
Title	How children become aware of patterns
Author(s)	Sumathi Lingesvaran and Ng Swee Fong
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How children become aware of patterns

Sumathi Lingesvaran
&
Ng Swee Fong
National Institute of Education
Singapore

This symposium

- Thinking algebraically include the ability to understand patterns, relations, and functions; to represent and analyze mathematical situations and structures using algebraic symbols, analyze change in various contexts. Each of these abilities evolve as children grow and mature (NCTM, 2001).
 - I will be presenting the work by Sumathi who looked at how three Primary 2 children recognized patterns.
 - Tan Hsing Hui explores how three Primary 6 EM3 students worked with function based tasks.
 - Anna Guerrero discusses how kindergarten, Primary 2 and Primary 3 developed in terms of balance scale tasks
 - Finally Wang Ming Hwee provides evidence of the different levels of understandings Secondary 3 students have of roots of quadratic equations.

Rationale

- Pattern recognition and generalization are central components of algebraic reasoning.
 - With repeating patterns, emphasis is placed on the cyclical nature of the repetition and the identification of the elements in the cycle – the structure as it were. Show examples
 - At simple level, the shapes follow a core repeating pattern. At a more complex level, the shapes are related to its position in the pattern, a growing pattern.

Research Questions

- Can Primary 2 pupils see the structure underpinning the pattern based tasks?
- Can these Primary 2 pupils see the relationship between the core repeating pattern and the growing number pattern?
- What types of knowledge and skills do Primary 2 pupils employ in their solution of pattern-based tasks?

Method

- 2 pattern based questions (list of questions)
- Three Primary 2 pupils from a better ability class were selected.
- The pupils were from the same class, but of different abilities (1 higher, 1 average, 1 lower)
- One-to-one interviews were conducted with each child.
- Interviews were taped for further analyses.

Findings: Can Primary 2 pupils see the structure underpinning the pattern based tasks?

- All three pupils were able to see the cyclical nature of the patterns.
- For both Questions 1 and 2, they were able to detect that it was a core repeating pattern.
 - Question 1 was a core repeating pattern of length 2.
 - Question 2 was a core repeating pattern of length 3.

Findings: Can these Primary 2 pupils see the relationship between the core repeating pattern and the growing number pattern?

- Both higher and average ability pupils were aware
 - that there was a connection between the shapes and the numbers, were able to make near and far prediction.
- Lower ability pupil, however,
 - was not aware that there was a connection between the shapes and the numbers. Hence this pupil was unable to make far prediction.

What types of knowledge and skills do Primary 2 pupils employ in their solution of pattern-based tasks?

- Both higher ability and average ability pupils had strong command of number facts.
- For Questions 1 and 2, their command of the multiplication tables 2 and 3 helped them to answer the following category of questions.
 - Questions 1 and 2 ended with number 10. Pupils were asked to make far prediction.
 - What shape is above a given number? (e.g. 21)
 - If there were n numbers, how many were of a specific shape was there?

What types of knowledge and skills do Primary 2 pupils employ in their solution of pattern-based tasks?

- The higher and average ability pupils used mathematics specific language to help them to explain how they arrived at the answers.
 - To each of items in Questions 1 and 2, higher ability pupil explained using the phrase “multiples of” a given number (taught by mother)
 - Average ability pupil still relied on counting strategies to make far prediction but was able to explain that shapes were in those specific positions because they followed the three times table for Question 2.
- Lower ability pupil had no recourse to such language.

Conclusions

- Important to offer such learning experiences to young children.
- Important to use language specific to mathematics to describe numbers.
- Even at Primary 2, the term multiple can be used to describe numbers.
- This may help prevent the confusion pupils have with factors and multiples when they are first introduced to these terms in Primary 4.

Excerpts of the Interview

Question 1



1 2 3 4 5 6 7 8 9 10

How do you know that the next shape is a triangle?



High Ability

- triangle, circle, triangle, circle, here is a circle pointing to the circle, so the next will be a triangle.

What is happening?

- The shapes are repeating.

What is repeating?

- 1 triangle and 1 circle are repeating.

Average Ability

- triangle, circle, triangle, circle and so after a circle, it is a triangle.

What is happening to the triangle and circle?

- a triangle, a circle, are repeating

Lower Ability

- It keeps going.

What keeps going?

- Triangle, circle.

What is happening to the triangle and circle?

- a triangle and a circle forming a shape.

After forming a shape what is happening?

a triangle and circle is forming a shape and it keeps going.

Excerpts of the Interview

Question 1

									
1	2	3	4	5	6	7	8	9	10

How do you know that 21 is a circle/triangle?

High Ability

- 13, 15, 17, 19. So 21 is triangle.

How do you know these numbers?

- I skip 1 number

Can you explain?

- Skip 12 then 13, skip 14 then 15.....

Average Ability

- I counted.

Can you count out loudly?

- 13 pointing to triangle, 14 alternating to circle. Counted up to 21 and point to the triangle. It is a triangle.

Lower Ability

- It is circle.

Can you tell me or show me how did you know that it is a circle?

Child continued to draw the shapes. Then found it to be a triangle. It is wrong, 21 is a triangle.

Question 1 **How do you know the 20, 40, 50 is a circle/triangle?**

High Ability

- I see a pattern

What is that you see?

- 10 is circle. Repeat the 10, 20 is circle, then you continue so 40 and 50 are a circles.

Read the numbers that has circle? What is the shape of 24? How do you know?

- circle. multiples of 2

Average Ability

- It is $10 + 10$ so circle.
 $10 + 10 + 10$ so circle
and the same way for 40 & 50.

Read the numbers that has circle? What is the shape of 24? How do you know?

- I know 20 is circle.
Then I counted 4.

Lower Ability

- I do 10, then 10 again so it is circle.

Read the numbers that has circle? What is the shape of 24? How do you know?

- triangle

**How do you know?
(after a long pause)**

- I plus 20 and 4

Can you show me?

- 20 circle, 21 triangle pointing and counted until 24. Oh! It is a circle.

Question 1 **How do you that the first 10 shapes have 5 circles and 5 triangles and 20 has 10 triangles and 10 circles?**

High Ability

- I divided by 2

Why did you divide by 2?

- because there are 2 shapes

What about 50?

50 divided by 2, 25 circles and 25 triangles

Average Ability

- 10 shapes, there are 5 circles and 5 triangles, so another 10 5 circles & 5 triangles. In 20 there will be $5+5$, 10 triangles and $5+5=10$ circles.

What about 50?

- I know 20 is 10 circles 10 triangles. So 40 will be 20 circles and 20 triangles. Then 10 has 5 circles and 5 triangles, so 25 triangles and circles.

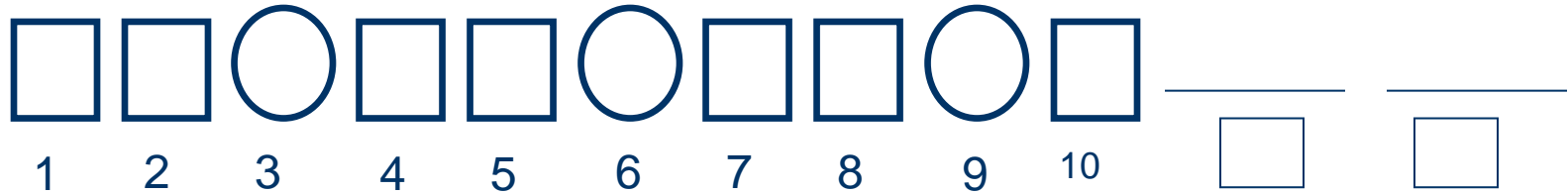
Lower Ability

- I counted 5 triangles and 5 circles.
- I counted 10 triangles and 10 circles.
- 40. I am not sure.

Let's try 2, 6, 8, 10, 12, 20

- Child answered correctly but 30 she was not sure
(child needed visual help)

Question 2



How do you know the next shape is a square?

High Ability

- 2 rectangle and 1 circle and the pattern goes on.

Average Ability

- 2 rectangles, 1 circle
2 rectangles 1 circle
and it is repeating, so
there is only 1
rectangle so the next is
a rectangle and this will
follow a circle.

Lower Ability

- It looks like a square.
- The shapes keep repeating

Why did you draw a square here?
What is the shape that is repeating?

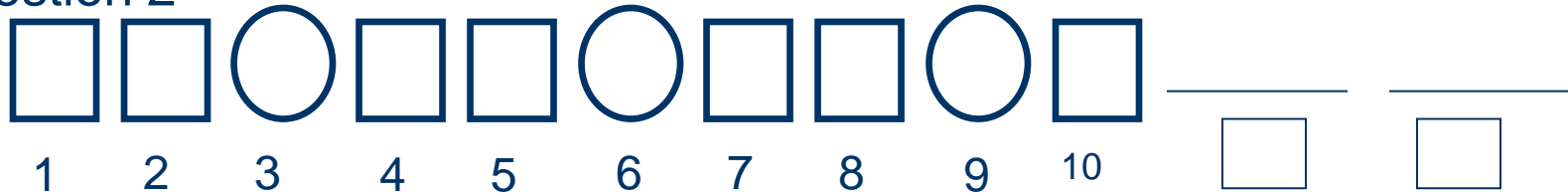
- The square and the circle is repeating.

Is the pattern in Qn 1 and 2 the same? Can you try describing qn2?No. 2 square and 1 circle pointing

What is happening to them?

2 square and 1 circle is repeating.

Question 2



What are the other number that has a circle besides 15? How do you know?

High Ability

- 15, 18. 12 you hop 2 numbers then 15. It is a circle. Then you hop 2 numbers, it is 18 again a circle.

What can you tell about these numbers?

- Multiples of 3

**What is above 30?
How do you know?**

Circle. Multiples of 3

Average Ability

- 15, 18. You skip 2 numbers.

What can you say

- 3 times table

**What is above 30?
How do you know?**

- Circle. 18 is circle, so 21, 24, 27 and 30 circles.

Lower Ability

- counting forward.

How did you count?

- 12 is circle, so 13 is square, 14 is square and 15 is circle.

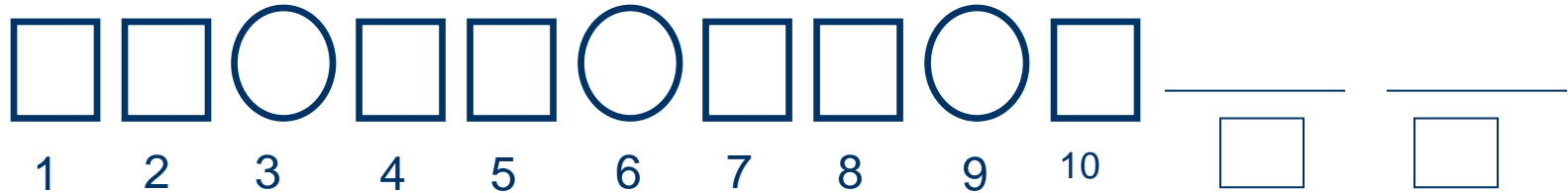
Can you give me another number that has a circle?

- Not sure

Do you want try counting?

(She had problems with counting. She drew to find out the answer)

Question 2



How do you know that there are 6 circles and 14 squares in the first 20 shapes?

High Ability

- 1st 10 shapes there are 3 circles and 7 rectangles. So 1 times by 2 . 3×2 , I get 6 circles and 7×2 , 14 rectangles.

Average Ability

- 10 has 3 circles and 7 rectangles. So another 10 will have another 3 circles and 7 rectangles. So you get $3+3$, 6 circles and $7+7$, 14 rectangles.

Lower Ability

- (not able to answer)
not sure

How many circles are there in the 1st 10 shapes? 3

How many circles are there in the 1st 20 shapes?

$3 + 3$, 6 circles

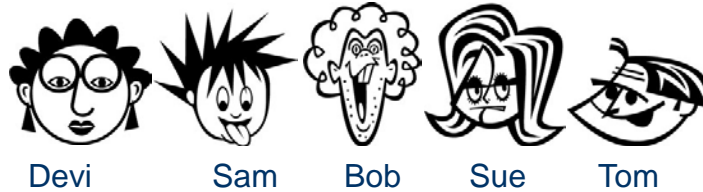
Now how many rectangles in the 1st 10? 7

What about the first 20?

Not sure.

Do you want to try counting?
Used fingers to count to give 14 .

Question 3



1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
21	17	18	19	20

Who will get these cards?

53	70	82	99	67
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Question 3

Who gets card 50? How do you know?

High Ability

- Tom. Because 5, 10, 15 so 50 will be in this group.

Who gets 53? How do you know?

Bob. Not sure.

Look at what Bob gets, what can you see?

Look at the number behind. He has 3 and 8. So any number with 3 or 8 behind will go to Bob.

Average Ability

- Tom. Because Tom gets 20, 25, 30, 35, 40, 45 and 50.

Who gets 53? How do you know?

- Bob. Because Tom gets 50. Then you count and Bob will get 53.

(All the rest was done the same manner.)

(After drawing the attention to the numbers explicitly, he was able to see the pattern)

Lower Ability

- Not sure.

What about 25? How do you know?

Tom. I counted. 21, 22, 23, 24 and 25

Without counting who gets 30? Sue.

Try counting. Tom.

What about 40? 50? 60?

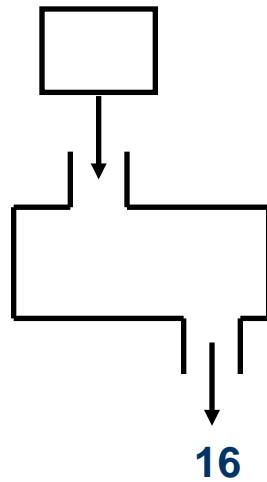
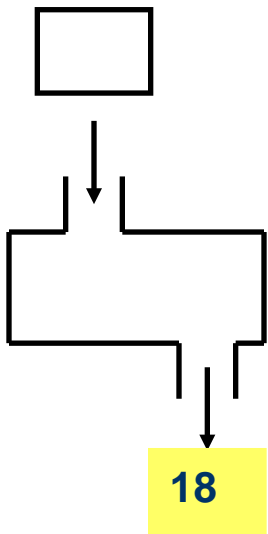
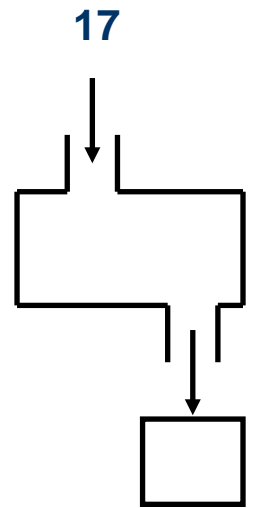
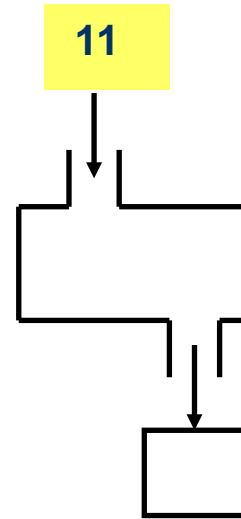
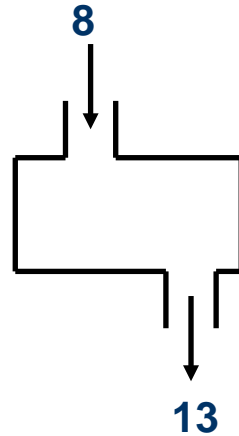
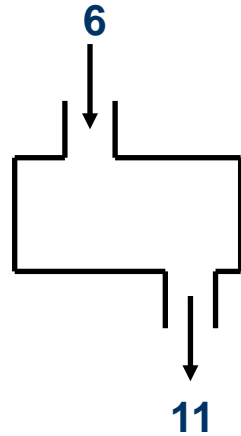
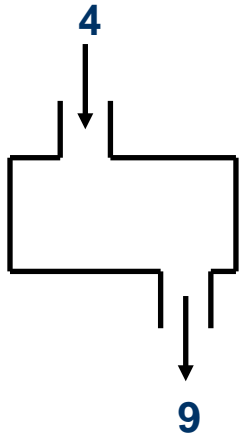
(Her answer was Tom)

How do you know?

Because he has 0.

What else you see? (after long pause) - 5

Question 4



Question 4

High Ability

- 16 & 21

How do you know?

I see a pattern 5.

What did you do to get the answer?

Plus 5

What is the number you put into the machine to get 18?

13

How did you get the answer?

I take away 5.

Average Ability

- 16 & 21.

How do you know?

- The number that goes in will be plus with 5.

What is the number you put into the machine to get 18?

13.

How did you get the answer?

$$18 - 5 = 13$$

Lower Ability

- It is going to plus.

What is it adding?

Add 5.

What will come out if I put in 11? Why? 16. I added 5 to 11.

What is the number you put into the machine to get 18?

After a long pause.

I will subtract 18 and 5. I will get 13.

Why did you do that?

I worked backwards

Findings

- Pupils were able to identify, construct and abstract the unit of repeat
- Pupils used different strategies to show the use of structure
- Pupils showed variation in their ability to identify patterns and structure depending on their understanding and learning styles.
- Questioning techniques encouraged pupils to see the structure of repetition using a unit of repeat. It also encouraged pupils to think and justify their answers .

Findings

The high ability pupil of was able to abstract patterns quickly, the average ability pupil needed more time and guiding questions, whereas the lower ability pupil needed even longer time, more guiding questions and most importantly visual help.

The Lower Ability pupils lack imagery skills and had difficulties with abstraction. The child found it difficult see the relationship between the numbers and the shapes.

The pupils' explanation gave a clear understanding of where each pupil was in his or her construction of knowledge.

Implications to teaching

- The awareness of how pupils' think and learn allows me to plan for their needs.
- The task given to pupils need to provide insights to children's representations of key concepts. This will aid the teacher to build concepts.
- Explicit and guided questioning will help improve pupils' understanding and solving of task. It will also help pupils' to monitor their own learning and construct meaning for themselves.

Implications to teaching

Draw pupils' attention to the mathematical features when engaged in a task. Focusing on each feature will help in integrating. E.g. what shape is above number 2? 4? 6?

What kinds of numbers are 2, 4, 6? At P2, teachers can accept answers such as from 2 times table. At higher grades, accept even numbers as an answer. When pupils see the structure of odd and even numbers underpinning the tasks they could learn to look for such similar structures in other related tasks.

Algebraic thinking in early years is essential. Provide meaningful tasks. Draw attention to the structure underpinning the task. This may help sensitize pupils to the structure underpinning the task.

Implications to teaching

- Patterning and structure build strong foundations in algebraic thinking. Algebraic tasks provide opportunity for children to practice important arithmetic skills and procedures. Algebra is problem based; it develops children's adaptive reasoning skills. Thus, it will excite children. But many teachers still do not believe that algebraic thinking is for early education.

Implications to teaching

- It is a matter of exposing students to patterning and structure, creating opportunity for them to practice what has been learnt and guiding students through questions challenge them and to see it as a fun element. It is unfair for students not to be exposed to these skills and to be tested during examination. We are not helping students, instead creating a phobia towards mathematics which in turn will not help them succeed in formal algebra.
- I feel that by giving students this kind of exposure makes them develop the conceptual understanding of numbers and operations well.

Conclusion

- Teachers need to know the importance of patterning and mathematical structures for productive instruction and habits of algebraic thinking can be learnt.