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Computer Algebra As A Tool In Problem Solving

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Abstract

This paper describes how a computer algebra system such as MAPLE can be effectively used as a tool in the solving of the monkey-coconut problem and in facilitating the further investigation of the problem for generalisation.

Problem solving is the heart of mathematics and some problems in mathematics are very appealing to students. One such problem is the following monkey-mango problem (Kraitchik, 1953).

> *Three men who had a monkey bought a pile of mangoes. At night one of the men came to the pile of mangoes while the others slept and finding that there was just one more mango than could be divided exactly by three, tossed the extra mango to the monkey and took away one third of the remainder. Then he went back to sleep. Presently another of them awoke and went to the pile of mangoes. He also found just one too many to be divided evenly by three, so he tossed the extra one to the monkey, took one third of the remainder and returned to sleep. After a while the third rose also, and he too gave one mango to the monkey and took away the number of whole mangoes which represented precisely one third of the rest. Next morning the men got up and went to the pile. Again they found just one too many, so they gave one to the monkey and divided the rest evenly. What is the least number of mangoes with which this can be done?*
>
> *(Answer : 79 mangoes)*

There are generally at least two methods of solution. One is working backwards using trial and error. The other involves the setting up of an equation and solving it manually or with technology.

The problem may be modified into many different versions. At a lower level than the monkey-mango problem, it may read something as follows :

After a mathematics quiz, Mr Lee gave a box of apples to the three winners to share. The first winner received 2/3 of the apples plus 1/3 of an apple. The second winner received 2/3 of the remainder plus 1/3 of an apple. The third winner received 2/3 of the remaining apples plus 1/3 of an apple. There was one apple left after this. How many apples were there in the box?

This is a simple problem suitable for upper primary or lower secondary students. There is only one answer (40 apples) to this problem. Most of the students use *working backwards with trial and error.* Some form the following equation where x is the number of apples and solve it:

$$(2x/3 + 1/3) + (2x/9 + 1/9) + (2x/27 + 1/27) + 1 = x.$$

At a higher level, one example of the problem is shown below :

Five sailors and a monkey were on an island. They gathered a pile of coconuts to be divided the next day. During the night one sailor woke up and divided the coconuts into 5 equal parts. There was one coconut left over which he gave to the monkey. He took one part as his share and went back to sleep. Each of the other four sailors repeated the performance. The next day, all the five sailors woke up and distributed the remaining coconuts equally among them. There was one coconut left over. How many coconuts were there originally?

This is a challenging problem to students at advanced level. There are infinitely many answers and the smallest solution to this problem is too large for the method of trial and error to be feasible. Some students can write an equation as follows:

Let x be the number of coconuts.

Sailor	took	left
1st	1/5(x-1)	4/5(x-1)
2nd	1/5(4/5(x-1)-1)	4/5(4/5(x-1)-1)
3rd	1/5(4/5(4/5(x-1)-1)-1)	4/5(4/5(4/5(x-1)-1)-1)
4th	1/5(4/5(4/5(4/5(x-1)-1)-1)-1)	4/5(4/5(4/5(4/5(x-1)-1)-1)-1)
5th	1/5(4/5(4/5(4/5(4/5(x-1)-1)-1)-1)-1)	4/5(4/5(4/5(4/5(4/5(x-1)-1)-1)-1)-1)

$$4/5(4/5(4/5(4/5(4/5(x-1)-1)-1)-1)-1)-1 = 5y$$

After setting up the equation, they find a computer algebra system such as MAPLE very useful (Abell & Braselton, 1994).

First of all, the equation can be simplified to:

```
> 4/5*(4/5*(4/5*(4/5*(4/5*(x-1)-1)-1)-1)-1)-1 = 5*y;
        (1024x - 11529)/15625 = y
```

There are two commands in MAPLE which can be used to solve the equation:

```
> msolve(1024*x - 11529, 15625);
        {x = 15621}
> isolve(1024*x - 15625*y = 11529);
        {x = 15625 _N1 - 55062504, y = 1024 _N1 - 3608577}
> 15625*3524-55062504;
        -4
> 15625*3525-55062504;
        15621
```

Since N1 is an integer, set N1 = 3525 for the smallest positive x and we have x = 15621.

MAPLE can also be used to tackle the problem using the following program:

```
> for z from 1 to 50000 do
> if type ((1024*z - 11529)/15625, integer) = true
> then print (z)
> fi; od:
```

and the output is 15621, 31246, 46871.

The use of MAPLE illustrates how a computer algebra system can be integrated with problem solving. It is certainly time consuming to solve the monkey-mango problem by trial and error without the use of technology. MAPLE facilitates the solution to the problem as follows:

```
> 2/3*(2/3*(2/3*(x-1)-1)-1)-1 = 3*y;
        8x - 65 = 81y
> msolve(8*x-65,81);
        {x = 79}
```

With a computer algebra system, it not only makes it easier for students to solve the monkey-coconut problem but also makes it possible to pursue the investigation further for generalisation.

For example, a generalisation that $x = n^n - n + 1$ for n sailors can be obtained from the following table.

n	x	n^n	$n^n - n + 1$
3	79	81	79
4	1021	1024	1021
5	15621	15625	15621

The conjecture is then confirmed by solving the diophantine equation as shown below:

Let $a = (n-1)/n$.

The equation becomes $a(a(a(a(a \,.....\, (x-1)-1)-1)-1)-1) \,.....\,)-1 = ny$.

$a^n(x-1) - (a^{n-1} + a^{n-2} + \,.....\, + a)-1 = ny$

$a^n(x-1) - (a^n-1)/(a-1)-1 = ny$

$a^n((x-1) - 1/(a-1)) + a/(a-1)-1 = ny$

Using MAPLE, the above equation can be simplified to

$(n-1)^n x - n^{n+1}y = (n-1)^{n+1} + n^{n+1}$

Solving it as a linear congruence, we obtain

$x = kn^{n+1} + 1 - n$

For $n = 3$, $k = 1$, $x = 79$; $k = 2$, $x = 160$;

$n = 4$, $k = 1$, $x = 1021$; $k = 2$, $x = 2045$;

$n = 5$, $k = 1$, $x = 15621$; $k = 2$, $x = 31246$.

References

Abell, M. L. & Braselton, J. P. (1994). *The MAPLE V Handbook.* Academic Press.

Kraitchik, M. (1953). *Mathematical Recreations.* New York: Dover.