Teaching the Language of Mathematics at Three Levels of an English-medium Primary School

Postprint

by Dr Sally Ann Jones and Dr Mark Fifer Seilhamer

Published as

Jones, S. A., & Seilhamer, M. F. (2019). Teaching the language of mathematics at three levels of an English-medium primary school. ***Oxford Review of Education, 45*****(5), 639-656. doi:10.1080/03054985.2019.1591944**

Abstract

This study, situated in a multilingual, English-medium educational context, draws on theory from mathematics and language education to capture teachers' perspectives on the place of language in their mathematics pedagogy. The benchmark study explored this topic through surveying and interviewing teachers. Additionally, it sought to relate teacher's views to their practice by focusing on observing three teachers' mathematics lessons at primary one, three, and five. Findings are that mathematics teachers placed importance on teaching language, being specifically concerned with language as input and comprehension. They taught vocabulary and reading skills in supportive ways explicitly yet differently at the three grade levels. Particularly at the lower levels, teachers contextualized language in the concrete examples employed for mathematics teaching. At all three levels, prominence was given to teaching pupils how to read word problems as well as how to solve them. However, at primary three, a tension was observed between the two aims of teaching mathematical vocabulary and teaching the reading skills for word problems. This paper illustrates the tension and discusses its possible causes.

Key words: English language in education; mathematics; content learning; primary school teaching; multilingual context; Singapore

1. Introduction

This study explores whether and how teachers of one Singapore school teach the language of mathematics in their classes at three different grade levels. It therefore deals with two equally educationally important symbolic systems, the one of English language, and the other of mathematics. As Middleton, Llamas-Flores, and Guerra-Lombardi (2013) point out, the language of mathematics is used for study within the discipline itself as well as to provide conceptual tools for the study of other disciplines. Therefore, children in primary school face the challenges of learning a new technical language as well as new modes of representation when working mathematically. This is compounded by the fact that some everyday language they already know is used in novel mathematical contexts with new technical functions and/or meanings (Middleton et al., 2013).

The issue of language is an important one in Singapore because in this multilingual country, English is the medium of teaching and learning for most subjects in the mainstream education system. In the most recent census (Department of Statistics, 2010), English was reported to be the main language used at home among five- to 14-year-olds for 52% of Chinese Singaporeans, 50% of Indian Singaporeans, and 26% of Malay Singaporeans, showing that for a great many Singaporean children, English is a non-dominant language.

While there is a long tradition of interest in language and mathematics internationally (e.g., Aiken, 1971; Pimm, 1991; Bernado & Calleja, 2005), attention to the issue in Singapore is more sporadic (e.g., Chan, 2005; Cheng, 2015). Recognizing this relative lack of attention, Kaur (2014) has recently argued for research into the enactment of the mathematics curriculum in Singapore. Moschkovich (2010) and Schleppegrell (2010) make similar calls for research on language in diverse mathematics classrooms. To our knowledge, there has been no exploratory research conducted with a comparative, developmental dimension over three grade levels in multilingual primary schools. As many commentators point out, multilingual pupils comprise a large proportion of children in classrooms worldwide (McKay, 2002; Middleton et al., 2013; Pimm, 1991). Singapore, therefore, provides an ideally representative context in which to examine the role of English in mathematics teaching and learning in the primary school.

2. Literature Review

2.1. *The language of mathematics*

The study starts from the Vygotskian premise that language constitutes thinking. According to Vygotsky (1962) every thought is a generalization or a concept which deepens and develops as a child uses language and becomes aware of its use. Applying Vygotskian notions to teaching, Mercer and Littleton (2007) conceptualize an intermental development zone among those in the classroom, where the teacher, as a discourse guide, promotes interthinking. An important consideration in this social, cultural, and psychological view of learning is that the language, thinking, and knowledge of school are seen as qualitatively different from that available at home. Educators consequently advocate the explicit teaching not only of subject content but also of the language which constructs it (Christie, 1998; Mercer & Sams, 2006; Sfard, 2002, for example). Although, by contrast, Gee (2000) sees the process of learning as one of apprenticeship or socialization, his argument that classrooms are distinct communities using specific registers of language is also applicable to learning mathematics in school. (See also Halliday, 2007).

Therefore, it is through acquiring the language of school that children develop the conceptual understandings necessary for learning. School language has been categorized into three types by Cummins and Man (2007): as discrete language skills (the foundational skills of word recognition, basic grammar, and spelling), interpersonal communication skills (everyday language), and academic language. The two latter forms were originally termed BICS (Basic Interpersonal Communication Skills) and CALP (Cognitive Academic Language Proficiency) by Cummins (2000). Cummins' (2000) typography of language may be somewhat related to children's learning of spontaneous and scientific concepts in the way that Vygotsky (1962) posits the development of scientific thinking as the purview of schooling.

Textual perspectives on the academic language of school from systemic functional linguistics (e.g., Halliday, 2007; Schlepegrell, 2001; Unsworth, 2001) show how academic language contrasts with everyday speech, evidencing a complexity and density of linguistic forms and distinct genres associated with subject disciplines. In focusing on the language of mathematics, Laborde (1990) discusses how the necessity within the discipline to be clear, precise, and unaffected by subjectivity results in a register characterized by nominalizations, heavy noun phrases and sequencing of information that is unfamiliar to children. Middleton et al. (2013) note how, in addition to a specialized mathematics vocabulary, everyday words, such as *table* are used differently in mathematical contexts. They further point out common syntactic features of the mathematical register, for example, the use of prepositions, comparatives, and conditionals, as well as the syntactic variation in the expression of one semantic idea, and a lack of a one-to-one correspondence between mathematical symbols and words; addition, for example, can be expressed by *plus, add, combine,* or *sum*. Symbols used in mathematics may also be co-opted from everyday language, such as the use of a hyphen and colon to respectively represent minus (or a range) and ratio. Apart from exposure to mathematical language, the ability to manipulate and connect it with concepts is integral to learning the subject (Middleton et al., 2013). To this end, Sfard (2002) makes an influential case for a 'communicational approach to cognition' (p. 13) in learning the discourse of mathematics. In conclusion, Middleton et al. (2013) state,

> The ability to interpret, differentiate, and manipulate the meaning(s) of words and symbols, as well as connect them with mathematical ideas and concepts is a critical aspect of developing the academic language necessary to be considered effective learners of mathematics (p. 210).

2. 2. *Learning language*

How children *use* mathematical language will enable their development of procedural understandings and the abilities necessary to develop habits of thinking and ways of approaching mathematical problems. Therefore, how children *learn* language will be of interest to mathematics educators. In applied linguistics, the language children are exposed to is generally termed input. For acquisition to take place, input should be comprehensible (Krashen, 1989), complex (Huttenlocher, Vasilyeva, Cymerman, & Levine, 2002), and available in large amounts (vanPatten, 2003). The frequency of vocabulary and grammatical patterns in the input acts to make them salient (Ellis, 2006). Additionally, the likelihood of learning is increased when particular language forms are noticed by learners (Schmidt, 1990), which can be due to their repetition in the input (Bybee, 2003) and teachers' use of other instructional and discourse strategies (Barnett-Clarke & Ramirez, 2004).

Sometimes, input may be provided, but children may not be able to access it because of lack of background knowledge or lack of familiarity with discipline-specific text genres. Shanahan and Shanahan (2008) researched the question of reading in different disciplines, and, through tasks and conversations with experts in the fields of history, chemistry, and mathematics, found that reading in mathematics required an extremely close focus with meticulous attention to function words as well as re-reading and memorization of formulae during reading. This type of reading contrasts with that needed for narrative, which is faster and involves more inference with the expectation of chronological sequencing, as shown by Jones (2012) in an analysis of English, mathematics, and science primary school textbooks. Therefore, while there are some similarities to the everyday reading pupils are used to, they have to apply reading strategies differently when reading mathematics.

2.3. *Empirical work on language in mathematics*

In the US, Martiniello (2008) analyzed word problems in a standardized test for the fourth grade, finding through think-out-loud protocols that these word problems showed 'disproportionate difficulty' (p. 333) for children whose English was non-dominant. In particular, grammatical structures, such as multiple clauses, long noun phrases, and only implicit relationships between syntactic units in the text, affected children's reading. Specific aspects of vocabulary were shown to be difficult, such as polysemous words, as well as everyday words expected to be understood mathematically. In Singapore, although Chan (2005) found English proficiency, determined by the end-of-year streaming examination at primary four, did not affect primary five children's performance in solving mathematics word problems, he did find that rewording to create a chronological order of events in the real-world problems and the 'repositioning of givens' (p. 85) had a positive effect on the performance of children less competent in mathematics. Notably, from an error analysis of primary three (P3) pupils' mathematics in Singapore, Cheng (2015) argues that reading and comprehending are 'major areas of difficulty' (p. 3) in solving word problems.

Tuveng and Wold (2005), in research in Norway with language-minority children in the third grade, found that the teacher was unaware of the pupils' comprehension difficulties, which emerged from interviews with the pupils, showing how they experienced problems in understanding the teacher's instructions and the language of textbooks. However, in order to be able to express their difficulties, they needed a higher level of metacognitive awareness than they possessed. The researchers found that 'the children do not focus on language comprehension, and *neither does the teacher'* (Tuveng & Wold 2005, p. 533, original italics). The accepted classroom norm was not to question language but treat it as transparent and understood. A reason suggested for this state of affairs by Adler (1998), in research in South Africa, is that some teachers felt uncomfortable about explicit teaching. She labelled this the 'dilemma of transparency' (p. 26).

Another strand of research focuses on the way concepts are represented in mathematics. While Barwell (2005) reports that the use of the narrative genre enabled pupils to connect their own experience to the contexts of mathematical word problems, other researchers express concerns. Marks and Mousley's (1990) research shows the need to teach pupils the wide range of specific mathematical genres rather than co-opting stories and recounts for mathematics;

Solomon and O'Neil (1998) go so far as to question the contextualization of math problems into pupil's everyday worlds, suggesting that doing so complicates mathematics for them.

2.4. *Mathematics education in Singapore*

The mathematics syllabus (MOE, 2012) was implemented up to primary four (P4) at the time of the project. It centers on mathematical problem solving supported by interrelated components of skills, processes, concepts, attitudes, and metacognition. Inspired by Bruner (1960) according to Leong, Ho, and Cheng (2015), it takes a spiral approach to the delineation of topics (MOE, 2012, p. 9). The syllabus gives language a prominent role, with one aim being 'to enable all students to develop thinking, reasoning, communication, application, and metacognitive skills through a mathematical approach to problem solving' (MOE, 2012, p. 8). Communication is defined as:

> ... the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely, and logically. It helps students develop their understanding of mathematics and sharpen their mathematical thinking (MOE 2012, p. 15).

The syllabus intends not only to delineate content but also to 'influence the way teachers teach and students learn' (MOE, 2012, p. 6). A particular heuristic for diagrammatically representing word problems known as the model method is recommended (Kho, Yeo, & Lim, 2009). Further, the syllabus proposes that students should be exposed to a variety of learning experiences, including hands-on activities and technological support to help them relate abstract mathematical concepts with concrete experiences and so deepen understanding (MOE 2012, p. 15). A progressive move from concrete through pictorial to abstract (CPA) learning experiences is envisaged. As Leong et al. (2015) point out, this notion is adapted from Bruner's (1966) proposition that modes of instructional representation should move from enactive to iconic to symbolic.

3. The Research Study

3.1. *The context of the research*

The study was conducted in one Singaporean primary school. The school followed the national mathematics syllabus and employed the popular 'My Pals are Here!' textbook series. The focus was on one primary five class (P5) of 39 pupils (aged 11), one primary three (P3) class of 35 (aged nine), and one primary one (P1) class of 29 (aged seven).

3.2. *The research aims*

The study sought to purposefully investigate whether and how teachers teach the language associated with the concepts of mathematics in mathematics lessons at different levels of the primary school. It also investigated how effectively pupils learned from this instruction. This article, however, focuses on the teaching, aiming to understand more deeply and from the ground up (Charmaz, 2006; Strauss & Corbin, 1998) the role of language in the regular practice of mathematics teaching in one Singaporean primary school.

In that the study aimed to explore the process connections between teaching, learning, and context, it employed a qualitative methodology, focusing on the case of one school in particular (Stake, 2008). The techniques of lesson observations, surveys, interviews, reflective tasks, and document analysis were employed in order to identify perceptions and attitudes that motivated behaviour. Considering that English is the medium of education and the study was about the role language might play in children's learning of mathematics, contextual information was gathered about pupils' home language use and teachers were asked about their pupils' use of English in surveys and interviews. Additionally, analysis of the pupils' mathematics tasks produced quantifiable data to situate how effectively the pupils learnt the mathematics topics covered in the focal lessons while analyses of syllabi and textbooks provided the policy perspective.

3.3. *The participants*

A survey administered to the school's 22 mathematics teachers was designed to reveal their attitudes and beliefs about teaching the language of mathematics at the different levels of the primary school. The survey included open questions as well as closed ones (e.g., about length of teaching service). These data situated the beliefs and attitudes of the three main teacher participants of the study who also completed the survey. Further validation was secured by means of interviewing the two senior teachers in the department separately for about one hour each.

The focal teacher participants of the study were three experienced teachers of mathematics: each taught a different grade level – P1, P3, and P5. One of each of their lessons was observed. The lessons observed were selected on the basis that they introduced a new mathematical topic to the pupils in each class. The teachers provided lesson plans before their lessons and were each interviewed for about 30 minutes after their lessons. The three lesson topics were ordinal numbers and position in P1, two-step word problems in P3, and ratio in P5. The P1 and P3 lessons were each about one hour while the P5 lesson was about 40 minutes.

3.4 *The method of analysis*

Each dataset was analysed in appropriate ways. The teachers' answers to questions on the surveys were extracted into digital tables for each teacher. From individual tables, answers were collated across the questions. The lessons were audio recorded and transcribed fully. Observers made notes and collected lesson materials, such as PowerPoint slides and activity sheets along with each teacher's lesson plans. The mathematics lesson plans indicated the intended learning outcomes of each lesson and the instructional activities. The first phase of the lesson analysis scrutinized the teacher language and activities from a pedagogic perspective using the transcripts, plans, and activity sheets. The second analytic phase focused on language and how it was used and drawn attention to in the lessons. In a third analytic phase, the constant comparison technique (Charmaz, 2009; Merriam, 1998) was employed to explore how language and mathematical concepts were related within and across the lessons. Finally, the transcripts were reread to examine whether and how themes from the teacher interviews and surveys were evident in practice.

The interviews with the three focal teachers and two senior teachers were transcribed fully. These were first read against the context provided by the survey data to ascertain consistency and anomalies. Teachers' perspectives on mathematics teaching and language were drawn out of the detail of the interviews. The teachers' perspectives were examined further to discover relationships between the interview data and lesson observations. In addition to the process of the researchers' data analysis, three meetings were held in school with the lead teachers at which time the data were further discussed, explored, and analyzed. These meetings acted as a form of member checking (Saldana, 2011). The examples from lessons presented in this article are those which illustrate the aspects of teaching mathematical language which were significant to teachers, evidenced in the themes drawn from surveys and interviews. The quotations from the teachers' interviews thus illuminate the lesson enactments which, in turn, exemplify teachers' rationales, all in relation to the research question about whether and how teachers teach the language of mathematics[i].

4. Results

4.1. *Teachers' thinking*

The 22 teachers surveyed had teaching experience ranging from one-and-a-half to 43 years. The main points emerging from the survey are:

1. All teachers thought language important in their pupils' learning of mathematics, generally mentioning this importance in the context of understanding word problems.
2. All except one thought of language in terms of children's understanding (reading and listening) rather than in terms of their own language for teaching.
3. Three conceptualized language production (writing and speaking) as communication or as language needed for learning. Otherwise, language production/output was not considered.
4. All teachers endorsed the provision of concrete and social experiences in learning throughout the levels. However, they thought these would diminish in importance with a move to greater abstraction, rigour, and complexity of terms as children matured.

The notion of students' understanding was also echoed among the focal teachers in the way they described the learning outcomes for their observed lessons.

> TP1: The aim of my lesson is for them to understand the difference between ordinal numbers and the numerals.
>
> TP3: The aim is to make them understand the operations to use for the word problem, and to highlight the key words, the clues that tell them what operations to use, and basically to just understand what the word problem is about.
>
> TP5: Okay it was to introduce ratio to them, to let them know that ratio is a comparison between two quantities. That was the main objective.

4.2. *Focusing on vocabulary*

At P1 and P3, teachers introduced vocabulary as input to be *learnt* and revised while the P5 lesson showed how vocabulary was to be *used*. The only feature of language the P5 teacher deliberately drew pupils' attention to was the colon, using the term *represent* to connect it to ratio. When asked for her reasoning, the teacher said:

> TP5: So, it's important to explain to the pupils what the symbols and numbers represent so that they don't just see the symbols and numbers as that. They really understand them conceptually, so that when they do word problems, they would know how to apply the ratio concept.

At P1 and P3, teachers used different strategies to introduce and talk about vocabulary. One strategy was linguistic signaling, which itself became more complex at the different grade levels. These were the terms used:

- P1 - *is called, word, spell*
- P3 - *is called, word, key word, terms, refers*
- P5 - *key word, phrase, concept, represent*

The focus on explicit signposting of vocabulary was most apparent at P1 where spoken vocabulary was labelled and linked to writing and spelling. In this lesson, the term *ordinal numbers* was introduced deliberately and the ordinal numbers were related to the previously learnt numerals. Examples of the linguistic highlighting of vocabulary in this lesson are, 'We're also going to learn the words *first* and *last*', 'These are words that you have learnt', 'spell out ten with the *th*', and 'these are the words you need to remember'. The numbers were spelled out in a list on the whiteboard and then the spellings of the ordinals were added by the side of each to spotlight the difference. The teacher explained the root of the word *ordinal* by saying, 'Ordinal numbers are numbers that give you an order. Can you see that these ordinal numbers are very close to the word order?', pointing to the word *ordinal* on the whiteboard. In addition to the signalling words used at P1, the P3 teacher used *terms* and provided initial letters to prompt pupils to spell words. The teacher deconstructed initialisms for pupils, for example, the intitialism UPDC is one used school-wide, standing for understand, plan, do, and check. In the lesson, the teacher revised pupils' understanding by eliciting the words represented by each initial letter and by applying each of the processes to one of the word problems of the lesson. At P3, the particular vocabulary focus was on synonyms and different forms of words related to the four mathematical operations. As Extract 1 shows, the teacher was concerned with extending the children's existing vocabulary of *plus* and *minus* to more technical, nominalized forms, signposting this with *word, terms* and *refers*.

Extract 1.

TP3:	Multiply. Now, I would not use the word plus. What is the math word//
C	Addition.
TP3:	That is supposed to be said for plus.
C:	Addition?
TP3	Addition. How about minus? How do we say it in math terms?
C:	Subtraction.
TP3:	Subtraction. So, when you see the four operations, it refers to addition, subtraction, multiplication, and?
M:	Division.

The second vocabulary-focusing strategy used by teachers at P1 and P3 was repetition. Table 4 below shows how the P1 teacher concentrated attention on the ordinal numbers through repetition. Additionally, the high frequency of prepositions suggests the link in the lesson between the ordinal numbers and the concept of relative position. Prepositions, such as *before* and *after*, were taught explicitly, not only through being highlighted as important *words* but also through the contexts provided by demonstrations, drawings, and discussions: paper labels were given to pupils who lined up and changed position; the finishing line of a race was drawn on the whiteboard; queuing in the school canteen was discussed.

Table 1. Frequency of ordinal numbers and prepositions in the P1 teacher's language

Frequency of ordinal numbers		**Frequency of prepositions**	
first	48	after	8
second	21	before	11
third	19	behind	9
fourth	15	between	14
fifth	6	in front (of)	14
sixth	5	left	9
seventh	3	next	1
eighth	2	right	8
ninth	2		
tenth	14		
eleventh	1		
twelfth	2		
last	23		

At P3, a focus on the forms of language related to mathematical operations was evident by their variety and frequency.

Table 2. Frequency of varied forms of vocabulary referring to the four mathematical operations in the P3 teacher's language

Frequency of teacher mentions of mathematical operations			**Frequency of teacher mentions of mathematical operations**	
Addition	7		Multiplication	5
○ add	4		○ multiply	9
○ altogether	14		○ times	8
○ plus	4		○ twice (as many)	8
○ sum	2			
○ total	1			
Subtraction	5		Division	8
○ difference	2		○ bring down	4
○ left	7		○ divide	10
○ minus	7		○ equally	3
○ subtract	8		○ quotient	4
○ take away	1		○ remainder	10

However, in addition to the focus on vocabulary input foregrounded by signalling words and repetition, evident in the teachers' language of instruction was some complex grammar. The P1 teacher used conditional structures while hypothesising about changing the position of people and pictures. The teacher said, for example, 'what happens if I were to change my position?', 'if I were to count from the left', 'what if I start counting from the right?' and assuming the condition in 'how many people will be in front of me?' Since, in the P3 lesson, the class was working on solving a comparison problem that required a particular sort of comparative diagram model, different forms of *compare* were used in the lesson such as 'comparison model', 'start comparing', and 'compares'. Added to those were comparisons made about numbers such as 'which number is bigger', 'which one is smaller' and the conditional, 'So if the number is smaller, what must you do with the size of the model?'

Comparatives were also used to refer to the process of learning; for example, the teacher asked, ‘Now, when I have already drawn the model like this, is it easier for you to solve the question?’

4.3 *Focusing on reading*

Every observed lesson engaged pupils in reading word problems in order to solve them. At P1 and P5, problems were used as a means of teaching the concepts of the lessons while, at P3, the lesson focused directly on teaching pupils how to draw models to solve problems. At P1, pupils had to read instructions and work in groups to manipulate pictures of animals into the appropriate order. The challenge was that the set of instructions did not reflect the procedural sequence of solving the problem. Another challenge was that some pupils could not read the word problem. At interview, the teacher explained the need to read questions out aloud and use choral reading to remediate this situation. The teacher also explained the meanings of the everyday word *vet* and the procedural word *clue* in the lesson. Extract 2 shows the teacher explaining while manipulating the animal pictures on the visualiser.

The P1 question.
Five animals are sick and are waiting to see the vet. How are they lined up?
Read the clues and put the animals in the correct position.
Clues:

a) Five animals (Tiger, Snake, Monkey, Dog and Horse) are in a queue.
b) Tiger is between Snake and Monkey.
c) Snake is 2nd.
d) Dog is last in the queue.

Extract 2

TP1 I’m going to explain to you what is this about. So, there are five animals who are sick. They are sick. And they are waiting to see the vet. Vet is the animal doctor. So how are they lined up? Read the clues and put the animals in the correct position.

C (…)

TP1 Okay? So, a). Five animals are in a queue. This is a clue. What are the animals? Tiger, snake, I will show you. We have the tiger, we have the snake, we have the monkey, we have the dog, and also the horse. Five animals. They are in a queue. Tiger is between snake and monkey.

During the P3 lesson, the teacher and pupils co-constructed models (Kho, Yeo, & Lim, 2009) to solve two-step word problems on the whiteboard. The diagrams were built through the teacher thinking-out-loud and questioning pupils as well as directly explaining while drawing the example models on the whiteboard. The teacher acknowledged that the questions were 'wordy' and 'long' and had pupils 'simplify' the problem by reading each line as 'it tells us clearly of the steps to answer the question'. The P3 teacher focused on the logic of problem solving by saying things such as, 'what is the next step?' and 'we'll do it one digit at a time'. In picking out the words of the problem which indicated the mathematical operations and procedures, the teacher was encouraging pupils to read every word of the question closely while ensuring that they used precise mathematical terminology, as Extracts 3 and 4 show.

Extract 3

TP3	So can you read aloud the word problem, one, two, three?
M	*April had four hundred and thirty-five dollars. Lin gave her three hundred and thirty-six dollars more. How much did April have altogether? April spent five hundred and forty-six dollars on an oven. How much did she have left?*
TP3	Okay, now can you identify some of the math words that you see in the word problem? C1?
C1	Altogether.
TP3	Altogether is one of them. C2?
C2	More.
TP3	More, okay. C3?
C3	Less.
TP3	Less. Okay, so these are the math words that we have seen. Now. Using your FAST kits[ii] that you have – don't tell me the answer, we have to use this to show – can you tell me, from what you have understood from the question, what you have understood, which model would you use for this question. A, is it a part-whole model? B – don't raise your hands – is it a comparison model, okay? Find the A and B in the FAST kits, at the count of three, you raise it up so that I can see. A or B, alright? Is everyone ready with the answer?

Extract 4

TP3:	Final? Statement. Okay? Whenever you have a number statement (…) there are two parts, part A and part B. I must have a number statement and a final statement for each part. Now, next part. *Rachel spent five hundred forty-six dollars on an oven. How much did she have left?* C1, what must I do next?

C1: (Minus)
TP3: Minus. What is the word that you must use?
C2: (Subtract)
TP3: Subtract. Which one tells you that it must be subtraction? C3? Which word tells you that it must be subtraction?
C3 Spent.
TP3: Are you sure? Which word. C4?
C4 (Left)
TP3: Left. So, the word left?

Additionally, the P3 teacher had the pupils engage with the text of the word problem by asking them to highlight, circle, and underline words in relation to the diagrams constructed on the board.

By contrast, in the P5 lesson, there was less extensive scaffolding through guiding questions and more of the teacher leading the step-by-step procedure of problem solving through self-questioning and thinking-out-loud. There was engagement with the abstraction of text in this P5 lesson as the word problem was shown on the visualizer and, through having pupils annotate and read aloud, the teacher kept the pupils thinking together, focusing on the text and completing the problem-solving process in their own workbooks. This was routine, as the teacher noted, 'I said before, in your homework there should be some form of annotation or highlighting to show that you are actually trying to understand the question.' These ways of engaging with the text meant that the thinking underlying the procedure was slowed and made the focus of the lesson. Through her pedagogy, the P5 teacher had the pupils examine the mathematical relationships coded in the language of the text and the mathematical operations that they indicated. In effect, the teacher was teaching the specific reading skills required to understand the mathematical word problem about ratio, which would ensure translation into the mathematical procedure and model. It was smoothly done in a class that had the necessary linguistic, conceptual, and procedural knowledge.

5. Discussion

This discussion centers on the enactment of the mathematics lessons in relation to teachers' perceptions from the interviews and surveys. First, we discuss the issue of transparency of language and the ways in which teachers encouraged noticing (Schmidt, 1990) of the specific terminology of mathematics at word level through repetition and signaling, in contrast to the way they assumed pupils' knowledge of complex grammatical structures particularly associated with mathematics in their instructional language. Second, we discuss the issue of translation from meaningful real world context to mathematical procedure necessary in teaching mathematics with word problems. We discuss pedagogic tensions at P3 when the outcome of the lesson was learning procedure, yet children's mathematical procedural vocabulary was insecure and the scaffolding around the context of the everyday world of the word problem was reduced. The tensions are particularly felt at P3, we argue, because of its mid-way stage in children's language development as well as their conceptual and procedural capacities in problem solving.

5.1 *Vocabulary as noticeable mathematical language input*

In this school, there was no 'dilemma of transparency' (Adler, 1998, p. 26). The teachers, concerned with deliberate language input, described their practices of explicit teaching of mathematical language as necessary, given the linguistic diversity among their pupils. In lessons, teachers provided the precise academic language needed for the mathematical concepts taught, particularly at P1 and P3, (Cummins, 2000; Huttenlocher et al., 2002; Laborde, 1990; Middleton et al., 2015), and they expected children to retain and use this vocabulary. The two main strategies observed were linguistic signaling and repetition at word level. It is likely that these strategies would make mathematical vocabulary salient and encourage pupils to notice them, in line with theories of language acquisition (Bybee, 2003; Ellis, 2006; Schmidt, 1990; VanPatten, 2003). The teachers argued that explicit teaching would lead to greater awareness of language among pupils and the development of metacognition – an aim of the mathematics syllabus and one which is shown not to be realized when language is treated as transparent (Tuveng & Wold, 2005). It is the explicit knowledge of the language of mathematics, which would prime pupils for the spiral syllabus (Bruner, 1960; Leong et al., 2015), according to the teachers.

In accordance with the literature, the lesson observations and interviews with teachers suggest that some aspects of language are particularly associated with mathematics. Middleton et al. (2013) note the prevalence of prepositions, comparatives, and conditionals while Laborde (1990) finds nominalizations to be frequent. Prepositions were explicitly taught at P1, as were synonyms, various forms of mathematical operation vocabulary items, and comparatives at P3. At P1 and P3, the teachers also highlighted the vocabulary associated with the concepts of the lesson, tending to focus on words rather than syntax. However, they also used the comparative and conditional, seemingly unnoticed and without commentary, in their instructional language, assuming children's understanding of these structures, and thus confirming the survey data (point 2) which shows that language is significant to teachers in relation to children's understanding rather than in the process of teaching. This confirms research by Tuveng and Wold (2005) and Cameron, Moon, and Bygate (1996) which suggests that pedagogic procedural language can be just as complex and problematic as technical, conceptual vocabulary for pupils for whom the language of instruction is non-dominant.

5.2 *Reading to understand mathematics and language*

The centrality of problem solving in the mathematics syllabus (MOE, 2012) is reflected in the teachers' concern with understanding. The lessons showed them teaching the appropriate comprehension strategies for mathematics in varied supportive ways. This, according to Shanahan and Shanahan (2008), entails a close, precise reading, focusing on individual function words. Teachers seemed, therefore, to anticipate Cheng's (2015) conclusion that reading and comprehending are 'major areas of difficulty' (p. 3) for pupils in solving word problems at P3 in Singapore. Apart from questions raised by Solomon and O'Neil (1998) and Marks and Mousley (1990) about the use of narrative genres in mathematics and their potential for confusing pupils, a reason for the difficulty may be the ways in which language functions in word problems. The word problems create imagined real-world situations in which mathematical operations and procedures are embedded. Thus, the teacher has to help pupils understand the real-world meaning and translate that understanding into a mathematical procedure that is often then instantiated in a model diagram.

Some difficulty appears in the P3 lesson which focused directly on the learning of model drawing. The teacher agrees with pupils that the problem is 'wordy' and 'long' (referring to the real-world context) but assures them that it clearly indicates 'the steps to answer the question' (referring to the mathematical procedure). An example of the difficulty occurs in the question and answer sequence in Extract 4, when the teacher, asking for a word indicating subtraction, rejects a pupil's response of 'spent' in preference to the word 'left'. This seemingly arbitrary choice can be explained by referring to the teacher's original question of 'what must I *do* next?' In phrasing her question thus, the teacher tries to elicit words that directly reference the mathematical procedure and accepts 'left'. The word 'spent' reflects the real-world situation and is therefore not preferred. The teacher here focused on translation while the pupil attended to the experiential context. At this middle stage of P3, the language needs of the pupils and the discipline impel the teacher to teach technical vocabulary, as would be done in P1. At the same time, the teacher has to introduce pupils into reading mathematically, focusing on procedure rather than context as would be done at P5. However, the conceptual transition necessary between the experiential world constructed in the language and the subtext of the mathematical procedure seems to create tension for her and a struggle for the pupils. Perhaps the pupils' language knowledge does not meet that expected by the wording of the problem, or perhaps the cognitive load created by learning how to represent the model is heavy for the pupils. Conversely, in the P1 lesson, the teacher moves fluidly between the real-world meaning of the word *vet* and the procedural meaning of the word *clue,* and, at P5, the reading and problem solving went smoothly, focusing, as it did, on the procedural aspects of the problem rather than the real-world context.

5.3 *Principles of teaching and learning from mathematics and language*

Contextualisation, an important principle of language teaching, involves situating language in meaningful contexts to facilitate learning (Halliday, 2007). The CPA principle of teaching in mathematics is used to scaffold pupils' thinking through the affordance of the material (or concrete) and pictorial as precursor to the abstract (Bruner, 1960; Leong et al., 2015). The three lessons showed different relations between these two principles and how they were drawn upon as teachers taught the language of mathematics.

At P1, there seemed to be a neat alliance between the two principles as the language served helpfully to label the concrete material, which was physically contextualised through demonstrations of pupils lining up and changing position as well as by pictures and drawings, the C of the CPA. Even the more complex conditional structures used by the teacher were demonstrated by the physical changes of pupils' positions, which may explain why teachers took pupils' linguistic understanding for granted. Thus, the principle of the CPA was a way in which language input was contextualized (Halliday, 2007) and made comprehensible (Krashen, 1989). At P5, word problems were used to teach the concept of ratio. While everyday situations about muffins and drinks were constructed by the language of the problems, the actual reading practised in the lesson concentrated on the procedure embedded within the word problems. Thus, the emphasis of the lesson was on translating the language of the word problem into its more abstract mathematical procedure. The A of the CPA was foregrounded; language was its transparent conduit.

In contrast to the alliance at P1 and the way in which language was used as a conduit at P5, these two principles appeared to collide at P3 when the lesson focused on learning about models and how to draw them (Leong et al., 2015; Ng & Lee, 2009). By P3, it seemed that the pupils were expected to have learnt the everyday language which contextualised the mathematical procedure, and therefore attention could be focused directly on the pictorial, the P of the CPA, that is, the model drawing procedure cued through mathematical vocabulary in the word problems. Attention to the real-world context was reduced, in the effort to gather the cues to translate the embedded procedure into the comparison model. This mid-way stage of learning when the scaffolding of understanding the real-world context was reduced in order to focus on representing procedure seemed to create the most difficulty for pupils and was a tension for the teacher.

6. Conclusion

This exploratory study has produced interesting findings about whether and how the language of mathematics is taught in a multilingual primary school. Teachers recognized the importance of language in mathematics learning. They taught vocabulary as input and reading for mathematics intentionally in varied and supportive ways. Some issues to explore in further research are the complexity of the teachers' instructional language at P1 and the extent to which this is made comprehensible by the CPA principle, especially when the language encodes important mathematical functions, for example, hypothesizing. Research attention may also be focused on the transitional stage of P3 since this study shows some potential areas of tension in teaching and subsequent confusion for pupils as they learn to represent mathematically through understanding a language which for them may be non-dominant.

Acknowledgements

This study was funded by Singapore Ministry of Education (MOE) under the Education Research Funding Programme (SUG14/15 SAJ) and administered by National Institute of Education (NIE), Nanyang Technological University, Singapore. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the Singapore MOE and NIE. IRB 2016-01-001.

Transcription conventions

T	teacher
C	child
M	many children
C1, C2, C3, and so on	named children
TP1, TP3, TP5	the teacher of primary one, three, and five
?	upward intonation
.	downward intonation
..	pause, relative length shown by the number of full stops
/	successive turns without pause
//	heard as an interruption
[ ]	heard as simultaneous
(….)	cannot be heard clearly enough to transcribe, sometimes the general area is given, e.g. (name)
(( ))	added by transcriber, e.g. participant's action
italics	text read aloud

References

Adler, J. (1998). A language of teaching dilemmas: Unlocking the complex multilingual secondary mathematics classroom. *For the Learning of Mathematics*, *18*(1), 24-33.

Aiken, L. R. (1971). Verbal factors and mathematics learning: A review of research. *Journal for Research in Mathematics Education*, *2*(4), 304-313.

Barnett-Clarke, C., & Ramirez, A. (2004). Language pitfalls and pathways to mathematics. In R. Rubenstein and G. W. Bright (Eds.), *Perspectives on the teaching of mathematics* (pp. 56-66). Reston, VA: National Council of Teachers of Mathematics.

Barwell, R. (2005). Working on arithmetic word problems when English is an additional language. *British Educational Research Journal*, *31*(3), 329-348.
Bernardo, A. B. I., & Calleja, M. (2005). The effects of stating problems in bilingual students' first and second languages on solving mathematical word problems. *The Journal of Genetic Psychology*, *166*(1), 117-128.
Bruner, J. S. (1960). *The process of education*. Cambridge, MA: Harvard University Press.
Bruner, J. S. (1966). *Toward a theory of instruction*. Cambridge, MA: Harvard University Press.
Bybee, J. (2003). Mechanisms in change in grammaticalization: The role of repetition. In R. Janda & B. Joseph (Eds.), *Handbook of historical linguistics* (pp. 602-623). Oxford: Blackwell.
Cameron, L., Moon, J. S., & Bygate, M. (1996). Language development of bilingual pupils in the mainstream: How do pupils and teachers use language? *Language and Education*, *10*(4), 221-236.
Chan, C. M. E. (2005). Language proficiency and rewording of semantic structures in P5 pupils' mathematical word problem solving. *The Mathematics Educator*, *9*(1), 84-99.
Charmaz, K. (2006). *Constructing grounded theory: A practical guide through qualitative analysis*. London: Sage.
Christie, F. (1998). Science and apprenticeship: The pedagogic discourse. In J. R. Martin & R. Veel (Eds.), *Reading science: Critical and functional perspectives on discourses of science* (pp. 152-177). London: Routledge.
Cummins, J. (2000). *Language, power and pedagogy: Bilingual children in the crossfire*. Clevedon: Multilingual Matters.
Cummins, J., & Man, E. Y.-F. (2007). Academic language: What is it and how do we acquire it?" In J. Cummins & C. Davison (Eds.), *International handbook of English language teaching, part two* (pp. 797-810). New York: Springer.
Department of Statistics. (2010). *Census of population statistical release 1: Demographic characteristics, education, language and religion*. Singapore: Singapore Department of Statistics.
Department of Statistics. (2016). *Population trends, 2016*. Singapore: Singapore Department of Statistics.
Ellis, N. C. (2006). Selective attention and transfer phenomena in L2 acquisition: Contingency, cue competition, salience, interference, overshadowing, blocking, and perceptual learning. *Applied Linguistics*, *27*(2), 164-194. doi:10.1093/applin/aml015
Evans, J. (2002). Talking about maths. *Education 3-13*, *30*(1), 66-71.
Gee, J. P. (2000). The new literacy studies: From 'socially situated' to the work of the social. In D. Barton, M. Hamilton, & R. Ivanic (Eds.), *Situated literacies: Reading and writing in context* (pp.180-196). London: Routledge.
Halliday, M. A. K. (2007). *Language and education.* Volume 9 of *Collected works of M. A. K. Halliday*. London: Continuum.
Huttenlocher, J., Vasilyeva, M., Cymerman, E., & Levine, S. (2002). Language input and child syntax. *Cognitive Psychology*, *45*, 337-374.
Jones, S. A. (2012). Recontextualising reading, rethinking teaching: Reading in the English medium primary school in Singapore. *Education 3-13, 40*(3), 243-258. doi:10.1080/03004279.2010.512564
Kaur, B. (2014). Enactment of school mathematics curriculum in Singapore: Whither research! *ZDM Mathematics Education*, *46*, 829-836. doi: 10.1007/s11858-014-0619-6.
Kho, T. H. (1987). Mathematical models for solving arithmetic problems. In *Proceedings of Fourth Southeast Asian Conference on Mathematical Education (ICMI-SEAMS):*

'Mathematical Education in the 1990's' (pp. 345-351). Singapore: Institute of Education.

Kho, T. H., Yeo, S. M., & Lim, J. (2009). *The Singapore model method for learning mathematics*. Singapore: EPB Pan Pacific.

Krashen, S. D. (1989). *Language acquisition and language education: Extensions and applications*. 2nd ed. New York: Prentice Hall International.

Laborde, C. (1990). Language and mathematics. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and cognition: A research synthesis by the international group for the psychology of mathematics education* (pp. 53-69). Cambridge: Cambridge University Press.

Leong, Y. H., Ho, W. K., & Cheng, L. P. (2015). Concrete-pictorial-abstract: Surveying its origins and charting its future. *The Mathematics Educator*, *16*(1), 1-19.

Loh, M. Y. (2005). *Using journal writing to assess primary five students' learning in fractions* (Unpublished master's dissertation). National Institute of Education, Nanyang Technological University, Singapore.

Marks, G., & Mousley, J. (1990). Mathematics education and genre: Dare we make the process writing mistake again? *Language and Education*, *4*(2), 117-135. doi:http://dx.doi.org.libproxy.nie.edu.sg/10.1080/09500789009541278

Martiniello, M. (2008). Language and the performance of English-language learners in math word problems. *Harvard Educational Review*, *78*(2), 333-368.

McKay, S. L. (2002). *Teaching English as an international language*. Oxford: Oxford University Press.

Mercer, N., & Littleton, K. S. (2007). *Dialogue and the development of children's thinking: A sociocultural approach*. London: Routledge.

Mercer, N., & Sams, C. (2006). Teaching children how to use language to solve maths problems. *Language and Education*, *20*(6), 507-528.

Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco: John Wiley and Sons.

Middleton, J. A., Llamas-Flores, S., & Guerra-Lombardi, P. (2013). English language learning and learning academic language in mathematics. In M. B. Arias & C. J. Faltis (Eds.), *Academic language in second language learning* (pp. 201-224). Charlotte, NC: Information Age Publishing.

Ministry of Education. (2012). *Mathematics syllabus primary one to four*. Singapore: Curriculum Planning and Development Division.

Moschkovich, J. (2010). Language(s) and learning mathematics: Resources, challenges, and issues for research. In J. Moschkovich (Ed.), *Language and mathematics education. Multiple perspectives and directions for research* (pp. 1-28). Charlotte, NC: Information Age Publishing.

Moschkovich, J. (2010). Recommendations for research on language and mathematics education. In J. Moschkovich (Ed.), *Language and mathematics education. Multiple perspectives and directions for research* (pp. 151-166). Charlotte, NC: Information Age Publishing.

Ng, S. F., & Lee, K. (2009). The model method: Singapore children's tool for representing and solving algebraic word problems. *Journal for Research in Mathematics Education*, *40*(3), 282-313.

Pimm, D. (1991). Communicating mathematically. In K. Durkin & B. Shire (Eds.), *Language in mathematical education: Research and practice* (pp. 17-23). Milton Keynes: Open University Press.

Saldana, J. (2011). *Fundamentals of qualitative research.* Oxford: Oxford University Press.

Schleppegrell, M. (2001). Linguistic features of the language of schooling. *Linguistics and Education*, *12*(4), 431-459.

Schmidt, R. (1990). The role of consciousness in second language learning. *Applied Linguistics*, *11*(2), 129-158. doi:http://dx.doi.org/10.1093/applin/11.2.129

Sfard, A. (2002). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. In C. Kieran, E. Foramn, & A. Sfard (Eds.), *Learning discourse* (pp. 13-57). Dordrecht, Netherlands: Kluwer Academic.

Shanahan, T., & Shanahan, C. (2008). Teaching disciplinary literacy to adolescents: Rethinking content-area literacy. *Harvard Educational Review*, *78*(1), 40-59.

Solomon, Y., & O'Neill, J. (1998). Mathematics and narrative. *Language and Education, 12*(3), 210-221.

Stake, R. E. (2008). Qualitative case studies. In N. J. Denzin & Y. S. Lincoln (Eds.), *Strategies of qualitative enquiry*. 3rd ed. (pp. 119-149). Thousand Oaks, CA: Sage.

Strauss, A., & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. Thousand Oaks, CA: Sage.

Tuveng, E., & Wold, A. H. (2005). The collaboration of teacher and language-minority children in masking comprehension problems in the language of instruction: A case study in an urban Norwegian school. *Language and Education*, *19*(6), 513-536.

Unsworth, L. (2001). *Teaching multiliteracies across the curriculum: Changing contexts of text and image in classroom practice*. Buckingham: Open University Press.

VanPatten, B. (2003). *From input to output: A teacher's guide to second language acquisition*. Boston: McGraw-Hill.

Vygotsky, L. S. (1962). *Thought and language.* (E. Hanfmann & G. Vakar, Trans. & Eds). Cambridge, MA: Massachusetts Institute of Technology.

---

[i] Since the research was exploratory and grounded, we report in this article on aspects of teaching the language of mathematics that appeared significant to teachers. They were preoccupied with teaching input, in particular, vocabulary and reading. However, we acknowledge the significant role of output in learning language and mathematics although it is not the focus of this article.

[ii] FAST kits are sets of cards, with one large capital letter written on each, for example, A, B, or C. As part of classroom assessment, a teacher may ask children to signal their thinking by holding up a card. All the cards can then be clearly seen by the teacher.