (hardly acknowledged) issue of uneven learning among members of Lesson Study teams. The site of research is a Singapore secondary school where the mathematics department had been involved in a number of Lesson Study projects over a five-year period. Based on interviews with two mathematics teachers who had different backgrounds and histories of participation in these projects, we examine the trajectories of their learning in a most recent Lesson Study on Number Patterns. We conjecture lessons for teacher development in relation to the framing concept of "standpoints".

Keywords: Lesson study, number patterns, teacher learning, mathematics problem solving

Lesson Study as a form of professional development for mathematics teachers

It is widely recognized that *Lesson Study* was popularised beyond Japan by Stigler and Hiebert (1999). In their book, the *Teaching Gap*, they highlighted the "gap" between mathematics teaching in the United States of America (USA) and in Japan. In particular, they surmised that the pervasive use of *Lesson Study* in Japan can serve as a reference for teacher development practices in the USA. Since then, there has been a proliferation in the use of *Lesson Study* in North America (Fernandez, 2005) across all education levels, from Elementary schools (eg., Taylor, Anderson, Meyer, Wagner, & West, 2005) to Tertiary institutes (eg., Alvine, Judson, Schein, & Yoshida, 2007). At the same time, *Lesson Study* was imported into a number of other countries. In the book *Japanese Lesson Study in Mathematics* edited

by Isoda, Stephens, Ohara, and Miyakawa (2007), there are reports of *Lesson Study* practices conducted in Thailand, Philippines, Cambodia, Laos, Indonesia, Egypt, Kenya, Ghana, South Africa, and Honduras. Based on more recent publications, *Lesson Study* remains a common enterprise in Asia (e.g, Leong, Kaur, & Kwon, 2017). A practice so widely adopted is surely a subject worthy of careful examination.

One would expect that such widespread practices would have evolved quite differently. Nevertheless, there are distinctive characteristics of *Lesson Study* that remain common across jurisdictions. One important feature is the observation and revision of a lesson by a team of teachers. This practice is premised upon the idea that "teachers can best learn from and improve their practice by seeing others teach" (p. xvi, Isoda, Stephens, Ohara, & Miyakawa, 2007). Another characteristic is the collaborative nature of the enterprise. "The significance of Lesson Study is that all these processes are performed in *collaboration* with other teachers" (p. 2, Baba, 2007, emphasis added). Teachers within the team identify goals and unit of study together; throughout the subsequent process of working towards refining a plan for the Research Lesson, teachers seek to draw on one another's expertise to arrive at a lesson design they can jointly own for implementation.

The key steps[2] of the *Lesson Study* process are summarized as follows:

1. *Identifying the problem and setting goals.* The first step of the process is for teachers to identify existing problems or difficulties encountered in their instructional practice. Goals for the team are then formulated. These goals can be more broad-grained or long-term—such as for the purpose of whole-school improvement—or more fine-grained and specific—such as improvement of teaching for a particular topic or unit.
2. *Designing the research lesson.* The team meets to design the lesson plan, including lesson objectives, sequences, and materials to be used in class.

[2] The actual process varies across different *Lesson Study* advocate groups. The steps listed here represent a sequence that is shared by most groups. For details on variants of the process, the reader is advised to consult the following: Lewis (2002); Stepanek, Appel, Leong, Mangan, and Mitchell (2007); Wang-Iverson and Yoshida (2005); Wiburg and Brown (2007).

3. *Teaching, observing, and refining.* Usually, one member of the team carries out the lesson while others take on different specific roles during observation of the lesson. Meeting(s) will then be held to reflect on the lesson and refine aspects of it. The focus of these post-lesson meetings is not on the actions of the teacher carrying out the plan; rather, the purpose is primarily to improve the lesson plan. Often, the refinement of the plan is carried out in "Cycle 2", looping Steps 2 and 3.
4. *Sharing of results*. The learning points of the team are shared, consolidated, and then disseminated to a wider audience of professionals in the form of either written reports or by verbal presentations.

There is also recognition within the *Lesson Study* setup of the need to tap on knowledge beyond local teacher communities. *Lesson Study* teams usually invite other professionals known as "knowledgeable others" (Lewis, 2002; Stepanek, Appel, Leong, Mangan, & Mitchell, 2007; Watanabe & Wang-Iverson, 2005; Wiburg & Brown, 2007). However, these authors view knowledgeable others (KO) as "outside the *Lesson Study* group" (Watanabe & Wang-Iverson, 2005) and as experts who "add value to [teachers'] work" (Stepanek, Appel, Leong, Mangan, & Mitchell, 2007). This portrayal of the role of KOs seems closer to the "guest" role (Oshima et al., 2006) than to the vision of "authentic partners" advocated by Hiebert, Gallimore, and Stigler (2002).

Determining the role of the KO in *Lesson Study* is a delicate matter: On one hand, if the KO assumes a central role to the point of completely driving the agenda, then the distinctive flavor of *Lesson Study* being a teacher-driven professional development platform will be compromised; on the other hand, if the KO is sidelined to merely a passive observer, he (or she) is unlikely to induce significant shifts in teachers' knowledge about the content or the pedagogy.

We take the view that the role of KO should avoid either extreme. It should safeguard both agendas—that of KO introducing knowledge that is useful to instructional innovations and that of retaining ownership of the innovations by the teachers in the team. The actual positioning of the role on the continuum between the poles would depend on local factors—such as the

capacity of the team and the history of collaboration between the school teachers and the KO. We think that teams who are novices to the *Lesson Study* enterprise or are not strong in the subject matter may require more intervention from the KO. As the team matures over a number of productive *Lesson Study* experiences, the KO can then shift his/her role to one that is closer to the middle of the continuum: that of genuine authentic partners.

Uneven teacher learning within *Lesson Study* teams

Although purveyors of *Lesson Study* present the vision of joint learning among team members—that is, every teacher in the group is actively involved in the module/lesson planning process and co-owns the outcomes, the reality is that it is hard to achieve equal ownership levels among the teachers.

Since literature on *Lesson Study* does not address this issue of uneven teacher learning, we draw upon our experience in working with numerous *Lesson Study* projects in our capacity as KOs. We observed this phenomenon: Members who would carry out the Research Lesson(s)—in any of the cycles—take strong ownership of the whole enterprise. This may have to do with the strong sense of wanting to implement a high quality lesson—especially when they would be observed by their peers, even superiors. It is therefore natural that they put in a lot more commitment to the process, resulting in significant learning for themselves.

In contrast, other members of the team (henceforth referred to as "supporting teachers") do not have to face the 'test' of their learning in the form of teaching the Research Lesson(s). Thus, it is not uncommon to detect a distinct difference in their levels of commitment and ownership. At its worst, these teachers see themselves as assuming a mere fringe role in the whole enterprise.

It is with this knowledge that we think that a study of their learning trajectory is all the more pertinent. Scarce literature which acknowledges such ownership divides—let alone deep analysis of different learning pathways—provides an added motivation for the research reported in this paper.

This study is thus about teacher learning within *Lesson Study* that has become a stable professional development structure among the mathematics teachers in the project school. In particular, we examine the learning trajectory of the supporting teachers in the context of participation in one *Lesson Study*.

Background

We had a longstanding working relationship with the research school. At the point of writing this paper, we had collaborated with them for over eight years. We worked together on a number of teacher development and instructional innovation projects—all built on the *Lesson Study* platform—which included concretising quadratic factorisation for Year 8 students who struggle with algebraic fluency, the use of Dynamic Geometry Software in the teaching of circle properties for Year 9 students, and teaching number operations involving negative numbers for Year 7 students in the [3]Normal Academic (NA) stream. Details of some of these projects can be found in Leong et al. (2010), Leong, Yap, and Chia (2011), and Leong et al. (2014).

From our perspective, this longstanding collaboration generates goodwill and trust between us and the mathematics teachers of the school which are necessary ingredients for embedding of innovations and sustainable teacher development practices. In other words, our working model with the school is one which situates *Lesson Study* within a broader context of sustainable teacher learning.

Lesson Study on problem solving disposition in number patterns

The research reported in this paper is based on the latest (at the point of writing) *Lesson Study* we conducted with some mathematics teachers in the same project school. The team identified problem solving as an area that needed to be attended to for the Year 7 NA students. We were conscious that, to help this group of students develop abilities in solving novel

[3] In Singapore, students who have completed primary education are channeled into three ability streams, according to their performance in the national examination. The streams are known as Express, Normal Academic, Normal Technical, and the percentage of students in each of these streams are roughly 60, 25, and 15 respectively.

mathematics problems, it was not sufficient that they built up content resources; they needed also to learn strategies to understand the problem, find a way forward, control their cognitive executive functions, get unstuck etc. in order to make progress. Instead of relying solely on the teacher each time they faced an unfamiliar problem, these students needed attack strategies that they personally owned to tackle the problems with a sufficient degree of independence. We termed this ability to devise a way forward in dealing with mathematics problems as "Problem Solving Disposition" (PSD). The choice of the word "Disposition" was to highlight the overall vision of PSD as not merely a simple reduction to a set of 'problem solving skills' (although learning these skills, such as "substitute values", "decomposition of a diagram" etc, may form part of the curricular implementation of this vision); rather, it was about students' high degree of self-agency in harnessing these tools appropriately whenever they were confronted with a mathematics problem.

Since the topic of "Number Patterns" lends itself easily to PSD, it was chosen as the focus for the *Lesson Study*. The teachers in the team readily supported this choice, with some admitting candidly that they had all along found it hard to teach the topic and were looking forward to learn more through the *Lesson Study* enterprise. We agreed from the start that the goal was not merely to teach students the techniques so that they can handle standard exam-type question on this topic well; rather we also wanted the topic to be a sort of vehicle to highlight PSD. Throughout the *Lesson Study*, we spoke of operating on "two planes": the "plane" of teaching techniques to deal with number pattern, and the "plane" of teaching PSD. These two planes were meant to be developed alongside each other throughout the module.

The overarching strategy in the planning of the module comprising seven lessons (of 45 minutes each) was to "foreground" PSD, using the examples of number pattern problems in the "background". This technique of foreground/background in the prioritisation of instructional goals was drawn from Leong, Chick and Moss (2007). This language is derived from performances where some actors are thrust at the fore of the stage, while other performers carry out their roles in the background—nevertheless doing important work although they are not the central focus of the audience. Similarly, we meant to thrust PSD into prominence to the students, but at all

times developing the skills needed to solve number patterns as they attempt one problem after another.

The embodiment of PSD is in the student worksheet containing the following features: (1) To keep the process simple but without over-compromise for the NA students, we reduce it to three sections combining Polya's (1945) "Devise a Plan" and "Carry out the plan" into a single step. In so doing, we were not unaware of the original distinction and importance of doing so between the two stages; rather, since the students were only beginning to acquaint with independent problem solving, we wanted to keep the focus on other stages first. Moreover, for the topic on number patterns, the "plan" does not vary significantly and thus is relatively mundane to insist on students writing the same "plan" for every problem. (2) We changed the language of the first stage to "Try it!" to encourage students to make attempts at the problem; the second section of the worksheet was entitled "My (first) solution" to highlight the need to present what they have initially drafted in the first section into a more formal representation as well as the awareness that it is nevertheless not necessarily a correct solution and thus a need to revise it, if necessary, after checking later. The last section "going beyond the (first) solution" emphasizes the disposition of not stopping at a first attempt at solving a problem; rather we wanted students to develop the habit of "going beyond" by way of checking the first solution and generalising the solution. For number patterns, generalisation normally means obtaining the expression for the general term of the pattern.

We adopted the features of *Lesson Study*—as depicted in the earlier sections of this paper—throughout the whole enterprise. All the Year 7 NA students (two classes) were taught the module by their resident mathematics teachers. Terence taught the first cycle and George—one or two days after, depending on the scheduled class times—taught the second cycle of lessons. Instead of confining ourselves to a single Research Lesson, all the lessons in the module were treated as Research Lessons in the sense that we made plans as a team for each lesson and observed every lesson. However, due to time-tabling constraints, we were only able to do post-lesson meetings at six junctures spread roughly evenly across the whole module for the two classes.

Method

In examining the teachers' learning trajectory, we began by considering their starting point prior to their actual involvement in the Number Patterns *Lesson Study*. The establishment of this baseline status is partly based on our knowledge of the teachers through our long-term engagement with them and also checked against the data from their responses in the first interview. The former was obtained from drawing upon our accumulated knowledge of them over years of interactions in earlier projects. The mutual understanding that we have developed with the mathematics teachers over the years and over a number of close-quarter interactions allowed us a more-than-cursory perception of their resources and orientations; the latter was collected from interviews we held before the commencement of this project. In the interviews, we focused on the teachers' resources, orientations, and goals that were relevant to the *Lesson Study* theme. This resources-orientations-goals triadic basis is derived from Schoenfeld's (2011) well-known Resources, Orientations, and Goals (ROG) framework.

We also use teacher interviews at two other points along the chronology of the *Lesson Study* enterprise: Middle, and after. The middle interview was carried out after the fourth post-lesson meetings and the last interview was done after all the six meetings. Each interview lasted about 25 minutes. Similar questions as the ones used in the first interview were asked during each of the subsequent semi-structured interviews and they centred on the teachers' thoughts about PSD, students' learning, their orientations regarding how to teach the topic, and their own learning both in content and in pedagogy related to number patterns. These components were each grouped under "resources", "goals" and "orientations" and they formed the initial categories used to code the transcripts of the interviews. However, we soon find that these categories were too broad and they were subsequently refined accordingly to the level of grain-size that is suitable for analysis. By "suitable" we meant a sufficiently adjusted theme that allowed tracking across the three interviews and that allowed comparison across the teachers' responses. The resulting strands are presented in the second column of Tables 1 and 2.

In line with aims of this study, we examined the textual transcripts of all the interviews for evidence of changes, learning, and shifts in the teachers' along the identified categories. These evidences, including actual extracts of interview responses which can fit within the limited spaces in the table

entries, are summarized in Columns 3, 4, and 5 (corresponding to the first, second, and the last interviews) in Tables 1 and 2.

Based on the analyses of data of each supporting teacher within each strand, we formed initial conjectures of their learning. We sought to confirm (or negate) these conjectures about changes by looking at other supporting (or counter) evidence from the rest of the teacher interviews.

In this paper, we report the learning trajectory of two supporting teachers: Mei Feng and Li Fei (both pseudonyms). Their selection is based on the intention to cast supporting teachers of different backgrounds in which they represent in terms of years of teaching experience, the roles they see themselves play in the team, and the knowledge they bring into the enterprise.

Our knowledge of the supporting teachers based on historical interactions. Mei Feng teaches only upper secondary (that is, Year 9 and 10) mathematics. This could be why she was not involved in all the earlier projects for lower secondary (that is, Year 7 and 8) students. The last project she was a member of was the one on using dynamic geometry for circle properties. She attended most of the lesson observations and post-lesson meetings, and remained usually quiet. In the few times she shared her views, she noted that the students she observed were able to follow the lessons and complete the worksheets; however, she questioned the suitability of the same set of materials, including the use of dynamic geometry, for "weaker students", such as those in the NA stream. We perceive that it is important to her that the proposed innovations must be placed against the acid test of direct implementability for her students. Unless this condition is fulfilled, she remained unconvinced of their usefulness. She has more than 20 years of experience teaching mathematics at the secondary levels and holds the rank of Senior Teacher[4].

Li Fei has about four years of experience teaching secondary mathematics for Years 7-10, with a heavy proportion of Normal Technical (NT) classes. She had been involved in a number of our previous *Lesson Studies* on lower

[4] Senior Teacher is a rank that is accorded by the Ministry of Education for teachers who are deemed as experienced in teaching and are expected to mentor other teachers in the craft. The progression along the teaching track is Teacher, Senior Teacher, Lead Teacher, Master Teacher.

secondary mathematics for NA students. We think this has to do with the school leaders' view of her as being a specialised teacher of mathematics to lower-achievers. We perceive her as positive about her role during *Lesson Study* meetings—open to learning about novel approaches to teaching and ready to share the relevant instructional experiences with her students, especially NT students. Although none of our earlier *Lesson Study* topic foci was for NT students, she shared about how she nonetheless carried out—usually in positive terms—some of those innovations to her NT classes. When she was a student teacher, three of the mathematics methods courses (totalling 72 hours) that she took were taught by the first author.

Learning trajectory of supporting teachers

The learning trajectories of Mei Feng and Li Fei are summarized in Tables 1 and 2 respectively. In the remaining sections of this paper, whenever references are made to entries in these tables, the following abbreviation is used: Name Strand Interview Number. For example, for the summary of Mei Feng's responses on students' learning in her second interview, the abbreviation MF SL2 is used.

Workshops Mei Feng's learning trajectory. Comparing the two data strands—our knowledge of Mei Feng based on historical interactions and the data from the pre-module interview (as shown in the corresponding column in Table 1)—we can see complementarities. For example, her MF NA1 and MF WA1 cohere with our perception of her view of low motivational levels for weaker students and her guide-rule of using classroom workability as the ultimate test for whether a particular instructional innovation should be adopted for these students; MF LS1 strengthened our hunch that she entered the project from a rather detached and elevated (as upper secondary teacher and as a Senior Teacher) position. In addition, the use of "They give me …" and "They want me …" in the extracts of the first interview signal a role she perceived was thrust upon her rather than one that she personally owned. In fact, she was not clear how she could participate or contribute apart from her acknowledgement that, as a Senior Teacher, she was expected to share her experiences to the group.

But we also learnt from the interview some specifics that we did not detect from previous encounters. She revealed some thoughts about the *Lesson Study* enterprise with respect to her growth as a teacher: She learnt about

different ways to solve problems involving number patterns (MF MC1); she found our involvement productive (MF KO1); she pinpointed one emphasis of PSD—getting students to "try" and not give up easily—that she concurred with (MF PS1).

At the point when the first interview was conducted, Mei Feng's standpoint remained primarily that of an upper secondary teacher. The discussions on lower secondary teaching of number patterns was seen as not directly applicable to her since she did not teach lower secondary classes, but she was open to learn new approaches to solve problems—especially from the KOs—because she would need to revise number pattern problems with her upper secondary classes. Approaches that she would adopt are those that would work well in the testbed of George's and Terence's forthcoming lessons.

Comparing the data from the pre-module and middle interviews, we can see similarities, but also significant changes. As summarized in MF SL2, she went beyond students' disposition of repeated trying to elaborate on other elements of PSD, such as the use of *decomposition* as a heuristic and for students to have a sense of *empowerment*—a term she used repeatedly in the interview and likely picked up from the *Lesson Study* post-lesson meetings as we used it often. For MF KO2, she gave more specific descriptions of how she thought the KOs were contributing to the discussions—through our depth of analysis and suggestions on the exact points to carry out certain instructional moves and transitions.

As to MF WA2, she seemed to have satisfied her need to see the plans enacted in class and interpreted the innovations as practicable. This is likely to be the primary motivation for shifts in MF NP2 and MF TM2: Although she still takes the standpoint as an upper secondary teacher, she is more open to the usefulness of these approaches to her own upper secondary students as well as to other NA students (MF NA2). She was intent on bringing the lessons she learnt from the *Lesson Study* observations and discussions into her own upper secondary classrooms. She had arrived at a point where she avowedly "bought in" to the innovations by intending to incorporate them into her own instructional practice.

In the last interview, Mei Feng's overall position with respect to the categories analysed did not differ significantly from what was expressed in

the middle interview. She maintained that she learnt from the *Lesson Study* enterprise—both from lesson observations and from the views expressed by the KOs in the meetings—in substantial ways along the lines of number pattern knowledge, ways to teach the topic, and what PSD entails.

Table 1
Summary of Mei Feng's learning trajectory

Categories	First Interview (1)	Middle Interview (2)	Last Interview (3)
Students' learning (SL)	"look forward" to see how students learn	"All students trying", not afraid to "dirty the paper". Students use "so many ways'.	"I really enjoy seeing how students learn, their thinking"
Mathematics content (MC)	"open my mind" to different ways of solving	No evidence	"I was very weak in this topic … I get to learn and increase my competency"
Her role in *Lesson Study* (LS)	"Don't feel so involved"	"Not taught lower sec before so this is an eye opener"	"Even though I am very busy, I do learn"
KO's role in *Lesson Study* (KO)	"Give good advice" when "stuck"	Provide deep analysis of the lessons	"They provide us with a lot" to think about in teaching
Workability of the approach (WA)	See how "students respond to it"	Students "able to do it"	Students "manage to solve" but "checking need to emphasise"
NA students' disposition (NA)	Lack "motivation"	"students enjoying, engaging, responsive … very good for NA"	"students more confident"
PSD (PS)	"Most important thing is for	"I like the part on students	"The focus in every lesson is to

	students to try" and not give up easily	constructing their own knowledge"	help students learn to be unstuck"
Teaching number patterns (NP)	Acknowledge she teaches only "one way" to solve	"I will try this approach but not the entire thing … because upper sec not the same". Time "very tight"	"I am thinking how I can modify" the different methods to help "my upper sec students"
Teaching mathematics (TM)	No evidence	From "efficiency" of coverage to students' learning. "I begin to buy in". "I like letting students try and empowering them to get the answer"	Make learning for students "enjoyable"

Nevertheless, there were more details provided in the last interview for MF WA3, MF NA3, and MF MC3. She added the significance of "checking" and noted that students were not yet able to internalise it and was thus an area that needed to be "emphasised". For beliefs about students' ability in and feelings towards mathematics, she was convinced that the students were "more confident" and that translated in their ability to solve problems involving non-figural number patterns. She also candidly confessed to being "weak" in number patterns prior to the study and believed that she had learnt much about different ways of solving the related problems—to the point that she felt more "confident" to teach the topic to her students.

A more careful scrutiny of her MF NP2 and MF NP3 reveals a constancy in her standpoint—that of an upper secondary teacher. However, the shift in her view of her role in the *Lesson Study* team is discernible: From that of uncertainty (MF LS1), to that of her experiences being an "eye-opener" (MF LS2), and being appreciative about what she learnt, despite being "very busy" (MF LS3).

She thought that the source of learning was not merely from the KOs, although she consistently mentioned the important contributions we made during discussions (MF KO1,2,3); rather, in MF SL3—that emerged only in

the last interview—she also attributed learning to the opportunity to observe in close quarters the working of students over a substantial period of time. We think that this window into the thinking of actual students-at-work, coupled with her observation that students were able over the course of the module to make good progress at the problems, shaped her motivation to change her instructional stance in the teaching of this topic to her upper secondary students (MF TM2, 3).

Li Fei's learning trajectory. Our perception of Li Fei as one who was open and ready to learn is confirmed in her responses during the pre-module interview. Her level of receptiveness to learn appeared to be higher than what was articulated by Mei Feng. First, unlike Mei Feng, who needed the actual workability in the classroom to test the usefulness of suggested innovations, Li Fei did not reveal such reservations (LF WA1); rather, she saw the interactions during *Lesson Study* meetings as opportunities for learning new methods of teaching (LF NP1). Second, not only did she share about learning within *Lesson Study* meetings, she also took the initiative of implementing some of the innovations that arose from a previous project with her NT class (LF TM1), even though her class was not officially included within the scope of the project. She looked up to the KOs as experts who not only set the stage for discussions but who also plumbed the depths of analyses that the teachers, on their own, were unable to reach. She also appreciated our guidance and knowledge on teaching (LF KO1, 2, 3). Perhaps, Li Fei's high view of us has to do with both her relatively less experience in teaching (only about four years) compared to her more experienced colleagues as well as her history of being a student to the first author (whom she still repeatedly made mention of in the interview).

There appeared to be some struggle in her consideration about PSD. While she shared the emphasis on getting students to give good tries on problems because she believed that this disposition was what the NA and NT students lacked (LF NA1 & LF SL1), she was unsure if confining PSD within the topic of number patterns would have the same effect as broadening it to include other types of novel problems (LF PS1). Yet, she understood that the limited resource of curriculum time will be further strained if it is taken up to venture into these 'out-of-syllabus' problems. To us, this ability to weigh different curricular options revealed her sensitivity towards instructional goals and how they impact on instructional decision-making.

The baseline portrait of Li Fei's view of this project can be summarized thus: She entered this project with a view of learning new methods of teaching, especially innovations that she can carry out directly in her NT classes. She had positive experiences of learning in previous *Lesson Study* teams and had high expectations of what the KOs bring to the team by way of content knowledge, ways of teaching, and probes that will help her think deeper about instructional matters. She retained some of the knowledge learnt in her pre-service mathematics methods courses (perhaps due to an abiding continuity through a human connection—the first author) which still influenced her instructional considerations.

There is a noticeable shift in Li Fei's conception of how PSD should be taught between the first interview and the middle interview. She earlier contemplated the use of out-of-syllabus problems as being more effective in bringing out the problem solving processes; in the middle interview, she could see that using within-the-syllabus topic of "Number patterns" as a way to foreground PSD was a practicable way to go (LF PS2). Moreover, based on her observations of how the students took to the lessons, she understood PSD to involve not just the "try it", but also the learning of "heuristics" (LF WA2).

With respect to learning new ways of teaching the topic, as was her aspiration expressed in the first interview (LF NP1), she reflected on her existing ways of teaching as being one-track and formula-driven (LF NP2 & LF TM2), and she saw the benefits of exposing students to alternative solution strategies (LF SL2). More specifically, she became convinced of the value of allowing students to discuss different attack routes among themselves and making public students' alternative strategies to the whole class. She resolved to adopt this instructional approach in the subsequent teaching of this topic (LF TM2).

There were a number of junctures in the last interview where Li Fei reiterated the points she made earlier in the middle interview: She continued to stress the importance of students' trying or "whacking" (a term commonly used by Terence in his class to mean "keep trying") when presented with novel problems and the need to learn different methods of tackling these problems (LF SL3); and she was convinced that the introduction of a

'formula' in this topic should be preceded by students' own exploration by way of PSD (LF TM3).

But there were significant differences in her responses between the last two interviews, especially in the grain-size and depth in which she analysed the various aspects of the *Lesson Study* process. She had a more expansive and thorough view of PSD in the last interview: Instead of focusing only on "Step One" and "Step Two"—as was the case in the middle interview, she commented on Step Three as well, and the lack of emphasis on "checking" components which we considered critical to building a PSD: Essentially, the students did not see the motivation for using the full PSD apparatus when some of the 'problems' became routine after a few repeated practice so that "PSD is of no use" (LF PS3). Li Fei was really questioning the sequence of problems in the module and suggested that PSD should only be foregrounded at junctures when students would find it motivating to use—when they are genuinely "stuck".

Closely linked to this analysis of PSD was her broad view of the "module" instead of confining her comments to selected lessons. In this last interview, it appeared that she took a step back and deliberately cast her sight on a broad sweep of what went on in the entire module of seven lessons. Under LF WA3, LF TM3, and LF NP3, Li Fei explained how she reconceived the flow of lessons if she was to "conduct the lessons [her]self next time": She will deal with the linear patterns and the quadratic patterns separately as two sub-modules; at the start of each sub-module, she will introduce PSD as a way to help students devise the formula for themselves before using the derived formula to practise on a number of similar patterns. In other words, Li Fei was able to envisage not only local hurdles (such as getting students to buy-in to the 3-step PSD worksheet), she was also mentally crafting the entire module development by intertwining the two planes of trajectory: Teaching PSD and teaching number patterns.

Table 2

Summary of Li Fei's learning trajectory

Categories	First Interview (1)	Middle Interview (2)	Last Interview (3)
Students' learning (SL)	"Expect them to be like my 2NT students" – leave lots of blanks	"I see pairs discussing" different methods and learning.	They "try when they are stuck" and are exposed to alternative solutions
Mathematics content (MC)	Basically not new to her as it was "what Author taught me"	No evidence	No evidence
Her role in *Lesson Study* (LS)	"See a different perspective. Helps me a lot"	"It helps that we are able to talk to the KOs"	Learnt "how to teach this topic because I had a lot of troubles teaching it to the 1E"
KO's role in *Lesson Study* (KO)	"Because [they] are around, we get so in-depth" in our discussions	"They provide guidance" but leave the decision to us	"We need their knowledge on how to teach" as they are more experienced
Workability of the approach (WA)	She was "not sure how they will react" as she had not taught 1NA before	The "trying" and the "heuristics part" have been "done quite nicely"	"The start of the module was fine, but at the end PSD was a hassle …"
NA students' disposition (NA)	"For NT and NA students, when they see problems, they will skip"	Second part of the worksheet is a challenge because "they thought they have done it in" the first part	"Most NA students need the visual", so putting figural patterns at the start "is fine"
PSD (PS)	Wonder if using more unfamiliar problems may be	"They are getting it quite well. I will use it"	"Use PSD (worksheets) only when they are

	better to highlight Polya's stages		stuck. But when the problems are too simple", PSD is of no use.
Teaching number patterns (NP)	Learn different solutions that we can "teach the students"	"I get to know there are other ways of doing it"	"We start with linear patterns using PSD. Use PSD again" when students confront an unfamiliar problem later
Teaching mathematics (TM)	"when it comes to learning new methods" of teaching, it "helps a lot for NT classes"	"Previously, I just give them the formula." Now that I have seen it done this way, "my lessons will be different"	"Let students explore first and come out with the formula themselves. I will do it this way next time"

Discussion

In view of the literature that points to teachers' beliefs and practices as rather resistant to change (e.g., Bibby, 1999; Clarke, 1994), it is encouraging to observe the noticeable shifts—in both Mei Feng and Li Fei—along some significant teacher characteristics. Through the history of engagements with the teachers over a number of *Lesson Studies*, we did expect that they would be positive towards our involvement in this current *Lesson Study*. Nevertheless, we are still surprised at the degree in which their participation in this *Lesson Study* had impacted them, despite their relatively less central roles as supporting members.

For Li Fei, she initially had an image of NA students as having little patience in trying problems. Through her observation of what the students could do, especially the diversity of solution strategies between working pairs of students, her belief about these students' capabilities and thus her own instructional strategies underwent revision: "[For student pairs] sitting side by side, two of them can have totally different ways of doing it but getting the same answers. … So they discuss it. I think [it] is good. ... [Previously] I

just give [the formula] to them. … But now … I get to know that there are other ways of doing it, I think my lesson[s] after these will be different. That's how it benefitted me." (Middle interview). Li Fei was also candid about acknowledging how she had "a lot of troubles" (last interview) teaching the topic in the past and how she learnt from the *Lesson Study* about ways to teach the module in the future. The frequent use of the first person pronoun during her interview responses (see Table 2) was to us an indication that Li Fei was not merely describing a supposed-to-be instructional ideal external to her sphere of personal belief; rather, she was looking for lessons about content and teaching that she can personally own and use.

In the case of Mei Feng, she went beyond general subscription of the innovation to a specific commitment to change her instructional practice: "I will try this approach. … I really like the part on the students constructing their own knowledge. This is something that I definitely will try. In fact, I am doing some of it in my class now" (Middle interview). Also, like Li Fei, but especially unusual for someone of Mei Feng's status as a Senior Teacher, she shed her sense of vulnerability in candidly confessing her prior deficiency in content knowledge and the appreciation for learning opportunities: "Just to let you know - I am actually quite weak in doing number pattern. This is one of my weaker areas" (Last interview). Moreover, Mei Feng's frequent use of emotive words such as "I enjoy …" and "I like …" in her interview responses (see Table 1) pointed to a "buy-in" that had perhaps reached the inner core of her teaching self. This positive emotional response was a signal that Mei Feng had "found a compelling reason to undertake the task of transforming [her] practice" (p. 46, Goldsmith & Schifter, 1997).

A closer look, especially from the data in the last interview, reveals a difference in depth and content of learning between Mei Feng and Li Fei. While Mei Feng's view of the innovation essentially stabilized after the middle interview, Li Fei's perspective—as seen from her responses in the last interview—broadened to consider not merely specific implements during particular lessons but also the flow of PSD and number pattern development over the *entire module*. She apparently had the whole module development mapped out in her mind.

We think that a useful way to understand how and what teachers derive learning opportunities from the *Lesson Study* enterprise is through analysing their participation based on the "standpoints" they took. An example is the standpoint of an upper secondary mathematics teacher that Mei Feng took throughout the whole study. From that standpoint, her scope of view limits as well as opens up the kinds of lessons she would learn. It helps to explain why she was interested in specific implements that she can directly import into her upper secondary mathematics classes; it also accounts for her relatively less interest when module-level considerations were discussed. In contrast, Li Fei's standpoint of a lower sec NA/NT teacher brings her very close to the overarching issues as well as details of the design—as one who would carry out the revisions in the next implementation in her classes; all the same, unlike Mei Feng, that standpoint does not naturally afford her the consideration of what revision lessons in the upper secondary level may look like. Upon reflecting on the impact of learning depending on the standpoints each of these teachers took, we undertook further work in expanding a 'standpoint theory'. Thus, in the final section of this paper, we propose a provisional theory of standpoints-within-a-learning-landscape metaphor as a way to advance our understanding of teacher learning (especially for supporting teachers) within *Lesson Study* setups.

Standpoints and teacher development.

Just as the different standpoints we take with respect to a physical landscape afford us different views (and at the same time, certain limitations of view), we found that, for Mei Feng and Li Fei, the standpoints they took similarly privileged certain viewpoints with respect to the 'learning landscape' provided by the entire *Lesson Study* enterprise. In other words, the learning benefits they derived from participation in the *Lesson Study* process were largely mediated through the standpoints they took. We are keen to explore and extend this metaphor to probe some important questions with regard to teacher development within *Lesson Study* teams. One such question is: From a teacher development perspective, we are interested not only in helping teachers take in new learning opportunities from their existing standpoints—something that showed up in the earlier sections that both Mei Feng and Li Fei experienced; in addition, we want to help teachers take *different standpoints* so that they can broaden their learning horizons and perhaps transform their resources, orientations, and goals (cf., Schoenfeld's ROG)

about teaching mathematics. How can that be done within *Lesson Study* setups?

We think that both teachers adopted a new standpoint some time during the *Lesson Study* process: That of a learner of students' learning. Both expressed clearly about how close and regular observation of students' work in class were triggers to help change their views about what students could accomplish in response to the instructional innovations. Mei Feng described how the manner in which students "enjoyed" the lessons and were "empowered" through productive attempts at the PSD worksheet contributed to her "buy-in"; similarly, Li Fei pointed that observing how students were able to come up with their own solutions changed her views of how to teach the topic.

From a teacher development perspective, this is a critical standpoint for teachers to take up if our goal is teacher change. Thus, professional development efforts should carefully consider providing teachers (compelling) opportunities to take up this observer-of-student-learning standpoint in its design. Thankfully, for *Lesson Study*, teacher observation is already an essential component of the overall design. However, it is naive to conclude that mere observation of students would bring about the kind of teacher change as evidenced by Mei Feng and Li Fei. We conjecture that ingredients that help translate teacher observation of student learning into committed instructional changes include: (1) observation of *productive* students' learning. This, in turn, is closely linked to the quality of instructional intervention. If the purported teaching innovation was perceived as 'not working' for students, the outcomes in terms of "buy-in" for teachers would have been quite different; (2) *sustained* observation of productive students' learning. We doubt if a mere one-off observation of students' learning would bring about a strong commitment for instructional change. In the case of Mei Feng and Li Fei, they observed the same pair of students over the entire module. They could see, through following the learning trajectories of the students, how the positive effects in them were built up alongside how the module developed.

Upon closer scrutiny, we can purport other standpoints that the teachers adopted along the *Lesson Study* process that helped them grow in their knowledge as mathematics teachers: Mei Feng's standpoint as a student teacher (a standpoint that was not evident before the module, perhaps over-

shadowed by the more dominant mentor standpoint as "Senior Teacher") and Li Fei's standpoint as a module designer (a standpoint that became more conspicuous towards the end of the module as we, as a team, looked back to evaluate the module and proposed revisions). We are aware that more evidence and analyses are needed to support the assertions about these standpoints. But what interests us more is this question, "How did these productive standpoints emerge in the course of the *Lesson Study*?"

The reality is that teachers come on board a *Lesson Study* with their varied instructional histories and assigned roles (in this case, both Mei Feng and Li Fei were supporting teachers), and hence standpoints. Through this study, we posit that positive changes to teachers' resources, orientations, and goals involve a shift or enlargement of standpoints. One critical standpoint that tips the balance is that of teacher as learner of students' productive learning. There is potential yet for adoption of other standpoints that are perhaps more teacher-dependent. We think that *Lesson Study* enterprises should provide a sufficiently rich 'learning landscape' so that teachers who enter with different backgrounds and goals (hence differing standpoints) can find suitable new 'locations' to stand upon to view teaching and learning in ways that they previously did not conceive of.

We conjecture some features of *Lesson Study* that help provide this rich landscape: (1) The need for a clear coherent focus of study to establish a landscape on which team members share common standpoints. In this study, the focus throughout was to design a module that fulfills the twin goals of teaching PSD and teaching number pattern; (2) While common goals are important, the landscape should not be so narrowly framed that it only attracts the interest of the teachers who are actually teaching the lessons; rather, the landscape has to be *expansive* enough to allow different teachers—including supporting teachers—to explore standpoints that are guided by their own resources, orientations, and goals. In this study, the landscape includes domains such as different methods of solving number pattern problems, different types of number patterns, problem solving heuristics, student learning, task design, whiteboard presentations, and considerations in module design, among others. Both Mei Feng and Li Fei, who brought their own interests into the fray, could find portions of this broad landscape in which they explored new standpoints; (3) linked closely to the previous point is the need to broaden the unit of study from a single

lesson to a suite of tightly-knitted module of lessons to allow teachers a wider space to locate and confirm recurring points of learning. This helps teachers view not just one-off occurrences but also gain opportunities to reinforce learning which can lead to an embedding of innovations into personal instructional practices; and (4) it is perhaps fitting to end on a reflective note as KOs. In this study, the KOs consciously took on the role of expanding the learning landscape as well as describing standpoints we took that some teachers might not have conceived before. As mentioned in an earlier section of this paper, the positioning of our roles as KOs is related to the state of maturity of *Lesson Study* as a professional development platform in the school or department. At this stage of our involvement with the mathematics department of the project school, we think that a clear articulation of the standpoints we take and relating them to the teachers' standpoints is one aspect of realizing the KO-as-authentic-partners role that we seek to play. We are encouraged that—based on the interview responses of Mei Feng and Li Fei—our efforts to fulfil this role was consistently received as positive by the teachers. We look forward to learning different aspects of our roles as KOs which contributes to building a lively culture of teacher learning through *Lesson Study*.

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