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Quantum Tomographic Cryptography with a Semiconductor Single Photon Source

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In this paper we analyze the security of the so-called quantum tomographic cryptography with the source producing entangled photons via an experimental scheme proposed in Phys. Rev. Lett. **92**, 37903 (2004). We determine the range of the experimental parameters for which the protocol is secure against the most general incoherent attacks.

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INTRODUCTION

The need to transmit information securely between two parties is off paramount importance in military and commercial communication. The discovery of the possibility of secure communication based on photons has triggered numerous investigations in this field. An important issue in quantum communication is the ability to distribute a key securely between distant parties, and a number of schemes such as the BB84 [1] and Ekert91 [2] have been proposed for this purpose. In particular, the tomographic quantum key distribution scheme proposed in Ref. [3] in which Alice and Bob utilize a tomographically complete set of observables to distribute the key as well as perform full state tomography on their key distribution source has been shown to be powerful as it severely limits Eve's eavesdropping possibilities, when compared to BB84 or Ekert91. An extension of this scheme to the more general class of Bell diagonal states has also been proposed in Ref. [4].

On the practical side, quantum key distribution (QKD) is a sufficiently advanced field so that there is already the possibility for commercialization of some of the QKD devices. However, security analysis of generic devices is not always straightforward.

Recently, using the pulsed optical excitation of a single quantum dot from a sample of self-assembled InAs quantum dots in a GaAs matrix, it was shown that it is possible to use linear optics techniques to induce polarization entanglement between single photons emitted independently from the source [5, 6, 7]. Such a technique can be feasibly applied to produce entangled photons for QKD. In this letter, we exploit the tomographic QKD scheme [3] to study the security of QKD based on such solid state devices.

TOMOGRAPHIC QKD

In the tomographic QKD scheme [3], a central source distributes entangled qubits to Alice and Bob. Here, we assume that these qubits arise from polarizationentangled photons generated using a quantum dot single photon source and the method described by Fattal et al. [5]. Alice and Bob independently and randomly choose to measure three tomographically complete observables σ_x , σ_y and σ_z (Pauli operators) on each qubit. At the end of the transmission, they publicly announce their choice of observables for each qubit pair. They then divide their measurement results according to those for which their measurement bases match, and those for which their measurement bases do not match. Exchanging a subset of their measurements allows Alice and Bob to tomographically reconstruct the density operator of the two-qubit state they share. Those measurements for which their bases match can be used for key generation.

According to the discussion presented in Ref. [5] the density matrix describing the photon source has the following form in the (say) σ_z basis:

$$\varrho^{(z)} = \frac{1}{2} \begin{pmatrix} 2\alpha & & \\ \beta_1 + \beta_2 + 2\gamma & \beta_1 - \beta_2 \\ \beta_1 - \beta_2 & \beta_1 + \beta_2 - 2\gamma \\ & & 2\alpha \end{pmatrix} (1)$$

where

$$\alpha = \frac{2g}{\frac{R}{T} + \frac{T}{R} + 4g}$$

$$\beta_1 = \frac{\frac{R}{T} + \frac{T}{R} - 2V}{2(\frac{R}{T} + \frac{T}{R}) + 8g}$$

$$\beta_2 = \frac{\frac{R}{T} + \frac{T}{R} + 2V}{2(\frac{R}{T} + \frac{T}{R}) + 8g}$$

$$\gamma = \frac{\frac{R}{T} - \frac{T}{R}}{2(\frac{R}{T} + \frac{T}{R}) + 8g}.$$
(2)

The meaning of the experimentally accessible parameters R, T, V and g is the following: R(T) denotes the reflectivity (transmittivity) of the beamsplitters in the Mach-Zehnder interferometer used in the experiment (the ratio $\frac{R}{T}$ reported in Ref. [5] was 1.1). The parameter V denotes the overlap of the wave packets of two consecutive photons emerging from the quantum dot, and g (denoted in Ref.[5] as $g^{(2)}$) is the equal time second-order correlation function. Furthermore, in order for entanglement to exist in the two photon state, we require that V > 2g.

From the point of view of the security analysis, it is more convenient to express the density matrix (1) in the Bell basis $\{|m_{ab}\rangle\}_{a,b=0,1}$. Here, $|m_{ab}\rangle = \sum_{k=0}^{1} \frac{1}{\sqrt{2}} \omega^{kb} |m_k m_{k+a}\rangle$ ($\omega = -1$) denotes the Bell state in the *m*th basis (m = x, y, z). We have

$$\varrho_{\text{Bell}}^{(z)} = \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \beta_1 & \gamma \\ & & \gamma & \beta_2 \end{pmatrix}$$
(3)

and in the remaining two other bases

$$\varrho_{\text{Bell}}^{(y)} = \varrho_{\text{Bell}}^{(x)} = \begin{pmatrix} \alpha & & \\ & \beta_1 & -\gamma \\ & & \alpha \\ & -\gamma & & \beta_2 \end{pmatrix}.$$
 (4)

From their state tomography, Alice and Bob can determine how the parameters $\frac{R}{T}$, g and V affect the security of their key. From these parameters, they can compute, for each basis, the maximal strength of correlations between Eve and any one of them. The Csiszár-Köner (CK) theorem [9] then guarantees that if the correlations between Alice and Bob are stronger than those between Eve and either of them, a secure key can be established through one-way error correcting codes, with the efficiency given by the *CK yield*. Thus for each basis, there is a *CK* yield for Alice and Bob's bit data, and they can find out which basis will give them a positive CK yield. They will then make use of data only from those bases with a positive yield to establish their key, rejecting the bits obtained from the remaining measurements. It is interesting to note that in the tomographic protocol presented in [3], Alice and Bob do not have to do this as the yield is the same regardless of the measurement basis.

EAVESDROPPING

Suppose we have an eavesdropper Eve in the channel. In order to make our analysis foolproof, we assume the worst-case scenario in which she is in full control of the qubit-distributing source, and that all the factors that contribute to experimental imperfections (parameters R, T, g and V) are due to her eavesdropping activities.

In order to obtain as much information as possible about the key generated by Alice and Bob, Eve entangles their qubits with ancilla states $|e_{ab}^{(z)}\rangle$ in her possession. She prepares the following state:

$$\begin{aligned} |\psi_{ABE}\rangle &= \sqrt{\alpha} |z_{00}\rangle |e_{00}^{(z)}\rangle + \sqrt{\alpha} |z_{01}\rangle |e_{01}^{(z)}\rangle \\ &+ \sqrt{\beta_1} |z_{10}\rangle |e_{10}^{(z)}\rangle + \sqrt{\beta_2} |z_{11}\rangle |e_{11}^{(z)}\rangle, \quad (5) \end{aligned}$$

where

$$\langle e_{a'b'} | e_{ab} \rangle = \begin{cases} \delta_{aa'} \delta_{bb'}, & a = 0, \\ \frac{\gamma}{\sqrt{\beta_1 \beta_2}} \delta_{aa'} (1 - \delta_{bb'}) + \delta_{aa'} \delta_{bb'}, & a = 1. \end{cases}$$
(6)

Tracing out Eve gives the mixed state Eq. (3) that Alice and Bob measure and this purification is the most general one as far as eavesdropping is concerned. Note that because of the tomography performed by Alice and Bob, Eve cannot prepare a state that would give her some additional correlations across different qubits emitted by the source. This considerably reduces the number of coherent eavesdropping strategies Eve can use. More precisely, the only possibility of a coherent attack for Eve is to collect her ancillas and perform some collective measurements on them. In this paper we do not investigate this scenario and assume that Eve measures her ancillas one by one.

Eve's purification, when expressed in different bases, reads

$$\begin{aligned} |\psi_{ABE}\rangle &= \sum_{k,a=0}^{1} \sqrt{\mu_{ak}^{(z)}} |z_k, z_{k+a}\rangle |f_{ak}^z\rangle \\ &= \sum_{k,a=0}^{1} \sqrt{\mu_{ak}^{(y)}} |y_k, y_{k+a}\rangle |f_{ak}^y\rangle \\ &= \sum_{k,a=0}^{1} \sqrt{\mu_{ak}^{(x)}} |x_k, x_{k+a}\rangle |f_{ak}^z\rangle, \end{aligned}$$
(7)

where

$$\mu_{ak}^{(z)} = \delta_{a,0}\alpha + \delta_{a,1}\left((\delta_{k,0} - \delta_{k,1})\gamma + \frac{\beta_1 + \beta_2}{2}\right)$$
$$\mu_{ak}^{(x)} = \mu_{ak}^{(y)} = \delta_{a,0}\frac{\alpha + \beta_1}{2} + \delta_{a,1}\frac{\alpha + \beta_2}{2}.$$
(8)

The ancilla kets have the following inner products

$$\langle f_{a'k'}^{z} | f_{ak}^{z} \rangle = \begin{cases} \delta_{kk'}, & \text{if } a = a' = 0, \\ \delta_{kk'} + (1 - \delta_{kk'}) \frac{\beta_{1-\beta_{2}}}{\sqrt{(\beta_{1}+\beta_{2})^{2}-4\gamma^{2}}}, \\ & \text{if } a = a' = 1, \\ 0, & \text{if } a \neq a'. \end{cases}$$

$$\langle f_{a'k'}^{x} | f_{ak}^{x} \rangle = \langle f_{a'k'}^{y} | f_{ak}^{y} \rangle$$

$$= \begin{cases} \delta_{kk'} + (1 - \delta_{kk'}) \frac{\alpha - \beta_{1}}{\alpha + \beta_{1}}, & \text{if } a = a' = 0; \\ \delta_{kk'} + (1 - \delta_{kk'}) \frac{\alpha - \beta_{1}}{\alpha + \beta_{2}}, & \text{if } a = a' = 1; \\ -\frac{\gamma}{\sqrt{(\alpha + \beta_{1})(\alpha + \beta_{2})}} \omega^{k+k'}, & \text{if } a \neq a'. \end{cases}$$

$$(9)$$

Eve's eavesdropping strategy then proceeds as follows. After Alice and Bob announce their measurement bases and the basis they intend to use for key generation, Eve knows which pairs of qubits contribute to the key and that her ancilla for each of those pairs is a mixture of four possible states. Formally this can be viewed as a transmission of information from Alice and Bob to Eve encoded in the quantum state of Eve's ancilla. To find the optimal eavesdropping strategy, she has to maximize this information transfer by a choice of a suitable generalized measurement known as a Positive Operator Valued Measure (POVM) [8]. For example, if Alice and Bob chose to obtain their key from the x basis, Eve would obtain the following mixed state of ancillas,

$$\varrho_E^{(x)} = \sum_{k,a=0}^{1} \mu_{ak}^{(x)} |f_{ak}^x\rangle \langle f_{ak}^x|.$$
(10)

Eve has to find the optimal measurement that will extract from the transmission as much information as possible, the so-called *accessible information*.

INCOHERENT ATTACK

The conditions for which our protocol is secure against Eve's incoherent eavesdropping attack is given by the CK theorem: a secure key can be generated from a raw key sequence by means of a suitably chosen error-correcting code and classical one-way communication between Alice and Bob if the mutual information between Alice and Bob ($\mathcal{I}(A; B)$) exceeds that between Eve and either one of them (the CK regime). For the protocol considered, the mutual information between Alice and Eve ($\mathcal{I}(A; E)$), and Bob and Eve, are the same so that security is assured as long as

$$\mathcal{I}(A;B) > \mathcal{I}(A;E). \tag{11}$$

Furthermore, the difference in mutual information $\mathcal{I}(A; B) - \mathcal{I}(A; E)$ gives the CK yield for the distilled key. Due to the asymmetric nature of the state in the σ_z and σ_x/σ_y bases, the yield is different for those bases. The yield is the same for σ_x and σ_y . As mentioned earlier, Alice and Bob will choose only those measurement bases which give them a positive yield and use the data from those bases for key generation.

We shall now present the POVM that maximizes the information transmitted from Alice and Bob to Eve for a given basis.

Suppose Eve receives a state in the σ_z basis:

$$\varrho_E^{(z)} = \sum_{k,a=0}^{1} \mu_{ak}^{(z)} |f_{ak}^z\rangle \langle f_{ak}^z|, \qquad (12)$$

where the kets have the structure given by Eq. (9). Ancillas from the correlation subspace (a = 0) are orthogonal to all other states; those from the anti-correlation subspace (a = 1) are in general non-orthogonal among themselves.

In the first step, Eve sorts the mixture of the ancillas into two sub-ensembles according to the index a. This

can easily be done using a projective measurement. After that, depending on the outcome of the projection (a = 0or a = 1), Eve has a mixture of two ancilla states each corresponding to Alice and Bob's result.

If she projects into the a = 0 subspace, Eve will possess a mixture of equiprobable orthogonal ancilla states

$$\varrho_{E,a=0}^{(z)} = \frac{1}{2} |f_{00}^z\rangle \langle f_{00}^z| + \frac{1}{2} |f_{01}^z\rangle \langle f_{01}^z|, \qquad (13)$$

which she can distinguish perfectly.

On the other hand, if she projects into the a = 1 subspace, she will obtain a mixture of non-orthogonal ancilla states instead:

$$\varrho_{E,a=1}^{(z)} = \left(\frac{1}{2} + \frac{\gamma}{2\alpha + \beta_1 + \beta_2}\right) |f_{10}^z\rangle\langle f_{10}^z| \\
+ \left(\frac{1}{2} - \frac{\gamma}{2\alpha + \beta_1 + \beta_2}\right) |f_{11}^z\rangle\langle f_{11}^z|. (14)$$

We shall denote the inner product of the two ancilla states by $\lambda \equiv \frac{\beta_1 - \beta_2}{\sqrt{(\beta_1 + \beta_2)^2 - 4\gamma^2}}$. If these states are equiprobable, which happens if $\gamma = 0$ or R = T, the optimal measurement for Eve would be the so-called square-root measurement [10, 11]. Its POVM is given by $\{|\omega_0\rangle\langle\omega_0|, |\omega_1\rangle\langle\omega_1|\}$, where

$$\begin{aligned} |\omega_{10}\rangle &= \frac{1}{1-2\eta} \left(-\sqrt{\eta} |f_{10}^z\rangle + \sqrt{1-\eta} |f_{11}^z\rangle \right) \\ |\omega_{11}\rangle &= \frac{1}{1-2\eta} \left(\sqrt{1-\eta} |f_{10}^z\rangle - \sqrt{\eta} |f_{11}^z\rangle \right), \end{aligned}$$
(15)

with $\eta = \frac{1}{2}(1 + \sqrt{1 - \lambda^2})$ being the probability of determining a given state correctly.

In general, the ancilla states will not occur with the same probability, and the optimal measurement for Eve will then not be the square-root measurement. Consider the POVM $\{|\tilde{\omega}_{10}\rangle\langle\tilde{\omega}_{10}|,|\tilde{\omega}_{11}\rangle\langle\tilde{\omega}_{11}|\}$, where

$$\begin{aligned} |\tilde{\omega}_{10}\rangle &= \cos\theta |\omega_{10}\rangle - \sin\theta |\omega_{11}\rangle \\ |\tilde{\omega}_{11}\rangle &= \sin\theta |\omega_{10}\rangle + \cos\theta |\omega_{11}\rangle. \end{aligned} \tag{16}$$

These states are rotated from the square-root measurement states by an angle θ . We then have the following conditional probabilities

$$p\left(\tilde{\omega}_{10}|f_{10}^{z}\right) = \left(\sqrt{\eta}\cos\theta - \sqrt{1-\eta}\sin\theta\right)^{2}$$

$$p\left(\tilde{\omega}_{11}|f_{10}^{z}\right) = \left(\sqrt{\eta}\sin\theta + \sqrt{1-\eta}\cos\theta\right)^{2}$$

$$p\left(\tilde{\omega}_{10}|f_{11}^{z}\right) = \left(\sqrt{1-\eta}\cos\theta - \sqrt{\eta}\sin\theta\right)^{2}$$

$$p\left(\tilde{\omega}_{11}|f_{11}^{z}\right) = \left(\sqrt{1-\eta}\sin\theta + \sqrt{\eta}\cos\theta\right)^{2}, \quad (17)$$

where, for instance, $p(\tilde{\omega}_{11}|f_{11}^z)$ denotes the probability of getting the result of the measurement corresponding to the projector $|\tilde{\omega}_{11}\rangle\langle\tilde{\omega}_{11}|$ provided the state $|f_{11}^z\rangle$ was sent.

Using the probabilities in Eq. (17), we can compute the mutual information between Alice and Eve as a function of θ . The optimal measurement for Eve is then given by the θ that maximizes the mutual information between them.

Suppose now that Eve receives ancillas from the σ_x basis. If Alice measured bit '0' $(k = 0, \text{ probability } \frac{1}{2})$, Eve will obtain the state

$$\varrho_{k=0}^{(x)} = (\alpha + \beta_1) |f_{00}^x\rangle \langle f_{00}^a| + (\alpha + \beta_2) |f_{10}^x\rangle \langle f_{10}^a|, (18)$$

and if Alice measured '1' $(k = 1, \text{ probability } \frac{1}{2})$, Eve will obtain the state

$$\varrho_{k=1}^{(x)} = (\alpha + \beta_1) |f_{01}^x\rangle \langle f_{01}^x| + (\alpha + \beta_2) |f_{11}^x\rangle \langle f_{11}^x|.$$
(19)

The structure of the (normalized) ancillas is given by Eq. (9):

$$\langle f_{00}^{x} | f_{01}^{x} \rangle = \frac{\alpha - \beta_{1}}{\alpha + \beta_{1}} \equiv \lambda_{0}$$

$$\langle f_{10}^{x} | f_{11}^{x} \rangle = \frac{\alpha - \beta_{2}}{\alpha + \beta_{2}} \equiv \lambda_{1}$$

$$\langle f_{0k'}^{x} | f_{1k}^{x} \rangle = -\frac{\gamma}{\sqrt{(\alpha + \beta_{1})(\alpha + \beta_{2})}} \omega^{k+k'} \equiv \mu \omega^{k+k'}.$$

$$(20)$$

Now, the *total* state describing Eve's ancillas is given by

$$\varrho^{(x)} = \frac{\alpha + \beta_1}{2} |f_{00}^x\rangle \langle f_{00}^x| + \frac{\alpha + \beta_1}{2} |f_{01}^x\rangle \langle f_{01}^x| \\
+ \frac{\alpha + \beta_2}{2} |f_{10}^x\rangle \langle f_{10}^x| + \frac{\alpha + \beta_2}{2} |f_{11}^x\rangle \langle f_{11}^x|,$$
(21)

which has the following eigenkets

$$\begin{aligned} |g_{0}\rangle &= \frac{1}{\mathcal{N}_{0}} \left(|f_{00}^{x}\rangle + |f_{01}^{x}\rangle \right) \\ |g_{1}\rangle &= \frac{1}{\mathcal{N}_{1}} \left(|f_{10}^{x}\rangle + |f_{11}^{x}\rangle \right) \\ |g_{2}\rangle &= \frac{1}{\mathcal{N}_{2}} \left(\kappa_{+} \left(|f_{00}^{x}\rangle - |f_{01}^{x}\rangle \right) + \eta \left(|f_{10}^{x}\rangle - |f_{11}^{x}\rangle \right) \right) \\ |g_{3}\rangle &= \frac{1}{\mathcal{N}_{3}} \left(\kappa_{-} \left(|f_{00}^{x}\rangle - |f_{01}^{x}\rangle \right) + \eta \left(|f_{10}^{x}\rangle - |f_{11}^{x}\rangle \right) \right), \end{aligned}$$
(22)

where

$$\kappa_{\pm} = \beta_2 - \beta_1 \pm \sqrt{(\beta_2 - \beta_1)^2 + 4\gamma^2}$$

$$\eta = 2\gamma \sqrt{\frac{\alpha + \beta_2}{\alpha + \beta_1}}.$$
(23)

The normalization constants \mathcal{N}_k (k = 0, 1, 2, 3) read:

$$\mathcal{N}_{0} = \sqrt{2(1+\lambda_{0})} \\ \mathcal{N}_{1} = \sqrt{2(1+\lambda_{1})} \\ \mathcal{N}_{2} = \sqrt{\frac{4}{\alpha+\beta_{1}} \left(\beta_{1}\kappa_{+}^{2}+4\gamma^{2} \left(\beta_{1}-\sqrt{(\beta_{2}-\beta_{1})^{2}+4\gamma^{2}}\right)\right)} \\ \mathcal{N}_{3} = \sqrt{\frac{4}{\alpha+\beta_{1}} \left(\beta_{1}\kappa_{-}^{2}+4\gamma^{2} \left(\beta_{1}+\sqrt{(\beta_{2}-\beta_{1})^{2}+4\gamma^{2}}\right)\right)}.$$
(24)

If we adopt $\{|g_0\rangle, |g_1\rangle, |g_2\rangle, |g_3\rangle\}$ as an orthonormal basis, the optimal measurement for Eve can then be expressed as $\{|\omega_0\rangle\langle\omega_0|, |\omega_1\rangle\langle\omega_1|, |\omega_2\rangle\langle\omega_2|, |\omega_3\rangle\langle\omega_3|\}$, where

$$(|\omega_0\rangle, |\omega_1\rangle, |\omega_2\rangle, |\omega_3\rangle) = (|g_0\rangle, |g_1\rangle, |g_2\rangle, |g_3\rangle) \begin{pmatrix} -a & a & b & b \\ b & -b & a & a \\ c & c & -d & d \\ d & d & c & -c \end{pmatrix}, (25)$$

and in which a, b, c, d are real numbers. These parameters are also related by

$$a^{2} + b^{2} = \frac{1}{2}$$

 $c^{2} + d^{2} = \frac{1}{2}$ (26)

so that the operators decompose the identity.

As before, we can compute the mutual information between Alice and Eve $\mathcal{I}(A; E)$ for this basis and maximize it over the two independent variables a and c to obtain the maximum information that Eve can obtain about Alice's measurements.

It should be mentioned here that the optimality of the above POVMs for the three bases was deduced and confirmed numerically using the algorithms presented in Refs. [12, 13].

Table I summarizes the results of the computations for various values of g and V. We fixed the ratio $\frac{R}{T} = 1.1$ (the value reported in Ref. [5]). We see that for certain values of q and V (the first row of the table), for which the state is still entangled, the CK yield (denoted as Δ) is negative in all the measurement bases. For such states, according to the CK theorem, one cannot extract secure bits by means of one-way communication (because the CK yield is zero). More interesting are cases where the CK yield is negative in one measurement basis and positive in another. In such cases, Alice and Bob reject the data obtained by measurements in the basis with negative yield and process only the data from the basis for which the CK yield is positive. The total CK yield is then the equally-weighted average of only the positive yields from the three measurement bases. In the case where all the CK yields are positive, Alice and Bob use the data from all the bases.

			σ_z			$\sigma_{x,y}$			
$\frac{R}{T}$	g	V	$\mathcal{I}(A;B)$	$\max \mathcal{I}(A; E)$	yield, Δ_z	$\mathcal{I}(A;B)$	$\max \mathcal{I}(A; E)$	yield, $\Delta_{x,y}$	overall yield
1.1	0.1	0.6	0.3478	0.6070	$-0.2592 (\times)$	0.1872	0.4320	-0.2448 (×)	0
	0.02	0.4	0.7598	0.7550	0.0048	0.1085	0.1088	-0.0003 (×)	$\frac{1}{3}\Delta_z = 0.0016$
	0.1	0.84	0.3478	0.3528	$-0.005 (\times)$	0.3869	0.3755	0.0114	$\frac{1}{3}\Delta_y + \frac{1}{3}\Delta_x = 0.0076$
	0.1	0.9	0.3478	0.2845	0.0633	0.4525	0.3321	0.1204	$\frac{1}{3}\Delta_z + \frac{1}{3}\Delta_y + \frac{1}{3}\Delta_x = 0.1014$

TABLE I: Table of yields in the three bases for $\frac{R}{T} = 1.1$, and different values of g and V. Due to the asymmetric nature of the state in the σ_z and σ_x/σ_y bases, the yield is different for those bases. The yield is the same in σ_x and σ_y . Crosses mean that the data from this basis is not used for the key generation.



FIG. 1: Three-dimensional plot of the CK yield for perfect beamsplitters R = T (*left*), and the corresponding contour plot (*right*). The threshold for security is given by the contour for which the CK yield is zero. For g = 0, security is guaranteed as long as V is greater than zero, although fewer secure bits can be distilled for smaller V.

In Fig. 1, we show the overall CK yield plotted against g and V in the case of perfect beamsplitters (R = T). In this case the state produced by the source is a mixture of Bell states. The security thresholds for such states have been analyzed in Ref. [4]. We observe that the protocol is always secure against incoherent attacks as long as g = 0 and V > 0, although fewer secure bits can be distilled for smaller V. If V is zero the state becomes separable and, of course, one cannot extract any secure bits. More detailed analysis reveals that these states (for which R = T, g = 0 and V > 0) have the interesting property that the mutual information between Alice and Eve is always zero when Alice performs measurements in σ_x or σ_y basis. This is due to the fact that Eve's ancillas corresponding to different outcomes of Alice's measurements in the σ_x and σ_y bases are the same, which means that they do not carry any information whatsoever about Alice's and Bob's correlations. Therefore, if Alice and Bob agree on using only the data from the σ_x and σ_y measurements (this reduces the efficiency), the protocol becomes secure against all possible attacks by Eve (unconditional security). In realistic situations however, the

value of g can be small but not exactly zero (for example, the value of g reported in [6] was 0.02). In this case, the protocol is secure over a smaller range of V. Even then, we conjecture that the information that Eve can extract from her ancillas in the σ_x or σ_y basis is negligible, and the protocol remains pretty robust against all possible attacks by her in those bases.

NOISY CHANNEL

So far, we have excluded the effects of noise in the channel so that Alice and Bob expect to receive the state 'as-is' from the source. In reality however, this is not the case: Alice and Bob can expect their quantum channel to be affected by interaction with the environment. We next consider what happens when there is symmetric white noise present in the channel, i.e. the state that Alice and



FIG. 2: Overall CK yield for $\frac{R}{T} = 1.1$ and g = 0.02 and different amounts of noise in the channel F. For a noiseless channel (F = 0), when $V \leq 0.39$, one can no longer extract secure bits by means of one-way communication because the CK yield is zero. As the amount of noise increases, the CK yield drops until for $F \geq 0.42$, where we will not be able to distill any secure bits at all (because the CK yield is 0 for all V).

Bob expect to receive is of the form:

$$\varrho^{(z)} = \frac{1-F}{2} \begin{pmatrix} 2\alpha & & \\ \beta_1 + \beta_2 + 2\gamma & \beta_1 - \beta_2 \\ \beta_1 - \beta_2 & \beta_1 + \beta_2 - 2\gamma \\ & & 2\alpha \end{pmatrix} + \frac{F}{4} 1 \otimes 1,$$
(27)

where we have a proportion F ($0 \le F \le 1$) of unbiased noise admixed to the original state from the source.

CONCLUSION

We have analyzed the security of the tomographic QKD protocol using a source of entangled photons produced in the experimental scheme proposed by Fattal *et.* al [5] against the most general incoherent attacks.

Using the analysis presented in this paper we can give the number of secure bits that can be distilled by means of one-way communication between Alice and Bob as a function of the experimentally accessible parameters R, T, g and V, and for different degrees of unbiased noise in the channel F.

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Analysis shows that the optimal POVM for Eve is of the same form as that presented earlier.

As before, we can obtain the condition for security and the CK yield for various proportions of noise and this is shown in Fig. 2, for fixed values of $\frac{R}{T} = 1.1$ and g = 0.02(values reported in Ref. [5, 6]). We can distill less secure bits as the amount of noise increases.

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- C. H. Bennett and G. Brassard, Proceedings of IEEE Conference on Computers, Systems, and Signal Processing, Bangalore, India (IEEE, New York, 1984), p. 175.
- [2] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991).
- [3] Yeong Cherng Liang, Dagomir Kaszlikowski, Berthold-Georg Englert, Leong Chuan Kwek, and C. H. Oh, Phys. Rev. A 68, 022324 (2003).
- [4] D. Kaszlikowski, J. Y. Lim, D. K. L. Oi, F. H. Willeboordse, Ajay Gopinathan, L. C. Kwek, eprint arXiv/quant-ph/0408088 (2004)
- [5] D. Fattal, K. Inoue, J. Vuĉković, C. Santori, G. S. Solomon, Y. Yamamoto, Phys. Rev. Lett. 92, 37903 (2004).
- [6] C. Santori, M. Plton, G. Solomon, Y. Dale, Y. Yamamoto, Phys. Rev. Lett. 86, 1502 (2001).

- [7] C. Santori, D. fattal, M. Plton, G. Solomon, Y. Yamamoto, Phys. Rev. B 66, 045308 (2002).
- [8] E. B. Davies, Quantum Theory of Open Systems (Academic Press, London, 1976).
- [9] I. Csiszár and J. Körner, IEEE-IT ${\bf 24}$ 339 (1978).
- [10] A. Chefles, Contemp. Phys. 41, 401 (2000).
- [11] C. W. Helstrom, Quantum Detection and Estimation Theory (Academic Press, New York, 1976).
- [12] Ajay Gopinathan, private communication.
- [13] J. Rehácek, B.-G. Englert, D. Kaszlikowski, eprint arXiv/quant-ph/0408134 (2004)