The Impact of Online Video Suite on the Singapore Pre-service Teachers' Buying-in to Innovative Teaching of Factorisation via Algecards

Weng Kin HO
National Institute of Education, Nanyang Technological University, Singapore
wengkin.ho@nie.edu.sg
+65-62196129

Yew Hoong LEONG
National Institute of Education, Nanyang Technological University, Singapore
yewhoong.leong@nie.edu.sg
+65-67903973

Foo Him HO
National Institute of Education, Nanyang Technological University, Singapore[1]
Foo_Him_Ho@moe.gov.sg
+65-97893297

Abstract

A group of pre-service teachers at one institute of education was assigned to record their reflections as they viewed an online video suite which recorded (i) a secondary mathematics teacher employing a Concrete-Pictorial-Abstract (CPA) based approach to teaching quadratic expansion and factorization to a group of low-ability secondary 2 (14+) mathematics students, (ii) students using concrete manipulative, Algecards, and pictorial representations, Rectangle Diagram, to perform quadratic expansion and factorization, and (iii) teachers, including a Head-Of-Department (HOD), sharing their teaching experience in the use of the teaching package based on the abovementioned CPA approach. This chapter studied the impact of the online video suite on the pre-service teachers' willingness to *buy-in* to innovative teaching of factorisation via Algecards through inspecting the written responses of these teachers to pre-set Milestone Tasks placed at different points along the progression of the video watching. The term 'buy-in' refers to the evidential shift of teacher's belief in the applicability of the lesson innovation. We identified the salient features of the video suite which were responsible for bringing about pre-service teachers' buying-in to the use of Algecards in teaching factorisation. Based on these findings, this chapter makes some recommendations on the design of teacher-preparation method-course using video technology.

Introduction

One of the persistent problems in pre-service teacher education is the theory-practice gap that is purportedly experienced by pre-service teachers (henceforth, PT for short) as they move from methods courses offered by institutes of teacher education (perceived as focusing primarily

[1] This co-author is now posted to Ministry of Education, Singapore.

on ‘theory’) to field placements in school (which they perceive as the arena for ‘practice’). This sense of disjunction is not helped by the traditional structure of pre-service programmes: frontloading theoretical ideas taught usually in university classroom settings with scarce opportunities to interact with authentic practice; then positioning field placement as a follow-up course where these theoretical ideas are *applied* in actual practice (Darling-Hammond, 2010; Zeichner, 2010). We think that one way to smoothen this perceived theory-practice divide is to restructure methods courses in such a way that authentic practices are featured alongside the introduction of theoretical ideas. In so doing, methods courses are injected with a certain degree of authenticity that can serve to increase the sense of direct relevance of these courses and to heighten anticipation for the realities of practice during field placements.

It is, however, not always easy to provide opportunities to observe real-time classroom instructions within the duration of the methods courses. In the case of the consistently large pre-service cohorts of over 150 pre-service teachers at the research site, henceforth known as the Institute the logistical cost of doing so arguably outweighs its benefits. Video records of authentic classroom instruction thus provide a potential avenue to realize the goal of bridging the theory-practice gap.

Videos for teacher development

Video has the potential of capturing the richness and complexity of classroom discourse. It allows one to enter the world of the classroom without being physically present in the teaching-in-the-moment (Sherin 2004). Videos afford repeated analyses of images and audio that take place within the classroom discourse—the detailed interactions between teacher and students and between groups of students—all at times and locations to the convenience of the user. Unlike live observation of a lesson, video-records are available on demand. Digital video clips can be processed and enhanced easily by embedding notes and instructions to aid learning and facilitate understanding. Also, video excerpts addressing a particular feature of teaching and learning can be isolated for analysis and discussion. Used in this way, video is a good tool to direct viewers’ attention to focus on the desired learning objectives (van Es & Sherin, 2008, 2010). The problem pre-service teachers have with factorisation was that they were unable to connect factorisation of quadratic expressions to a concrete meaning that was visible to beginners.

We, the authors of this chapter, using our experiences in teaching the mathematics methods course in the Institute to several cohorts of PTs, agreed that most PTs who taught factorisation of quadratic expressions in their micro-teaching sessions (where PTs practised teaching certain topics to their peers) continued to use the usual "cross-method" of factorisation. This was despite the fact that PTs were introduced the use of algecards (or algetiles -- a similar manipulative without algebraic labels) to teach factorisation of quadratic expressions. It therefore prompted us to investigate if the use of video suites for this particular topic has any positive effect in changing the beliefs of PTs. In particular we wanted to ascertain whether the PTs would be convinced of the efficacy of the use of algecards in the teaching of factorisation if they observed established teachers teaching using such tools to the extent that they express their willingness to try this teaching approach in their own lesson in the future. In addition, since video technology is most suitable for capturing the real-life use of the algecards, the choice of this topic amongst others seems compelling.

Design considerations of a video suite

Although there is growing literature on task design for teacher education in recent years (Thompson, Carlson & Silverman 2007) there is a lack of discussion on the following domains which are the foci (and constraints) of our work as mathematics teacher educators: (1) use of videos (2) on authentic classroom instructional work (3) for the purpose of teacher development, (4) based on tasks that are in-built with features which exemplify or lead to an awareness of sound pedagogical ideas in the teaching of mathematics, and (5) that are suitable for use by a large cohort of PTs.

For Domain (1), the use of videos is a technological means. Domain (2) demands the use of genuine classroom teaching to show sound pedagogical practices. In pre-service teacher training, relevant theoretical framework and assumptions of several learning theories were taught to the pre-service teachers. Usually these theories were supported by some case studies documented in the literature. *However*, pre-service teachers did not have the opportunity to see a live application of these theories in an authentic classroom situation. The contents of the video necessarily address this specific need that was not addressed by the theoretical phase of their training. Domain (3) refers to teacher training. For Domain (4), task design principles must be present to guide PTs in their training as teachers. Finally Domain (5) demands that whatever the method of pre-service teacher training, it must be applicable to cater for large numbers of PTs who were undergoing the methods course at one time. In addition, Domain (5) demands that the means of instruction must also be available on demand for training of the PTs. Notice that the current methodology of using video for pre-service teacher training presents itself as a natural intersection of all these five domains.

The models of video use for professional development of teachers attend to some but not all of the requirements listed above.

The following six principles guided us in designing the video suite.

(a) The videos capture authentic classroom practices. In the Singapore context, local classroom conditions typically include (i) one teacher giving instructions to a class of size not exceeding forty, (ii) grouping of students of roughly the same overall academic ability, and (iii) requirement for the teacher to complete a fixed set of topics listed in the mathematics syllabus via a commonly-shared (among other teachers) scheme of work.

(b) The classroom practices exemplify instructional innovation that is based on sound pedagogical principles.

(c) The materials contain sufficient evidence that directs the learner to assess the applicability of the innovation and to infer the pedagogical principles underpinning the innovation.

(d) Within the overarching anticipated learning trajectory embodied in a sequence of tasks, there is room for individual learner's preferences and interests.

(e) There are opportunities for learners to reflect on their learning.

(f) The suite is fully accessible online.

The motivation for (a) and (b) was to show PTs how teaching approaches and learning theories taught in the methods course could be applied to a typical classroom in a local school. The emphasis here was about applicability. One common concern of teachers is that the innovation may be time consuming. They may choose not to use the innovation if the trade-off is not so favourable. More precisely, they have to decide if the innovation is too time consuming and the learning outcome is not obvious or too insignificant for them. We now give elaborations on (c) – (f). Video is only a tool for learning (LeFevre, 2004). So, for (c), the tasks offered for learning and the nature of facilitation with video need to be coordinated in such a way as to capitalize on what video has to offer. Thus, directing the learners to assess an instructional innovation and to look for the theoretical underpinnings provide this needed focus to their work on the video-based tasks. At the same time, we wanted to retain a sense of flexibility within the task sequences so as to allow elements of self-directedness in the learning. This balance was provided for in (d).

Reflection is an integral component for teacher thinking and development (Schön 1983, 1987 and Dewey 1933). Hence (e) is an important component of this study and the use of reflections will be elaborated in a later section of this chapter. Through purposeful reflections, learners were given the opportunity to compare and contrast what they have observed against their existing beliefs (Stockero 2008, Alsawaie & Alghazo 2010). This pairing could result in coherence, confrontation, or changes. In any case, the task of reflection was intended to provoke some of these outcomes that can facilitate deeper thinking about mathematics teaching. As for (f), given the current technology and infrastructure available, online was a mode that was appropriate to allow learners access to the videos. It allowed for multiple entries at different time junctures (throughout the day) to the video suite and thus offers flexibility that could help circumvent problems associated to time scheduling and diversities in learning styles. We were also motivated by the ease in importing this video suite as a standalone unit across relevant methods courses that were offered concurrently.

Constructing contents for the video suite

The second and third authors of this chapter had just completed a *Lesson Study* project with some mathematics teachers of a local secondary school. The teachers were implementing an instructional innovation that was based on the Concrete-Pictorial-Abstract (CPA) approach to the teaching of Secondary 2 (to students of 14+) mathematics. CPA is adapted from Bruner's (Bruner 1966) enactive-iconic-symbolic theory and is an approach commonly advocated (Leong et al, 2013) for mathematics teaching in Singapore schools. Readers may refer to Leong et al. (2010) for full details of the study.

The intervention in the *Lesson Study* project involved the use of a manipulative—known to the teachers in the project school as AlgeCards —consisting of three types of 'tiles': a square tile with dimensions of x units and its area duly labelled as "x^2"; a rectangle tile with dimensions 1 unit by x unit and it area duly labelled as "x"; a square tile with dimension of 1 unit and its area duly labelled as “1”. Students were given these tiles and they worked on them to form a rectangle with combined areas represented by appropriate quadratic expressions. Figure 1 provides one example of a rectangle where the area is represented by the quadratic expression $x^2 + 3x + 2$. The length and breadth of the rectangle are the factors of the trinomial. The basic objective of the task was for the students to see the association between the area of the rectangle with the product of the two linear factors. The higher objective was for students to achieve the level of abstraction, i.e., the product of linear factors (dimensions of the rectangles) gives the value of the area of the rectangle, namely the quadratic expression. After substantial experimenting with the AlgeCards, the students then proceeded to represent the rectangles pictorially by drawing an abstracted form using paper and pencil. Finally, they expressed the factorization algebraically. The three modes described correspond roughly to the concrete, pictorial, and abstract stages of CPA and they are illustrated in Figure 1. The resident teacher taught the three lessons, each lesson of 45 minutes duration.

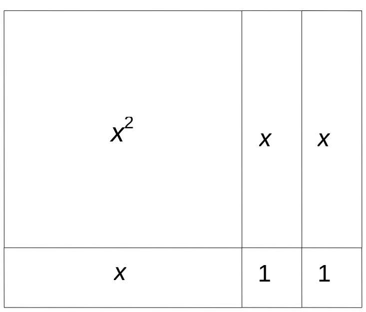
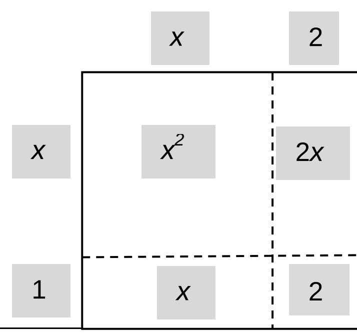
AlgeCards	**Rectangle Diagram**	**Algebra**
		$x^2 + 3x + 2$ $= (x+1)(x+2)$

Figure 10.1. Linking *AlgeCards* to Rectangle Diagram and to the algebraic factorisation

The secondary mathematics teachers had chosen to apply the innovative approach to the teaching of factorisation of quadratic trinomials because the students found factorisation of a quadratic trinomial challenging. In particular they failed to see the relationship between the two linear factors and the trinomial. The lessons were innovative because they showed how a teacher in a typical Singapore classroom was able to employ the CPA approach, using algecards, to help students establish the aforementioned relationship. Hence the use of algecards as an instructional innovation made the lessons more meaningful to the students than in a conventional classroom where the teacher would show the algorithmic procedure for finding the factors of a quadratic trinomial.

Furthermore, the work of this teacher was carried out in real-time, under the usual local classroom conditions (e.g., 35 students), and exemplified a lesson implementation that was designed and supported by sound pedagogical principles (i.e., CPA approach), and thus fulfilled the Design Principles (a) and (b) for selecting materials for the videos.

All the relevant data from the project school—videos of classroom happenings, videos of teachers' and students' reflections, lesson plans and worksheets—we collected from the *Lesson Study* enterprise were processed for the purpose of packaging them in a way that would fulfil Design Principle (c), that is, through accessing the videos, the learner will not only see a limited view of the lessons; rather, they will be able to take different angles (such as students' learning, teacher-observers' points-of-view) to enable them to follow the entire flow of the lessons and thus assess whether it is feasible to deploy the CPA approach in their own teaching of factorisation.

The contents were packaged into an e-learning suite which consists of five sequential sections. The structure and contents are summarized in Table 1. Videos were cut into snippets of no more than four minutes each. In particular, the video snippets in Section 2 demonstrated in a step-by-step fashion how the teacher exemplified the use of the algecards over a large number of trinomials. In line with Design Principle (d), the PTs has the flexibility to view as many of these demonstrations as was needed to gain an understanding of how the algecards helped students learn about factorisation and how they can be deployed in a typical classroom situation with 35 students. Similarly, in the Section 4, a number of teachers who had used the same teaching innovation with their students were interviewed; the PTs may choose to view all or some of these interviews to get as much information as was needed to appreciate the feasibility of this teaching innovation through the lens of the teachers themselves.

The PTs were required to proceed with each Section sequentially. To ensure that the PTs spent sufficient time viewing a number of video snippets so as to have a more wholistic idea of the actual innovation as it was carried out in the project class, an administrative device was set in the video suite to prevent PTs to proceed to the next section at will. Also, the exact duration of each section was not made known to the PTs; otherwise, such knowledge of these durations may result in some PTs skipping the entire segment of the video within a Section and only returning to the end of it for transit to the next Section. Moreover, the Milestone Tasks at the end of each section was intended to provide focus and goal to what the PTs would notice from the videos. The Milestone Tasks were also built in for the purpose of providing opportunities for their reflections (Design Principle (d)). In line with Design Principle (e), the PTs accessed all the videos and attempted the Milestone Tasks solely through electronic means over a recommended total time—not necessarily continuous in one-go—of six hours. Prior to the implementation of this video-suite reported in this chapter, a pilot version was implemented to an earlier cohort of PTs for the same methods course. From this preliminary implementation, we gathered sufficient feedback from the PTs concerning both the time spent in each Section and the total time spent in working through the entire video-suite. A majority of those PTs indicated that both these durations were long enough for them to work comfortably towards the completion of the Milestone Tasks. Reassured by this feedback, we decided that the timing set was about right and made no changes to these durations.

Table 10.1.

Structure of the e-learning suite that Pre-service Teachers accessed.

Section	Main content	Data from Lesson Study project	Post-section milestone task for student teachers
1	Overview of instructional innovation	Lesson plans and worksheets used; article by Leong et al. (2010).	Milestone Task 1: Tasks that check PTs' proficiency in the use of the Rectangle Diagram in expansion and factorisation
2	Zoom-in to Lesson 2	Selected video snippets of Lesson 2 that highlight the key moves of the lesson, including selected snippets of students' seatwork and discussions.	Milestone Task 2: A series of questions about Lesson 2
3	Zoom out to before and after Lesson 2: Lessons 1 and 3	Selected video snippets of Lessons 1 and 3 that highlight the key moves of the lesson, including selected snippets of students' seatwork and discussions.	Milestone Task 3: A question about PTs' overall response to the module
4	Views of teachers and students	Video snippets of interview sessions conducted with selected teachers in the Lesson Study team. Video	Milestone Task 4: A review of PTs' response in Milestone Task 3 in the light of feedback

		snippets of interview sessions with two students selected by the teachers as among the most mathematically-challenged.	from teachers and students
5	Overall reflections	None	Overall response to the package in relation to ideas about teaching mathematics covered in the Mathematics methods course at NIE.

The study

The focus of this study is to examine how video technology could be used to influence pre-service teachers' belief so that they were more likely to buy-in to innovative teaching of factorisation via Algecards. In the research literature on teachers' belief, it has been well-established that a teacher's set of core beliefs are very resistant to changes (Ball and Cohen 1999, Schoenfeld 1998, 2003, Törner 2002, Törner et al 2006, 2010). Such resistance could be partly explained by Lortie's notion of "apprenticeship of observation" (Lortie 1975, p. 61), i.e., PTs wanted to teach the way they were taught. Once PTs' have acquired a set of beliefs of what works in their class, they are very resistant to changes. Because beliefs are very difficult to change, it is important to present to PTs alternative pedagogies which are powerful and produce results that are significant. In this study we propose that video technology can be harnessed to influence the PTs' belief in the usefulness of the teaching innovation of using Algecards in teaching factorisation. Using this video-suite, together with the Milestone Tasks, we want to identify which aspects of the suite that brought about observable changes, if any, in the PTs' belief about the usefulness of the teaching innovation of the algecards. The Milestone Tasks were designed with the specific aim of eliciting the PTs' views about the CPA approach (via algecards) as they progressed through the different Sections of the suite. We hypothesised that the PTs' responses to those questions in the Milestone Tasks would provide evidence for shifts, if any, of the PT's beliefs.

This chapter focus on how, with the aid of Algecards and the CPA approach, teachers were able to help students acquire a more meaningful understanding of factorization and to make sense of the three representations, concrete, pictorial and algebraic (in this case, the CPA approach).

Analysis

We adapted Manouchehri's (2002) five levels of reflection: *describing*, *explaining*, *theorizing*, *confronting* and *restructuring*, to classify the levels of reflection manifested in the PTs' responses. Reflection at the *describing* level was a surface description of the happenings in the classroom. Reflection at the *explaining* level was an attempt to relate PTs' actions to a teacher's approach to teaching; in other words a cause-and-effect explanation was dominant at this level. At the *theorizing* level, a PT's reflection on certain trajectory of teaching or teaching approaches was supported by research findings or learning theories. Reflections in the c*onfronting* category referred to PTs' ability to provide deeper analysis whereby they expressed different opinion about certain teaching approaches and teachers' actions. In the confronting stage, the PT questions about the existence of alternative theories or explanations that justify what has been observed in a certain learning-teaching moment. In the original formulation of "confronting", Manouchehri 2002 restricted its scope to those activities that involved the PT challenging one's own views about one's teaching or a peer's teaching. To be relevant to our present context in which the PT views the teaching practice of the videoed teacher, we have widened the scope of interpretation for "confronting" to include those reflective processes of challenging one's own views about the teaching carried out by the videoed teacher. For example, those PTs who reflected by asking "Did she do it right?" or "Is there another way to explain what she was doing?", were classified as engaging at the confronting level. At the fifth level, *restructuring* referred to PTs' reflection that focused on "how an experience (either their own or another teacher's) can be redesigned to avoid potential problems or better support learning" (p. 377, Stockero 2008). In this study restructuring occurred when a PT undergoes a change of beliefs that fit with modified conceptions of teaching. For example, some PTs, after watching how the teacher in the video snippet used the algecards, began thinking of ways to modify the approach used by the teacher filmed in the video in order to suit to their own students' needs (e.g., PT's students may be of a different profile from the one shown in the video) because they might think that the teaching style demonstrated in the video was more suitable for a lower-ability class. A typical question such PTs would ask at this level of reflection is "I think her method of teaching applies better to lower-ability students, but my students are higher-ability ones, and so what can I do differently from this teacher that would work better for my students?" Then such PTs would be considered to have experienced "restructuring" in their own reflective stance.

Method

The participants in this study were eighty-two pre-service Secondary mathematics teachers who were undergoing a one-year Postgraduate Diploma in Education programme in one institute of teacher education. A mandatory component of this programme was a 66-hour methods course in which pedagogical theories related to teaching and learning of mathematics at the secondary school level are taught. In particular, the Concrete-Pictorial-Abstract approach underpinned by Bruner's "enactive-iconic-symbolic" modes of representation was a feature of this course. We use these modes of representation as a convenient subject of discussion upon which the PTs' reflection can be anchored. To familiarize the PTs with the CPA approach to teaching mathematics the PTs attended an introductory lecture that illustrated how this approach could be adopted in conventional classrooms.

It was unreasonable to assume that the PTs would have acquired a clear idea of the CPA approach from one mass lecture. As a follow-up activity, the PTs were required to access online learning suite that contained video-recordings of lessons on quadratic expansion and factorisation using the CPA approach that were implemented in the lesson study described previously.

Formulation of the Milestone Tasks

Because the raw data for this study were the PTs' online responses to the tasks which were placed at different time junctures of their learning journey (see Table 1), the way in which the questions were crafted and phrased in these tasks was of significance. We used the task design principles listed above to design the Milestone tasks. Not only should the questions in each Milestone Task meet the learning objective of reinforcing the PTs' learning of how CPA approach could be applied in the teaching of factorization of quadratic expressions and expansion of two linear factors but also elicit the responses that would adequately address our research questions stated in the purpose of our present study. Our present formulation was guided by the existing preliminary study (Leong et al, 2010) conducted with an earlier cohort of

PTs who underwent the same programme and the same contents in the online suite, with this Milestone Task 2:

1. Discuss the role Algecards in the instructional process in this lesson.

2. How is the smooth transition from Algecards to Rectangle Diagram critical to students' learning about quadratic factorisation in this lesson? Make reference(s) to the videos in your answer to this question.

3. Is there evidence to suggest that students in the video benefitted from this mode of learning? If so, what do you think are the main ingredients that contributed to its success?

In that study, although they were not asked to discuss the CPA approach most of the responses focused on its applications. These PTs' responses were evidence that the online suite had the potential to be used as an example of how schools applied the CPA approach. Since the online suite was intended to deepen the PTs understanding of the CPA approach and how it can be applied in the actual classroom, the task design principles invited the PTs into a deeper state of reflection about the use of algecards. To ensure that the PTs provided more in-depth reflections of the CPA approach, we revised the questions in Milestone Task 2. The revisions should leave no doubts in the minds of the PTs that their reflections should focus on CPA approach but still provided them with ample opportunities to express their thoughts and observations of the applications of the CPA approach to teach factorization and expansion of quadratic expressions in the project school. To achieve this end, Questions 1 and 3 were retained but Question 2 was amended to the following set of instructions.

2. View the lesson through the Concrete-Pictorial-Abstract (CPA) sequence, identify (i) the "Concrete" element(s), (ii) the "Pictorial" element(s); the "Abstract" element(s); and (iv) the connections among these modes.

Furthermore to guide the PTs to provide higher-order reflections Milestone Task 3 provided PTs with a hypothetical situation where they were challenged to consider how they would implement the CPA approach to their own teaching.

Milestone task 3: Imagine you are a team member in this teaching innovation project and you are about carry out the module of lessons for your resident Sec 2NA Mathematics class. What are some things you buy-in to the innovation? What are some things you will change/add?

In each case, provide supporting reasons by way of appealing to sound practices or theories you have learnt in the last semester.

With these amendments it was hypothesized that the PTs, as they progressed from Milestones Task 2 to 3, the emphasis of their reflections would begin to shift from *theorizing* with the application of the CPA approach to *confronting* their own beliefs how to teach factorization and expansion of quadratic expressions. The intent of the question “What are some things you buy-in to the innovation?” was an attempt to capture changes, if any, in the teacher’s belief system, which may come in the form of reinforcement or a complete change of viewpoint. Section IV provides the final video-suite, where the PTs view the feedback provided by the teachers who taught these lessons, the Head of the Mathematics Department of the participating school and selected students who participated in these lessons.

Milestone Task 4: Have the feedback of the teachers and students about their experiences in the project altered or reinforced your views about this teaching approach as expressed in your comments above? Explicate your thoughts/observations below:

The objective of Milestone Task 4, was to capture possible shifts in PTs’ beliefs and to guide the student teacher into experiencing *restructuring* in his or her reflective stance. Before exiting from the entire learning suite, the participating PTs were required to write down their overall reflections of the innovations featured in the video suite. The PTs reflections were crucial part of this study as they provided evidence which parts of the video suite heightened the PTs’ reflective noticing, and which parts were responsible for the PTs’ buy-in to the innovation.

This research design would allow us to use the their feedback as evidence of (i) the PTs’ reflective stance and noticing ability, and (ii) how PTs’ beliefs about the application of the CPA approach to teaching based on these grounded video images. Alongside this qualitative analysis of the students’ reflection notes, we concurrently identify those salient features of the video suite, be it the structure of the suite or the way in which the Milestone Tasks were phrased, etc., that are connected to those significant reflective activities observed in the PTs’ comments.

Analysis

Each PT was identified using a numerical code, 1-82. PTs’ responses to the questions from Sections II - IV were extracted from the online web-based platform specially designed for the implementation and monitoring of this study. Each PT's responses were analysed section-by-section against the five reflective noticing levels: (i) *describing*, (ii) *explaining*, (iii) *theorizing*, (iv) *confronting* and, (v) *restructuring* (Manouchehri, 2002).

Table 10.2. Some sample responses classified into the five levels of reflection.

Level	Examples of reflective noticing by PT
Describing	PT 9 : The students seemed to have enjoyed using the manipulatives more than I had imagined. PT 23 : I saw some students were still hesitant in arranging AlgeCards even though they had been taught the systematic way of arranging it. PT 41 : The students in the videos generally look engaged in the lesson, and most seemed to follow the lesson well. Some were even able to quickly skip the AlgeCards and move on to the tile diagram or rectangle diagram.
Explaining	PT25 : The AlgeCards also helps the teacher to relate the area of a rectangle to the concept of expansion, and similarly the length and breadth of a rectangle to the concept of factorization. PT41 : I think one of the reasons for the success is because it has concretised an abstract algebraic procedure, and made it into a more tangible process for students. For students, it seems easier to move pieces around to form a rectangle, than to work out different combinations of possible factors to find the correct factorisation. PT44 : The students firstly experiments with the concrete element "hands-on", developing some understanding through placing the AlgeCards at different positions to form a rectangle for each problem. Subsequently, with the help of the pictorial elements, students are able to visualise the concrete element without physically manipulating the cards and thereby begin to use the abstract elements to solve the mathematical problems related to expansion and factorisation.
Theorising	PT40 : This innovation has tapped on the a multi-modal representation i.e According to Bruner's cognitive modes of enactive (concrete), iconic(visual) and symbolic(algebraic). PT44 : According to Piaget's cognitive process of assimilation and accommodation, this approach uses previously learnt mathematical concepts which students are fluent and familiar with to integrate new perspectives and create new learning outcomes. PT82: According to the paper "Concretising factorisation of quadratic equations", it was mentioned that the building of the rectangle has to begin with placing the constant term (term without and 'x') at the corner of the x^2 term.
Confronting	PT18 : will consider grouping the students into pairs such that every pair will work on one set of AlgeCards to promote collaborative work. PT44 : : Perhaps, students should be encouraged to discuss more with their desk partners as peer learning has been proven to be relatively effective in strengthening learning outcomes for student. PT61 : The rectangle diagram is somewhat similar to the "cross method" but I am afraid the students will be stuck upon finding areas (and) fail to understand

	the way to bring out common terms or grouping of like terms.
Restructuring	PT23 : Personally, I think it would be nice if at the end of these lessons, teachers can compare the "cross-method" for factorization with the rectangle diagram. As such, whatever they had been taught would not be left as different pieces of mathematics. PT25: However, I will not buy-in the use of AlgeCards and rectangular diagram throughout all sessions. Students should wean off the use of such representations otherwise they will be over-reliant on it which is not beneficial for exam purposes. PT62: The teacher could make the lesson more interesting by asking students to go to the board to do some examples and correct them if there is mistake. .. teachers can bring students to computer lab to use an interactive Algebradisc program to let students explore and learn algebra and virtual manipulatives.

Coding was done by two of the authors of this chapter. Before proceeding with the actual coding process, the two coders classified and coded some responses. This was to ensure that there was consistency between the two coders. Those ambiguous responses that were difficult to code were singled out for discussion and re-coding by both coders. Table 2 shows some example of responses belonging to different levels of reflective noticing.

Findings

Reflection levels

Table 3 shows the proportion of PTs reflecting the various levels of reflection. Since PTs may reflect at some or all the levels of reflection, the total responses *need not* be100%. The reflections in the "Describing", "Explaining" or "Theorizing" level came mainly from Milestone-task II questions whereas Milestone-task III and IV questions elicited majority of the responses from the next two reflection levels, viz.: "Confronting" and "Restructuring". On average about 15% more PTs provided high-levels (confronting and restructuring) than the low-levels reflections. It is interesting that the PTs in this study provided more at higher-level than lower-level ones reflections. This finding is in stark contrast to the findings reported by Stockero (2008) where PTs reflections about others and their own teaching were predominantly at the low-levels. One possible reason for this difference could be that the sequential nature of the tasks and the way the milestone-task questions were crafted and worded may have helped to channel and direct the PTs' attention on noticing the essential components of the teaching approach.

Table 10.3
Proportion of Pre-service Teachers Reflecting at Different Levels of Reflection. (n = 82)

Reflection Level	Describing	Explaining	Theorizing	Confronting	Restructuring
Percentage	59%	67%	57%	80%	73%

"Buy-in" referred to PTs who believed that the CPA approach was effective for teaching a class of academically-challenged students the factorization and expansion process. Table 4 shows that approximately 72% (59 out of 82) of the PTs indicated "buy-in" to the teaching approach. Table 4 shows the proportion of those who buy into the approach and their reflections at each of these levels.

Table 10.4
Proportion of Pre-service Teachers who buy in to the Teaching Approach and the Different levels of Reflection (*n = 59*)

Reflection Level	Describing	Explaining	Theorizing	Confronting	Restructuring
Percentage	66%	75%	68%	80%	80%

The "buy-in" PTs were sufficiently engaged at each reflection level, with an average proportion of 73% engagement across the five levels. The highest frequency of reflection occurred at the "Confronting" level, followed by the "Restructuring". One would have expected that for the non-"buy-in" group, the higher levels of reflection were not as frequently engaged in contrast to the lower levels. But this is not so (see Table 5 below).

Table 10.5
Proportion of Pre-service Teachers who do not buy in to the Teaching Approach and the Different levels of Reflection (n = 23)

Reflection Level	Describing	Explaining	Theorizing	Confronting	Restructuring
Percentage	39%	44%	30%	83%	57%

Tables 4 and 5 indicate that (1) the non-"buy-in" group has a generally much shallower engagement in the lower levels of reflection, and (2) the "buy-in" group was more reflective at the theorizing level than the non-"buy-in" group.

It is important to examine carefully the underlying assumptions made in Manoucherhi's 5-level model concerning the transitions from Explaining to Theorizing, and from Theorizing to Restructuring. At the Explaining level, the PT "identifies and discusses the causal factors in the course of an interaction" but the PT's acknowledgement of these factors "is not supplemented by a description of how one can analyze the event or further study the problem" (Manouchehri, 2002, p. 721). At the third level of reflection, Theorizing, the PT attempts to substantiate his/her explanation by reasoning from data, and invokes known theories from research or experience from previous coursework to justify his/her explanation. At the fourth level of reflection, Confronting, the PT uses the reasoning obtained at the level of Theorizing to revisit his/her explanation made in the Explaining stage and checks its validity. During the stage of Confronting, the PT questions the alternative ways of assessing the situation. Thus, confronting can manifest itself as a challenge to one's own views or a challenge to the views and practices of others.

For the buy-in group, there was evidence that before transiting to the next level of reflection, a relatively high level of engagement had already occurred in the present (and preceding) level(s). This explains why, on the average, the buy-in group exercised more reflection at each level as compared to the non-"buy-in" group (i.e., 73% versus 51%). This somehow suggests that the buy-in group has been more conscientious in completing each Milestone Tasks, and progressing through the different levels of reflection, i.e., paying adequate attention to each of the levels, and caring not to skip to the next level until sufficient thinking has been carried out at the present level.

Although the non-"buy-in" group seemed to exercise a high frequency in Confronting, a careful look into the comments given by PTs belonging to this group revealed that they were merely challenging the teaching practice of the teacher in the video basing their arguments mostly on their own views or experiences, rather than on established learning theories. Evidently, 70% of the non-"buy-in" group did not engage Theorizing at all.

Reasons for buy-in

The 59 PTs who were willing to "buy-in" to the CPA approach wrote about their "support" at either Milestone-Tasks III or IV. One of the reasons for the "buy-in" was the presence of the "insider's opinion" or an "internal reflection". All the PTs entered their comments based on their observation of the lesson. It was only at Section IV that they heard the "other side of the story", i.e., how the teachers who taught the lessons felt, how the students thought about their learning experience, etc. For instance, PT7's belief in this teaching innovation was reaffirmed when his/her opinions resonated with students' and teachers' views.

> I think the feedback of the teachers and students have reinforced my views about this teaching approach, .. is more meaningful to the students to make sense of what they are learning.

More importantly it would appear the opinions of those in authority may influence PTs beliefs. Many PTs' beliefs in the efficacy of the CPA approach were reinforced by the comments made by the Head of Department (HOD) and students. More than 85% of the PTs made some comments on what they thought based on the HOD's comments. The following are examples of responses from four PTs:

> After hearing the HOD, I think that the rectangle method helps promote understanding and this helps students learn better as they are able to make sense out of it. (PT13)

> It is insightful to learn from the HOD of Maths that the stronger students should also be exposed to this method so that they can make sense of what they are doing and not just apply methods mechanically. It has altered my view on whom to expose this method to and now I felt that it would be good to expose every student to this method. (PT14)

> However, what struck me the most is the words of the HOD. She did acknowledge the fact that this method might not appeal to the better students, but the bottom line is that the rectangular method can give meaning to the mathematics that the students do. (PT51)

In fact unlike many preceding studies on the use of video technology in pre-service teacher education, the video suite incorporates the concept of "internal reflection". This "internal reflection" came in the form of the video-clips in Section IV, where the teachers, the students and the HOD talked about their experience in the teaching package. While the remarks made by the teachers and the students had some impact on the reflective stance of the PTs, the data seem to point heavily towards the presence of the HOD in Section IV. In the Singapore school system, the HOD is seen to be a school leader who not only is well-versed in both the content and pedagogical knowledge but also has an influential role in shaping the curriculum and approaches to teaching in the school. Thus, it was not surprising to see that PTs' beliefs were, to a great extent, influenced by the view-points of the HOD; and in certain cases, such an impact results in a shift in the PT's attitudes and opinions about the teaching approach. While we must be careful not to claim that "buy-in" equates to a more desirable outcome, it is perhaps wise to realize that the presence of this “internal reflection” somewhat widens the PTs’ perspective, reduces the possibility of biasedness and encourages an open mindset that we educators have always been advocating.

Implications and conclusions

Implications

Contrary to the findings by Stockero (2008) which showed that PTs’ reflections about others and their own teaching were usually at the low-levels. The PTs in this study showed more engagement at higher-level than lower-level reflections. This difference in findings could be a consequence of the nature of the tasks which were designed created scaffolds for the PTs to focus on their reflective thinking.

The structure of the e-learning suite may have aided the PTs by encouraging them to think and reflect more deeply about teaching, students’ learning and the use of the Algecards in the CPA approach. The design and the structure of the video were guided by six principles. Because the online suite did engage the PTs in deeper reflection, it is then instructive to examine more closely the usefulness of the guiding Task Design Principles (a)-(f) in ensuring effectiveness of the online learning suite. Thus it is useful to discuss the impact the six Design Principles individually.

(a) *The videos capture authentic classroom practices.* If a teaching innovation is shown to be effective in a typical local classroom, the PTs are more likely to be convinced about its applicability. The following PT’s view supports this point:

> I do believe that these ingredients can be incorporated into typical lesson in Singapore schools consisting of mathematically-challenged students. (PT22)

(b) *The classroom practices exemplify instructional innovation that is based on sound pedagogical principles.* With Bruner’s CPA as a well-founded theory of instruction, the teacher in the video showed how lessons can be conducted in the usual Singapore classroom context that made used of algecards.

(c) *The materials contain sufficient evidence that directs the PTs to access the applicability of the innovation and to infer the pedagogical principles underpinning the innovation.* The video clips capture the essential parts of the teaching package, recording how the teacher used the algecards in a step-by-step manner, and how the students engaged in using this manipulative to learn factorisation of quadratic trinomials. This claim is supported by evidence from the PTs' description of what they observed took place in the classroom captured in the video; for instance:

> The role of the Algecards is to concretize the concept of expansion and factorization. Students are able to *have hands-on experience and opportunities to arrange the cards in order to obtain the 'best' rectangle.* The introduction of Algecards to help the students to raise questions on 'what is the best rectangle?' and it helps the students to better visualize the process of factorisation and expansion.

Because the video snippets were arranged in such a way that the PTs could generate a continuum of what, in their mind, would a very close approximation to the entire implementation, the PTs were able to judge for themselves what would be, in their opinion, the effective features of the teaching innovation, and to make connections with a relevant learning theory associated to the teaching innovation. One PT's reflection notes supported this claim:

> The concrete element (Algecards) and pictorial element serve as strategies to concretize and scaffold the abstract algebraic representations of quadratic expressions which most students find it hard to understand. The concrete element in particular brings in the abstract concepts into forms that allow the students to 'do' and 'see' for themselves (i.e. help the students to achieve another perspective of the abstract concepts). The use of the concrete representations will allow the students to better learn and understand the basics using tools that are more developmentally appropriate for students who are concrete operational.

(d) *Within the overarching anticipated learning trajectory embodied in a sequence of tasks, there is room for individual learners' preferences and interests.* The Milestone Tasks focus the PT on some aspects of the teaching innovation shown in the video but allows the PT to freely choose those observations that were of interest to the PT, and to comment on them.

(e) *There are opportunities for learners to reflect on their learning.* The Milestone Tasks scaffold the reflection process throughout the video-suite. The questions were phrased in such a way to invite the various levels of reflection to occur. Milestone Task 1 leads PTs into Describing and Explaining; Milestone Task 2 leads them to Theorizing, and Milestone Tasks 3 and 4 lead them to Confronting and Restructuring.

(f) *The suite is fully accessible online.* This feature allows users to access the video-suite at any time they felt convenient – an exploitation of modern technology to circumvent problems of time-scheduling and to promote e-learning.

Our statistics indicated that among the "buy-in" group, not only were the highest occurrences of the reflection taking place at the higher level, there were sufficiently high average engagement in the lower levels of reflections. The situation was different for the non-"buy-in" group in that the average engagement in the lower levels of reflections were substantially lower than that observed in the "buy-in group", while there still was a high proportion of engagement in "Confronting" and "Restructuring". This interesting observation somewhat suggests that "buy-in" is more likely to take place when a PT experiences a more wholistic range of (and hence more authentic) reflection levels, beginning from the basic levels before proceeding to the advanced levels. From the responses supplied by the PTs in the "buy-in" group, one can see that these participants took more effort to give detailed descriptions of what they reflected upon in each Section. In short, these PTs were conscientious about the entire reflection process as they continued to stay focused on what was asked of them in all the Milestone Tasks.

Stockero (2008) raised an important open question of "whether it is desirable, or even possible, to move entirely away from this type of reflection [i.e., at the describing and explaining levels]", and further cautioned that it may be the case that some amount of lower level reflection (i.e., describing and explaining) is a *necessary* part of reflective discourse, as "it sets the stage for higher level reflections" (p.389, Stockero 2008). To a certain extent, our present finding answers negatively to Stockero's question. Indeed, more than just being necessary, lower levels of reflection should occur at a substantially high amount before a PT gains enough "energy" to jump to the higher levels of reflection. The upshot of this conclusion is that in designing tasks which are intended to develop and deepen PTs' reflecting stance during their pre-service teacher education, the instructor should help PTs build a sufficiently rich experience in the lower levels of reflections by giving them appropriate scaffolding prior to exercising their reflection at the higher levels.

It is important to realize that PTs in the "buy-in" group were stronger in Theorizing than those in the non-"buy-in" group. PTs who "buy-in" are likely to support their explanations of what they have observed by referring to a well-known learning theory that was taught to them earlier in the methods course. Based on this theory, such PTs would use this theory they considered to confront the explanation or views given earlier on. Then reassessing other possible approaches, factors and theories, such PTs would then move on to suggest what improvement or modification could be made if they were the one teaching the lesson. Here is a sample of a "buy-in" PT who passed to the levels of Confronting and Restructuring via Theorizing:

> E: It [CPA approach] is also easier for students to construct their own links to how length and breadth of the rectangles actually has more purpose than they already know.
>
> T: Using the approach of perceiving rectangle area and its side with expansion and factorisation made it easier for the students to relate to. It is similar to engaging the students' prior knowledge and linking what they already know with something new. According to *Piaget's cognitive processes*, this approach falls under assimilation where the teaching of new knowledge (expansion and factorisation) is done by expanding existing schemata (area of rectangle).
>
> C: Although the students found it easier to solve their factorisation problems using the new method, they were unable to give a clearer description to it. In fact there is still a risk this approach would be similar to cross-method approach where students know how to use but do not really understand what and why they are doing it. From Zaki's, it was evident that he did not catch the relationship of area of a rectangle to expansion while Hariss felt that he found it easier to get his answers using the method.
>
> R: When introducing the individual AlgeCards, it might have been better if students are given some time to think how the card area can be x and let them identify by themselves that if the area is x, $(x \times 1 = x)$ the sides of it are x and 1.

Our findings suggest that deeper levels of reflection can only take place when PTs engage sufficiently in the lower levels and must make meaningful connections of what has been observed (Describing) and explained (Explaining) with sound pedagogical theories that they have learnt in the methods course (Theorizing). Without going through Theorizing rigorously, some PTs might appear to be engaging in Confronting but what they may be merely doing is a baseless "fighting" against the views and practices of others rather than that of their own. Therefore, it is important for PTs, during their pre-service teacher training, to acquire a sound knowledge of learning theories so that they can begin to think of how their teaching can be restructured after having observed how others teach.

Theorizing is therefore a crucial piece in this jigsaw puzzle of teacher's reflective stance and teacher's belief system. Our present findings suggest that a change of teacher's belief towards “buying-in” a new teaching innovation or approach is most likely to occur if the teacher can invoke relevant learning theories to challenge his or her own explanation of what was being observed and then make creative modifications to what the new teaching innovation or approach. This realization then brings us back to begin where we talked about the theory-practice divide that we singled out as a common deficiency in most methods courses. The implication is therefore to integrate an essential exercise of developing PTs’ reflective abilities taking advantage of video technology and the six task design principles into the methods course so that PTs are given a natural setting to invoke the learning theories they acquired earlier in the methods course and translate them into new teaching approaches and overall improvement in the quality of their own teaching.

In conclusion, we would like to re-iterate the importance for PTs to have sound theories of learning, supported by research and to provide examples which modelled good pedagogical practices *and* for significant others supporting the innovation. Given the social set-up in a school where HODs are those in authority, those in authority *must* themselves be highly engaged in every level of reflection so as that they can also buy-in to the innovation.

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