Teachers' pedagogies and their development of mathematical problem solving

HO Kai Fai
Centre for Research in Pedagogy and Practice, National Institute of Education,
Nanyang Technological University, Singapore
Email: kfho@nie.edu.sg

John HEDBERG
Australian Centre for Educational Studies, Macquarie University, Australia

Luis T. LIOE
Centre for Research in Pedagogy and Practice, National Institute of Education,
Nanyang Technological University, Singapore

This paper attempts to address the question of the extent emphasis on mathematical problem solving as spelt out in the syllabus gets enacted in the classrooms of three primary 5 teachers. It reports on the initial development of a video coding scheme and offers a glimpse into the teachers' pedagogies and practices. Through the coding scheme, results suggest that the focus on problem solving, the range of heuristics observed and types of problems used were somewhat limited.

Introduction

'Mathematical problem solving' (MPS) is at the centre of the Framework of the mathematics curriculum in Singapore (Ministry of Education, 2000). Its primary aim 'is to enable pupils to develop their ability in MPS' and deal with problems that cover 'a wide range of situations from routine mathematical problems to problems in unfamiliar contexts and open-ended investigations that make use of the relevant mathematics and thinking processes.'

Anecdotal evidence has suggested that students in Singapore schools are not exposed to a wide variety of problems and the full range of problem solving heuristics. In a two-year study (Chang, Kaur, Koay and Lee, 2001) of four teachers, two each from two Singapore schools, investigating the pedagogical practices in primary mathematics classroom, a traditional teaching approach was found to predominate amongst the teachers – expository teaching, followed by pupils practicing routine exercises to consolidate the concepts, knowledge and skills. This suggests that the classroom focus is very much on acquiring skills and concepts, and that students are not exposed to a wider range of situations to include unfamiliar contexts and open-ended investigations as mandated by the syllabus. In other words, it appears that an important component of MPS, i.e. the processes incorporating thinking skills and heuristics are not given due emphasis in classrooms. Although the study was limited in scale and the sample small, other evidence (see Hedberg, Wong, Ho, Lioe & Tiong, 2005) further support the view that such an approach to teaching was prevalent in many primary schools – students are not exposed to a wide variety of problems and the full range of problem solving heuristics as mandated in the curriculum.

However, Cai (2003)'s exploratory study suggests that that may not be the case. He explored fourth, fifth and sixth grade students MPS skills, using four tasks which were mathematically rich and were "embedded in different content areas and contexts and allow for examining Singaporean students' thinking from various perspectives." He found most students were "able to select appropriate solution strategies to solve" the tasks, and chose "appropriate solution representations to clearly communicate their solution processes" (p.733). Further, Singaporean students' top ranking performance in mathematics – first in the Third International Mathematics and Science Study (TIMSS) (Kelly, Mullis & Martin, 2000), then in the Third International Mathematics and Science Study – Repeat (TIMSS-R) (Mullis, Martin, Gonzalez, Gregory, Garden, O'Connor, Chrostowski & Smith, 2000), and again in the Trends in International Mathematics and Science Study (TIMSS-2003) (Mullis, Martin, Gonzalez, & Chrostowski, 2004) suggests that the extant syllabus is working well and the school mathematics curriculum in place has somehow *delivered*.

Such seemingly conflicting findings warrant further investigation. While the curriculum emphasizes MPS and a more dynamic, inquiry-based, problem-solving paradigm to promote thinking skills and independent learning (MOE, 2001, p.5), the questions of how the emphasis is enacted in classroom practices if at all, and what sort of effects such practices have on the learning outcomes of students remain. Such concerns warrant a development of a systematic, evidence base that describes current practices of mathematics instruction, and suggests guidelines for practice and policy through the examination of interventions specifically designed to support mathematical problem solving.

This paper takes an exploratory look at some aspects of the evidence base, namely, the classroom practices of Primary 5 teachers teaching mathematics, the type of mathematical word problems used in class and the range of heuristics that can be used to solve those problems. It reports on the development of a video coding scheme designed to look for main pedagogical features of classroom practices and some of the initial findings.

Methods

The sample

The focus is on three teachers from three different schools who were involved in a larger study. Their involvement came about on an opportunistic basis through a meeting with the schools' superintendent. The three schools were from an upper band, an average band and a lower band (in national academic ranking terms). Each teacher either taught in the top or the near-top class of their school. All three had been teaching for at least eight years when the study began.

The video coding scheme

The scheme was developed using the Grounded Theory approach (Glaser & Strauss, 1967) where ideas of pedagogical phases *emerged* as the videos were reviewed. After several iterations, a coding scheme that divided each lesson to five phases comprising *problem solving*, *teaching of skills and concepts*, *going over assigned work*, *student activities* and *others*, was developed. The *problem solving* phase happens when the teacher presents a problem to teach problem solving. It usually involves teacher's explication or teacher initiated question-and-answer. This phase is further broken into Polya (1988)'s four stages of problem solving namely understanding, planning, executing and reflecting. Teaching of skills occur when the teacher uses an example usually from the textbook to teach *specifically* skills such as arithmetic or algebraic manipulation, estimation/approximation, mental calculation, communication, use of mathematical tools, handling data, etc. This teaching of skills is to be distinguished from the problem solving *Execution* phase where skills are *used* to solve a problem after some preceding *understanding/planning* has taken place. For the teaching of concepts, time is spent showing, demonstrating, defining, explaining numerical, geometrical, algebraic or statistical concepts. For example a teacher tells students what parallel lines are, and show how alternate angles are equal. The *Going Over Assigned Work* phase refers to a time when the teacher goes over work that had been assigned. Students would have had spent some time on the assigned work. Common examples include given exercises or worksheets, tasks/homework, test paper/assessment, etc. The *Student Activites* phase refers to students spending class time doing assigned tasks. This phase is divided into: Presentation, Group Work and Seat Work. Presentation is when student(s) are tasked to present their solutions of assigned problems to their classmates/teacher using the white board or overhead projector, with or without their oral presentation. Group work happens when groups of students are given tasks to do as a group. Seat work refers to students doing assigned tasks individually (sometimes with short consultation with the teacher). The *Other Events* is a catch-all category to include all other events not listed in the previous phases. Examples can be teacher reorganizing the class, making an unrelated announcement, telling a story, disciplining students, etc.

Two other aspects important to the development of the evidence base are the type of mathematical word problems used in class and the range of heuristics that can be used to solve those problems. Both are included in the coding scheme. In identifying types of problems, an adaptation from Foong (2002) and Frobisher (1994)'s classification of problem types is used. There are four types of problems, namely closed-routine, closed non-routine, open-ended with goals known and open-ended investigations/projects. The range of heuristics to be coded in the scheme taken directly from the official syllabus (Ministry of Education, 2000, p. 6) is listed in Table 3.

Results and Discussion

At the time of the scheduled observations, the teachers were teaching whole numbers and fractions. A total of 24 lessons, averaging about 50 minutes each were video recorded: 9 lessons from *P*, a male teacher from the upper band school, 8 from *Q*, a female teacher from the middle band school and 7 from *R*, a female teacher from the lower band school. Lessons were recorded as a continuous series for each teacher over a period of about two weeks. The following Table 1 summarizes the time each teacher spent on each phase:

Table 1: Amount and Proportion of Time Each Teacher Spent on Each of the Five Phases

Teacher	Problem Solving (% of total)	Teaching Concepts/Skills (% of total)	Going Over Assigned Work (% of total)	Student activities (% of total)	Other Events (% of total)	Total Time Observed (100%)
P	0:53:00 (12%)	1:03:30 (15%)	2:25:30 (34%)	1:17:30 (18%)	1:26:30 (20%)	7:06:00
Q	0:07:00 (2%)	1:13:00 (17%)	0:33:30 (8%)	3:27:00 (48%)	1:51:00 (26%)	7:11:30
R	0:26:30 (8%)	0:41:00 (12%)	1:15:00 (22%)	2:32:30 (46%)	0:38:30 (12%)	5:33:30

The three teachers shared some common ways of teaching but differed in the amount of time spent in the various phases. Focusing on the problem solving phase, the proportion of time the teachers spent on giving instructions via the Polya's four stages of problem solving was low, at 12%, 2% and 8% for *P*, *Q* and *R* respectively. These low percentages suggest that the main focus of the series of recorded lessons was *not* on problem solving per se (in the Polya sense). Instead teachers *P* and *R* spent nearly three times as much time *Going Over Assigned Work* where the main emphasis was to show the procedures needed to solve the problems. Teacher *Q* also spent a comparable proportion of time going over assigned work. Based on the proportion of time, when it comes to giving instructions on problem solving, the teachers seemed to approach it through going over assigned work rather than tackling problems afresh via the four stages recommended by Polya.

Looking further at the problem solving phase, the following Table 2 shows the time each teacher spent on each of the Polya's four stages:

Table 2: Time Spent on Each of the Four Stages of Problem Solving

Teacher	Understanding (% of total)	Planning (% of total)	Executing (% of total)	Reflecting (% of total)	Total Time Spent on Problem Solving Phase
P	0:16:00 (30%)	0:01:00 (2%)	0:20:30 (39%)	0:15:30 (29%)	0:53:00 (100%)
Q	0:01:00 (14%)	0:00:00 (0%)	0:04:00 (57%)	0:02:00 (29%)	0:07:00 (100%)
R	0:04:30 (17%)	0:00:00 (0%)	0:06:30 (25%)	0:15:30 (58%)	0:26:30 (100%)

One clear point that emerges from the table is that the three teachers spent little or no time in the *planning* stage when giving instructions on problem solving. Generally the questions were read (i.e. *understanding* the problem), and executed immediately after. This was followed by looking back and checking (i.e. *reflecting*) on the correctness of the solutions. Qualitative aspects about the *way* the teacher went about the four Polya's phases of problem solving were not reflected in these numbers. Notwithstanding, these observations highlighted a need for teachers to spend more class time on solving mathematical problems, going through more thoroughly the four phases recommended by Polya. In particular, the need to be more explicit during the *planning stage*, expressing what they had in mind and perhaps also explore other possible plans before proceeding to *execute* the plan. The *reflection stage* can also be enhanced to include not just checking the correctness of the solutions but also exploring the use of other heuristics, to derive the result differently.

The range of heuristics observed in the *Problem Solving* and the *Going Over Assigned Work* phases is summarized in Table 3:

Table 3: Range of Heuristics

Range of Heuristics in Syllabus	Teachers			Total
	P	*Q*	*R*	Range of Heuristics observed
A. Act-it-out				
B. Use a Diagram/Model	20	2	9	31
C. Use Guess & Check				
D. Make a Systematic Lists				

E. Look for Pattern(s)				
F. Work backwards	1			1
G. Use Before-After concept	1			1
H. Make Suppositions				
I. Restate the problem	1			1
J. Simplify the problem				
K. Solve part of the problem				

From the table it is clear that the predominating heuristics used by the teachers is *Use a Diagram/Model*. Only *P* tried using the Work Backwards, Use Before-After Concept and Restate the Problem heuristics albeit only once. About 95% of problems used were closed routine taken mainly from the textbook, while the rest were closed non-routine taken from non-textbook sources such as commercially assessment books. Neither open-ended nor investigative problems were observed. The limited range of heuristics and problems highlighted the need for teachers to widen the variety of problems to include more non-routine and open-ended types and to develop their repertoire of problem solving heuristics to extend beyond Using a Diagram/Model.

For the teaching of concepts and skills, teachers spent between 12 to 17 percent of their class time. The main emphases were the acquisition of concepts related to fractions and the skills needed to perform the related arithmetic operations.

The Student Activities Phases of the three teachers were very different. The following Table 4 gives the details:

Table 4: Time Spent on the Three Types of Student Activities

Teacher	Student Presentation (% of total)	Group Work (% of total)	Seat Work (% of total)	Total time Spent on Student Activities
P	0:09:00 (12%)	0:00:00 (0%)	1:08:30 (88%)	1:17:30 (100%)
Q	0:30:30 (15%)	0:46:00 (22%)	2:10:30 (63%)	3:27:00 (100%)
R	0:54:00 (35%)	0:00:00 (0%)	1:38:30 (65%)	2:32:30 (100%)

In *P*'s case, the main student activity was seatwork although some students showed their work on the white board without verbally presenting their solutions. Seatwork took up about sixty odd percent of the Student Activities time in *Q* and *R*. In the main seatwork involved students doing assigned exercises individually and copying solutions written on the board for corrections. Only *Q* did group work with her students where each group solved problems and presented their solutions as a group. In *R*'s case, her students' presentation involved individual students going up to the white board to write their solutions to assigned problems. Like *P*, she would then go over the students' board work to check the correctness of the solutions.

Reliability

Percentage agreement, defined as the proportion of the number of agreements to the number of agreements and disagreements, was used to estimate inter-rater reliability. Out of the 24 lessons, five were randomly selected to be coded by two coders. The percentage agreement was about 78%.

Limitations

Some limitations of the coding scheme emerged from this initial phase of development. For example the *Going Over Assigned Work* phase can be further broken down to reworking the problem using Polya's four stages, focusing on the procedural and quick answer checking, to yield richer details. The *Other Events* phase could also be further refined to distinguish between those that relate to the on-going lesson directly such as the teacher giving instructions to reorganize the class and those that are not related such as the teacher making a sports day announcement. This would give a better account of on- and off-task class time.

At this phase of development of the coding scheme, only the main features were identified and coded. Qualitative nuances such as the way the teacher went about *understanding* the problem in the *problem* *solving phase* or teaching and linking of important mathematical concepts were not captured. The question of whether such qualitative aspects which are important classroom pedagogical processes can be worked into the scheme remains to be explored.

Summary and Conclusion

Some general pedagogical features of classroom teaching of mathematics were identified. Using the coding scheme with the main phases of teaching problem solving via Polya's four recommended phases, teaching concepts and skills, going over assigned work and student activities, a glimpse of the classroom practices of three teachers were captured. Through it, the three teachers were observed to use only about 8 to 12 percent of their class time teaching problem solving skills. Of this time, much of it were spent on reading/understanding the problem, executing the steps to solve and looking back to check the correctness of the answer. There was very little or no time spent on the planning part of the problem solving process. The range of heuristics used by the teachers was limited mainly to the *Using a diagram/model* heuristic and the types of problems were confined to those found in the textbooks, mainly closed routine.

These findings suggest that the emphasis on MPS as enacted in the three classrooms was somewhat limited. The *Use a diagram/model* heuristic that was observed most often was just one of the eleven heuristics outlined in the syllabus. The ten remaining heuristics were either used infrequently or not at all. The limited range of problems observed was not the "wide range of situations from routine mathematical problems to problems in unfamiliar contexts and open-ended investigations (MOE, 2000, p.5)" as mandated in the syllabus.

This study offers a glimpse into the classrooms of three teachers. While the findings cannot be generalized beyond what was observed, it nevertheless provides some ideas about pedagogical features that relate to the teaching of MPS. It gives a picture that would hopefully contribute to a better understanding of teachers' pedagogies and practices. With it, there is some basis to work towards teacher development of MPS.

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