Development of a Framework for Analysing Mathematical Problem Solving Behaviours

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Abstract

The taxonomy described in this paper was developed to investigate the process of mathematical problem solving in terms of definable behaviours. It was also used as an instrument to classify and encode behaviours in their sequence of observed occurrence during the process of mathematical problem solving. It is a behavioural analysis framework formulated to examine the "thinking-aloud" protocols of individuals for comprehensive information about the problem solving process itself, the individual differences in the behaviours of subjects and the strategies applied by each in dealing with non-routine mathematical problems.

Descriptors: Mathematics problem solving, thinking aloud, protocol analysis.

Introduction

Much of the recent research on mathematical problem solving attempts to describe and characterise the problem solving process, derived from verbal reports. It has become increasingly important for research to develop valid and reliable instruments which reflect actual behaviours in the problem solving process. Kilpatrick (1967), Lucas et al. (1980), Rowe (1980), Schoenfeld (1983), Putt and Pountney (1989) and many others have devised protocol analysis frameworks to record and analyse behaviours in a sequence of observed occurrence.

This paper describes a framework for analysing the "thinking aloud" data from problem solving performance on certain non-routine mathematical problems. The methods of data collection and protocol analysis were derived from research within the framework of information processing influenced by Newell and Simon's (1972) theory of problem solving.

The present study was able to develop a taxonomy comprising cognitive, metacognitive and affective behaviours manifested in individuals during problem solving activities.

The taxonomy identified 28 behaviour categories which could be coded reliably. These behaviours could be considered as low inference measures that were objective, observable and most importantly could be defined in operational terms. Inter-coder agreement was used to establish reliability and to refine concepts which had to be reconciled with the actual data under the conditions set up by the methodology in the present study.

Design

The Sample

The nine volunteer subjects in this study were adults enrolled as pre-service trainee teachers at the Singapore Institute of Education. A range of mathematical abilities for this adult level was chosen so that all possible behaviours could be expected to occur. The nine subjects were from three levels of mathematical background. Two had General Certificate of Education (GCE) 'O' (Ordinary) level secondary school mathematics with low credits, two GCE 'A' (Advanced) level post-secondary mathematics with average credits and five had university degrees in mathematics with two having upper class honours. They were recommended by their respective mathematics education course lecturers who considered that the subjects' mathematical backgrounds were suited to the investigator's purpose. These volunteers were briefed by the investigator on the aim of the project and given training in "thinking aloud".

All the participants were proficient in the use of the English Language. This proficiency was both a prerequisite for their selection into the teacher training programme and had been the language of instruction throughout their mathematical education. This was an important consideration as this investigation utilised verbal reports as data and the subjects, who were not native speakers of the English Language, needed to be confident in the use of the language in order to participate in the "thinking aloud" procedure.

The "Thinking Aloud" Technique

As the processes during cognitive performance tend not to be directly observable, the first task was to find a methodology which would make it possible to identify and monitor valid elements, in terms of operationally defined behaviours. A promising method was the "thinking aloud" (TA) technique which generates data from the concurrent verbal reports of subjects while engaged in problem solving. The main concern was that the TA instructions should be given to the subjects so that they could vocalise all their problem solving activities with minimal interruption while performing the tasks.

Previous research, using a similar methodology, commonly gave training to subjects before the study began so that during problem solving they could "think aloud" in an ongoing manner and without time delay. It was found that "thinking aloud" was relatively easy to learn given sufficient practice and as long as the subjects were motivated to cooperate. A standard procedure was essential for all subjects and tasks. The experimenter should not say more than absolutely necessary; neither direct the subject's approach, ask leading questions or reinforce while the subjects were "thinking aloud".

The TA instructions were adapted from previous studies reported in Ericsson and Simon (1984). The subjects were asked to vocalise their thoughts as though they were talking to themselves. They were told to constantly vocalise whatever came to mind and that the experimenter was not primarily only interested in their final solution but also in their thinking processes. To prevent subjects from explaining their solutions aloud, the experimenter followed Krutetskii's (1976) warning to subjects at the beginning against confusing the instruction to think aloud with that of explaining the solution:

"Do not try to explain anything to anyone else. Pretend there is no one here but yourself. Do not tell about the solution but solve it." (Krutetskii, 1976, p 93)

Procedures for Data Collection

The subjects were arranged in individual recording booths in a language laboratory to take part in the experiment. The experimenter started each subject off with a problem presented on a typed sheet of paper. The subject was asked to begin work on each task by reading the problem statement aloud. Once this had begun, the experimenter moved on to the next subject to start on the same problem. The subjects were told to put up their hands when they had finished with a problem and the experimenter would come to present them with the next problem. Each problem was presented on a separate sheet of paper and subjects were also provided with pencil and paper for their scratch work. Five problems (Appendix A) were presented, one at a time in the same order, for every subject. No time limit was imposed and the subjects were reminded at the beginning of the session of the TA instructions.

All the subjects attempted to solve the same five problems. Each problem solving session was recorded in its entirety and the audiotape protocols were transcribed verbatim. Forty-five protocols were collected, each protocol consisted of one complete transcription of a subject's audiotaped recording of his or her solution process to a problem. Each transcribed protocol was divided into segments of behaviours. Each segment was then classified and encoded according to a pre-determined taxonomy of behaviours and a process-coding scheme. For each subject and task, the data consisted of the types, the frequencies and the sequence of coded behaviours used during the problem solving process.

Protocol Analysis

Protocol analysis refers to the use of a systematic event-by-event record of an individual's behaviours while engaged on a cognitive task, as a source of data. The analysis of the "think aloud" protocols in this study were based on a framework that consisted of two components:

(a) a taxonomy with a process-coding scheme to code all observable problem solving behaviours in actual sequence of their occurrence.

(b) a working model for episode-analysis of the protocols to identify global patterns of metacognitive processes.

The Taxonomy

The term taxonomy used in this research refers to a predetermined set of problem solving behavioural variables that are classifiable into five distinct categories. It serves as a dictionary for characterising the mathematical problem solving behaviours in the subjects' protocols. Encoding decisions made by a coder were based on this dictionary of coding categories. A review of the problem solving literature provided the investigator with a list of frequently recurring behaviours that could be classified into five major categories:

(1) PROBLEM-ORIENTATION HEURISTICS: strategies through which a problem solver attempts to analyse and understand the problem situation.

(2) PROBLEM-SOLUTION HEURISTICS: rule-of-thumb strategies through which a problem solver moves towards a solution.

(3) DOMAIN-SPECIFIC KNOWLEDGE: the inventory of mathematical facts, procedures and skills that a problem solver is able to use in the solution process.

(4) METACOGNITION: a problem-solver's awareness and monitoring progress of his or her own thinking during the task.

(5) AFFECTIVE BEHAVIOURS: self-related expressions and emotional responses that are aroused in the problem solver.

A preliminary list consisting of all the possible behaviours in mathematical problem solving was prepared from the study of the literature. Altogether 40 behaviours were identified as relevant to the tasks used for this research. They formed the initial taxonomy which was used as a preliminary guide to identify observable behaviours in the protocols of the subjects in this study.

After a series of modifications through tests of reliability a final taxonomy (Appendix B), with a coding system was derived. Although it was initially based on prior assumptions about problem solving processes from previous research, the taxonomy was modified empirically, through the consensus of different coders after they had applied it to actual problem solving protocols of different subjects across different problems. In order for the coding judgments of any coder to be as objective as possible, the definitions of each specific behaviour in the taxonomy were stated in operational terms and were supported by examples from the protocols (see Appendix C for some definitions and examples).

Process-Sequence Encoding Procedures

Encoding was carried out on the transcripts of the subjects' audiotaped protocols, together with their written work, to provide a more comprehensive analysis of the solution path. The analysis of the protocols was based on the taxonomy. The overall strategy was to divide a protocol into segments of behaviour. Each segment was matched against the behaviours in the taxonomy and assigned a code. A whole protocol was eventually recorded as a sequence of encodings in a horizontal string corresponding to the order of the actual problem solution of an individual. Conversely, a string of encodings could provide a relatively clear description of what had actually happened during the problem solving process. (A sample of the protocol analysis is shown in Appendix D.)

The coding of individual segments of the protocols was intended to facilitate later comparison of encodings by different coders to establish the reliability of the taxonomy. Each segment of a protocol corresponded to a statement. According to Ericsson and Simon (1984), if the verbalisations were completely grammatical, a statement would essentially be a clause or a sentence, but in the thinking aloud protocols which were close to normal speech, statements were often abbreviated to phrases or even single words. It was also found in the protocols that some subjects often verbalised a single behaviour or operation ungrammatically as an aggregate of phrases. Each segment or an aggregated segment of behaviour was then encoded from the information contained in it.

A Working Model for the Episode-Analysis of the Protocols

The flow chart in Figure 1 represents a working model of a global problem solving process where "chunks" of consistent behaviours identified in the taxonomy could be parsed into six types of **episodes** which were modified from Schoenfeld (1983) for the purpose of this study as:

1 SCANNING
2 ANALYSIS
3 EXPLORATION
4 IMPLEMENTATION
5 REVIEW
6 VERIFICATION

An **episode** defined for this research is a period of activity during which the problem solver is engaged in a set of consistent behaviours that can be described as a certain event, such as analysis of the problem or exploration of strategies. The episodes can occur in any sequence or they can be recurrent. For example, some problem solvers may, after **scanning** the problem, have gone straight to **implementation** without **analysis** or **exploration** and in **review** became "stuck" or "does not work" and might go back to **scanning** or **analysis** and work through the process again, or they may "give-up", as illustrated by the variety of possible paths in the flow chart. The **review** episodes are junctures where evidence of metacognitive activities are most apparent.

Each episode consists of specific behaviours that are based on the taxonomy. Table 1 shows the list of predominant behaviours in the six episodes which can be classified into three main components of the mathematical process: "**orientation to the problem**", "**execute the solution**" and "**evaluation**". Each protocol sequence was parsed by examining the behaviours and dividing them into episodes that were enclosed and labelled in boxes. An example is shown in Figure 2. The episodes were parsed in sequential order of occurrence and were presented in the form of a flow-chart. The behaviours within each episode were presented in code symbols and the interpretations of these codes are shown in the key.

Establishing the Reliability of the Taxonomy

Issues of reliability and replicability in the coding scheme based on the taxonomy are concerned with the questions: "Would others see the same behaviours in the protocol?" and "Would they ascribe to them the same coding categories?". To address these issues tests of reliability based on agreement between different persons encoding independently of each other were used. The correlation between the categorisations made by different coders in relation to the same task provides an index of the objectivity of the taxonomy for categorisation of problem solving behaviours.

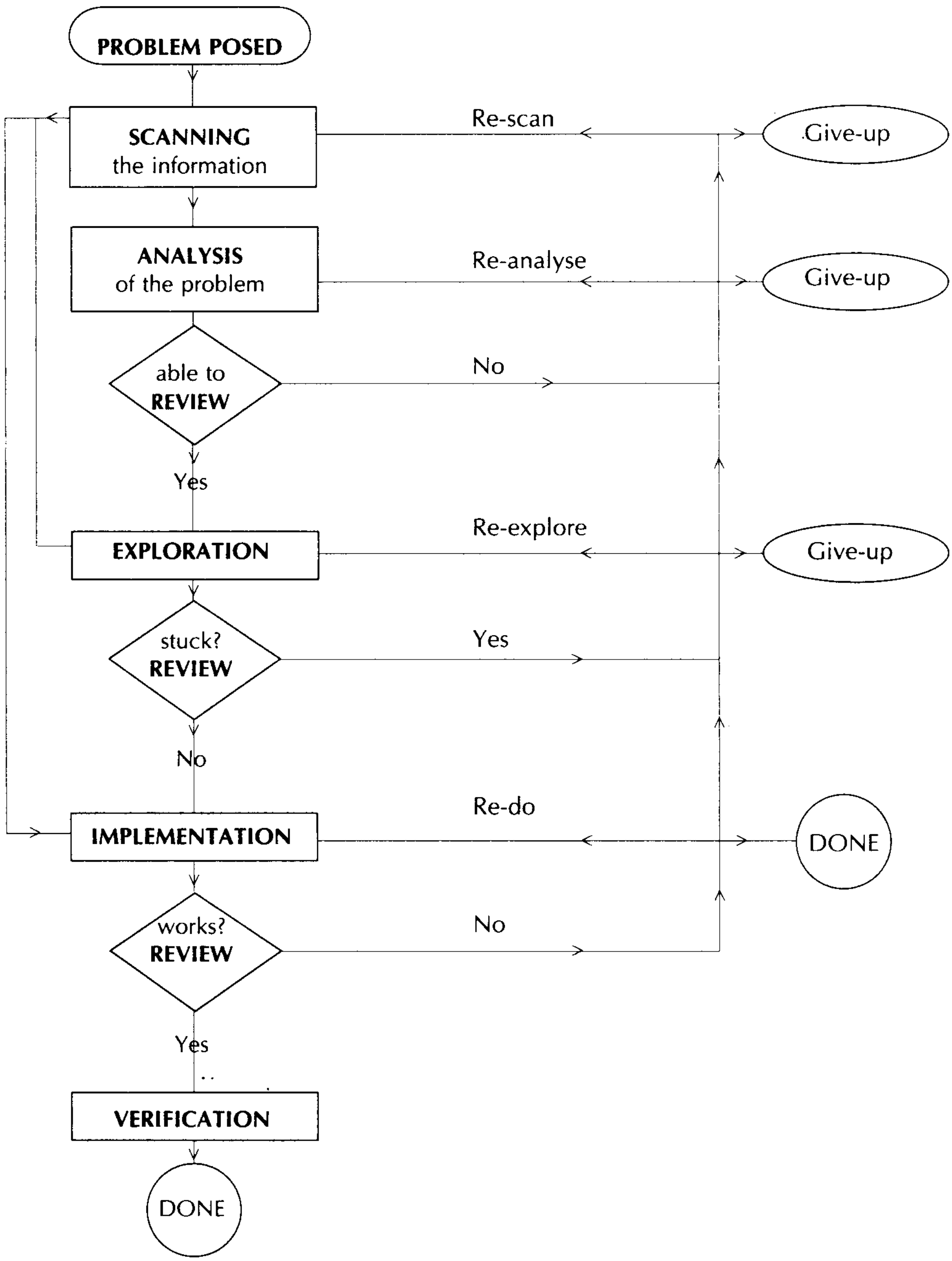

Figure 1: A working model for episode-analysis of the mathematical problem solving process

Table 1: Predominant behaviours in the six episodes of the three main problem solving components

I ORIENTATION TO THE PROBLEM	
1 Scanning	**2 Analysis**
P2 - re-reading of problem P3 - paraphrase statement	P4 - examine dimensions P5 - construct representation H2 - draw diagrams
II EXECUTING THE SOLUTION	
3 Exploration	**4 Implementation**
H4 - look at cases H5 - guess and check H6 - search for patterns K1 - computation K2 - state a fact or rule	H7 - generalise H8 - make a deduction K3 - use relevant mathematical procedures
III EVALUATION	
5 Review	**6 Verification**
M1 - suggest a plan M2 - assess difficulty M3 - review progress M4 - recognise error M5 - new development Q1 - self-question	H9 - check computation and final answer; some metacognition and P, H and K behaviours

EPISODES	EXCERPTS OF THE VERBAL PROTOCOL
Analyse **P1, P3, P5** recognise the general term	"...that means in general a 4-digit palindrome can be written as ABBA..."
Explore 1 **H4, K1, H6**	"...2 digit number, *xy* multiply by 11, something *xy* plus something *xy*..."
Review **M4, M1**	"...no it cannot be...let's find counter examples of palindromes."
Explore 2 **H4, K1, H4 K1**	"How about 3223...divide by 11...uhmm no...3443 by 11 ....313..uhmm..."
Review **M4, M3, M3**	"...wait..there must be some properties of 11 that I can make use of...let's go back to this 4...ABBA."
Explore 3 **P4, H4, K2, K1** search for 4 digits	"...so we need a 2 digit number greater or equal to 91...so when multiply by 11 you get a 4 digit number..."
Review **M4, M3, M5**	"...what I'm doing...no...not necessarily..." "let's try again...I must start with 1001..." "Oh!...I see...ABBA, I can do it this way ..."
Implement **H7, K3, Q1, K3** Ans. state answer	"ABBA is equal to 1000A + 100B + 10B = 1A, alright...then I group them together..." "100A + 1A + 100B + 10B = A (100) + 10B (10 + 1)" "...I can take out a common factor ...11"

Key:

- P1 - first reading of problem
- P3 - rephrase or give meaning
- P4 - analyse given
- P5 - correct representation
- K1 - computation (division)
- K3 - use math procedures
- M1 - suggest a plan
- M5 - recognise new development
- H6 - search for pattern
- Q1 - self-question
- H4 - look at examples (of palindromes)
- H7 - generalise
- K2 - state a fact
- M3 - review progress
- M4 - recognise error

Figure 2: An example of the Episode-Analysis of a process-sequence - A protocol for the palindromes-problem (Time Taken: 7 min)

The protocols collected in this study were divided into individual segments in order that direct comparisons between different encodings of the same segments by different coders were made relatively easy. A percentage of agreement between coders was calculated to give an aggregate measure of reliability. However, this reliability mainly reflects the very frequent coding categories, while the encoding of infrequent categories may be more or less reliable than the average.

To further test the replicability of each of the individual behavioural variables in the taxonomy by different coders, a **coefficient of agreement** calculation modified by Putt and Pountney (1989) from Lucas et al. (1980) was used. Inter-coder agreement was assessed systematically in two stages. In the first stage, tests of reliability were performed for the five categorical constructs of the taxonomy. This led to a refinement of the preliminary taxonomy by either eliminating, combining, or redefining behaviours which were redundant or vague. The second stage involved tests of reliability for every of the 28 specific behavioural construct defined in the final taxonomy.

On the whole, the majority of the twenty eight behaviours had relatively high coefficients of agreement. Of the total 943 encodings across all problems, inter-coder agreement was relatively high at 83.4%. The most frequent behaviour "H4-Looking at particular case(s)" had a coefficient of 0.86. The mean coefficient values of all the behaviours in each of the five categories were: P-category 0.89, K-category 0.77, H-category 0.76, M-category 0.74 and A-category 0.93. The higher values of the P-category and A-category indicated that these categories of behaviours by problem solvers to analyse the problem situation and behaviours that expressed emotions and self were easier to identify reliably.

Limitations

A number of limitations for this study have resulted directly from the chosen method of the "think aloud" data collection and protocol analysis based on a predetermined taxonomy for encoding behaviours. The contents of the subjects' problem solving protocols were restricted to what subjects were able to verbalise during the process. The possibility of observer bias in making inferences from the encodings of the subjects' responses cannot be denied. It is a well known fact that no matter how carefully a research strategy has been designed to be objective, the method of behavioural analysis through the use of verbal data and encoding categories has to contend with some distortions and incompleteness on the part of the subjects' verbalisation and the inconsistency in inferences on the part of the coder. The taxonomy that was developed is just one of the possibilities which could have been developed in such a study, depending on the position adopted by the investigator on the phenomena under discussion. This would eventually determine what the investigator observed and to some degree influence the regularities and patterns of behaviours that might be identified.

The task of coding was time consuming and required great concentration on the part of the coder. It was not feasible to engage an inexperienced coder for the task. A good level of mathematical background and teaching experience to detect the nuances and complexity of thinking in the students' problem solving processes for a semantically rich domain like mathematics was found to be essential. The coding system developed for this study required coders to be able to identify a wide range of behaviours, including those that deviated from the ideal. Coders needed to be able to detect any errors in mathematical structure, strategies that could lead the problem solver astray or mathematical insights which an untrained person might not have.

Except for some behaviours which were relatively difficult to code or which had few occurrences, the majority of the 28 behaviours in the taxonomy yielded consistent judgement between the different coders. The inter-coder agreements resulted from the two stages suggest that the problem solving behaviours investigated in the present study can be regarded as having been assessed reliably. The final taxonomy that evolved was intended to be used as a tool for investigating problem solving processes in a variety of non-routine mathematical tasks for a variety of problem solvers in the main part of a research project.

Conclusion

The notion of "cognitive process analysis" is seen by Schoenfeld (1987) as central to the cognitive science approach to explore problem solving. In general, the approach involves the construction of a "process model" specifying the particular knowledge accessible to the problem solver, the thinking strategies the problem solver appears to have and the nature of the interaction between the two. Studies using this approach are carried out in great depth and the number of subjects involved is usually quite small. This same research strategy was used in the present study.

For too long mathematics teachers and educational researchers have traditionally evaluated students' performance with results obtained from product-oriented testing. Very often a series of multiple choice tests have been designed and used to measure some dimensions of mathematical understanding or ability. In such tests the only aspect of the student's work that received attention is whether or not the correct answer was selected or whether a certain step or rule had been used correctly. Such assessment of students' performance usually has very little information to offer to the teachers who want to make their students better problem solvers.

However, if mathematics teachers could be trained to listen to students solving problems aloud and to then analyse the processes that were used, they would gain greater insight into their students' thinking. If teachers used some of the classifications of the problem solving behaviours, similar to the taxonomy developed in this study, they might also be able to diagnose the nature of the difficulties encountered by the problem solver along the solution path. With this facility teachers could use more effective diagnostic teaching to improve their students' problem solving performance.

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APPENDIX A

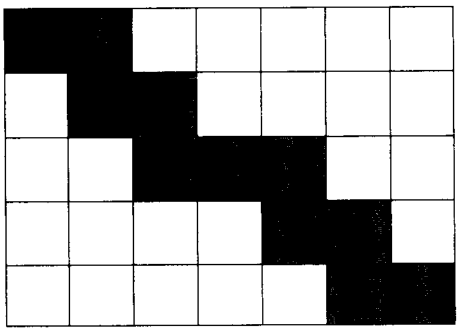
THE PROBLEM SOLVING TASKS - Five Non-Routine Mathematical Problems

1 **Loans**
Sally loaned $7 to Betty. But Sally borrowed $15 from Estella and $32 from Joan. Moreover, Joan owes $3 to Estella and $7 to Betty. One day the girls got together at Betty's house to straighten their accounts. Which girl left with $18 more than she came with?

2 **Palindromes**
A number like 12321 is called a palindrome because it reads the same backwards and forwards. A friend of mine claims that all palindromes with four digits are exactly divisible by eleven. Are they?

3 **Chicken-Coop**
The Smith family wants to build a chicken coop and they have bought enough wire for 19 metres of fence and one gate that is one metre wide. They have decided to make the coop rectangular or square. What width and length would they make the coop so that the chicken can have the largest possible area inside the coop?

4 **Diagonals**
A diagonal of this 5 x 7 rectangle passes through 11 squares (shaded). Can you find a way of forecasting the number of squares passed through if you know the dimensions of the rectangle? How many squares will the diagonal of a 1000 x 800 rectangle pass through?



5 **Christmas Presents** (this task was used only in the pilot study)
Mrs Tan is buying Christmas presents for her six children to give one another. Each child gives a present to each of the others. How many presents must she buy?

APPENDIX B

THE FINAL TAXONOMY WITH A SYSTEM OF CODES

P-Category: Problem-Orientation Heuristics

P1 - First reading of the whole problem
P2 - Re-reading the whole or part of the problem
P3 - Paraphrase problem statement
P4 - Examine dimensions
P5 - Construct Representation

H-Category: Problem-Solution Heuristics

H1 - Recall similar problem
H2 - Draw diagram(s)
H3 - State an answer
H4 - Look at particular or simple cases
H5 - Make a guess and check
H6 - Search for a pattern
H7 - Generalise to a rule
H8 - Make a logical deduction
H9 - Check computation or final answer

K-Category: Domain-Specific Knowledge

K1 - Apply arithmetic algorithm
K2 - State a fact, principle or theorem
K3 - Apply routine mathematical procedures
K4 - Mathematical misconceptions

M-Category: Metacognitive Behaviour

M1 - Suggest a plan
M2 - Assess task facility
M3 - Review progress
M4 - Recognise error
M5 - Recognise new development
Q1 - Task-relevant self-question
N1 - Task-irrelevant rhetoric

A-Category: Affective Behaviour

A1 - Negative self-evaluation
A2 - Giving up
A3 - Emotional expression

APPENDIX C

SOME OPERATIONAL DEFINITIONS OF PROBLEM SOLVING BEHAVIOURS

The operational definitions helped to specify the phenomena i.e., the problem solving behaviours under investigation in the present study, to provide a basis for distinguishing between various items of the taxonomy. For a better introduction to the general nature of the phenomenon to be observed, some examples from the subjects' problem solving protocols were extracted to provide a frame of reference for the operational definitions.

Every behaviour was assigned a code, eg P3 for "Paraphrase problem statement" and if the behaviour was incorrect, inappropriate or irrelevant to the solution then a bar was drawn over the code, eg $\overline{\text{P3}}$ for "Misinterpret problem statement". The following are sample definitions of selected behaviours in the five categories of the taxonomy:

PROBLEM-ORIENTATION HEURISTICS

Eg **P3 - Paraphrase problem statement.**
Attempt to give meaning to a certain word or to rephrase the problem in their own words.

$\overline{\text{P3}}$ - Misinterpret problem statement.
A subject sometimes misinterpreted the meaning of a certain word or of a certain part of the text from the problem.

Some examples of code P3 :
(a) "...loan and borrow..owes and borrow is the same."
(b) "..Can you find a way of forecasting ....forecasting means what.?...ah..is it to predict?........Yes."

PROBLEM-SOLUTION HEURISTICS

Eg **H5 - Make a guess and check**
Subject may predict or expect a certain result while considering a particular case and then proceed to check it.

An example of code H5:
(a) "....so, I'll expect 3 by 7 to be 9...let's check that 1,2,3...1,2,3,4,5,6,7,...Ok, I expect 9 squares cut ...1,2,3,4,5,6,7,8,9..yes."

Eg **H6 - Search for pattern**
This behaviour follows from H4 or H5 where subjects, after trying some cases, attempted to identify relationships for a pattern or common property among them.

Some examples of code H6:
(a) "....Ok, let me see the relationship again..if it's 5 x 7 is 11, 10 by 8 is 16, 6 by 4 is 8...."
(b) "....let's try and get pattern...2 by 3 you have 4 squares, 2 by 4 you have 4 squares, 2 by 5 ....6."

DOMAIN-SPECIFIC KNOWLEDGE

Eg **K3 - Apply routine mathematical procedures**
Subjects may apply domain specific routine procedures in algebra, geometry, calculus or others in their solution paths.

$\overline{\text{K3}}$ - Inappropriate application of procedures
Structural errors may stem from a misunderstanding of the problem or some of the principles necessary for its solution. However, any computational error made while manipulating these procedures is coded as **$\overline{\text{K1}}$**. Sometimes subjects may try to use certain mathematical procedures which are inappropriate to the solution of the problem.

Some examples of code K3:
(a) "....I can do it this way...ABBA is equal to 1000A + 100A +10B +1A , alright..and then I group them together and get 1001A + 110B, right."
(b) "....can use calculus to calculate...so that gives me ...differentiating with respect to small *a* ,we get 10 - 2*a* with zero, we get *a*=5 and *b*=5.."

METACOGNITION

Eg **Q1 - Self-questioning**
In a problem-solving situation it is quite natural for subjects to ask questions of themselves. These self-questions could be task-specific and were often junctures at which decisions had to be made, eg "....let's say if it's a rectangle in shape..can I have other possible numbers?"

Eg **M1 - Suggest a plan or subgoal operation**
Subjects might consider the possibility of using a particular plan or subgoal operation. This might be a global plan or hypothesis where the subjects could see the solution and sometimes it could be just an exploration of ideas.

Some examples of M1:
(a) "So we need to generalise first and then find answer for the specific dimensions of 1000 x 800.."
(b) "....Ok, let's draw a table and see if I can generalise anything from there."

Eg **M2 - Assess the task facility**
When presented with the task some subjects would quite spontaneously perceive it as easy, difficult or confusing. Sometimes in the middle of the solution the subject may say a certain operation is easy or hard to do.

Some examples of code M2:
(a) "let me see...it's simple...straightforward..."
(b) "If it's a square...then it'll be easier."

Eg **M3 - Review Progress**
Subjects may review and evaluate progress at any stage of the solution path, in terms of adequacy and reasonableness of any results obtained; method used; or outcome of any action or decision made, for example: "What I'm doing is I'm adding it this way...OK... no, that's not very good."

AFFECTIVE BEHAVIOURS

Eg **A1 - Negative Self-evaluation**

Subjects assess their own ability, confidence or personality. Express self-doubt, they say they don't know how to do; can't remember; don't understand; not sure etc.

Some examples of code A1:

(a) "....Umm..not again!..I can't do!"
(b) "....Aiyah, my math is not working!"

Eg **A2 - Giving Up**

Lack of persistence. Subjects in this study were not given the chance to ask for help if they were stuck. This behaviour was coded when subjects indicated their intention to quit the task.

Some examples of code A2:

(a) "Hiyah, I think I give up, I just don't know how to do!"
(b) "I surrender already...Ok, I surrender here."

APPENDIX D

A SAMPLE OF THE PROTOCOL ANALYSIS ON THE LOAN-PROBLEM

Segment	Code	Verbatim Transcript
1	P1:	"Sally loaned $7 to Betty. But Sally borrowed $15 from Estella and $32 from Joan. Moreover, ...Which girl left with $18 more than she came with?"
2	P2:	"Ok, one more time...Sally owes $7 to Betty.....(repeats reading the question)."
3	M2:	"Umm..this is quite confusing."
4	M1:	"Ok, I'll consider each person one at a time."
5	H2:	"S..B..E..and J.. (drawing a table) "
6	P4:	"Sally loans $7 to Betty, so Sally will receive $7 in the end and Betty returns $7, so plus 7 here and minus 7 here."
7	P4:	"But Sally borrow 15..borrow 15..so minus 15 from Estella..so from E put +15.."
8	P4:	"$32 from Joan..so minus $32 for Sally and Joan plus $32."
9	P4:	"Moreover, Joan owes $3, so she must minus 3 to E, E must plus 3 and owes $7 to Betty so another minus 7 to B, B must plus 7, alright"
10	P2:	"One day the girls got together and ...which girl left with $18 ? "
11	M3:	"Now is easy .. to add up for each one."
12	H4:	"Sally, cannot because she's still in debt."
13	H4:	"Er..Betty..is zero..ya..."
14	H4:	"Estella got $18 right, plus..."
15	H4:	"and Joan has 32 minus 10, $22 to take home."
16	H8:	"So looks like Estella is the girl."
17	H3:	"The answer is Estella left with $18 more than she came with."

Sequence of behaviours in string of codes in Protocol 1A:

P1 —> P2 —> M2 —> M1 —> H2 —> P4 —> P4 —> P4 —> P4
—> P2 —> M3 —> H4 —> H4 —> H4 —> H4 —> H8 —> H3

Conversely the sequence of codes can be interpreted as:

> Subject first read the problem (P1), reread the problem (P2), found it confusing (M2), suggested a plan to organise the given information (M1), by tabulating (H2), then examined the dimensions of the problem statement by breaking down into the table drawn (P4, P4, P4, P4,), reread part of the question (P2) then reviewed to sum up the numerical relationships (M3) by looking at specific cases of the borrowers and lenders (H4, H4, H4, H4) and making a logical deductions (H8) and finally stated the correct answer (H3). [time taken : 2 min. 15 sec.]