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# **Bridging the gap between secondary and primary mathematics**

FONG HO KHEONG

## **Issues**

In line with the recommendations of the National Council Teachers of Mathematics (NCTM), the local mathematics curriculum placed emphasis on problem solving and mathematical investigation at both the primary and secondary levels. The Primary Mathematics Project's model basically uses two psychological paradigms: the Piaget's and Bruner's models. According to the Project, pupils who learn mathematics should be led from concrete to abstract experiences so that abstract mathematics could be conceptualised by them. The Project has even moved a bit further by attempting to introduce the 'model' approach for solving problems which are meant for secondary schools. As a result of the bold move, some problem solving approaches used at the primary level (based on Piaget's and Bruner's paradigms) differ from the secondary school mathematics. One can see two problems faced by the pupils and teachers at the secondary level. Many secondary teachers are unaware of the shift in the problem solving approaches at the primary level. To insist that pupils use abstract approaches for solving mathematics problems puts a lot of pressures on them to adopt the changes.

The other problem which many secondary teachers are unaware of is the repetition of some topics which have been taught at the primary level. The problem could be caused by secondary mathematics textbook writers who are not familiar with the primary mathematics syllabus. Very often secondary one teachers rely on the textbooks and follow strictly the scheme there. Many questions found in the textbooks used at the secondary one level are often repeated, or are too simple when compared with the questions used at the primary 5 or 6 levels. Pupils at the lower secondary level may find it boring to go through again the same types of curriculum materials.

There seems to be a need to relook into the secondary syllabus or to revise the lower secondary curriculum materials in view of this problem.

## Problem Solving Approaches

The previous paragraphs indicate the differences in problem solving approaches used in the primary and the secondary pupils. The following paragraphs illustrate some examples to show their differences. Five categories of questions are described below: the difference problems, the multiple problems, the two-variable problems, the constant-value problems, and the fraction problems.

### (a) The Difference Problems

#### ***Problem***

A ruler and 2 pencils cost \$1.40. A ruler cost 20 cents more than a pencil. Find the cost of a ruler. (Extracted from Fong, H.K.(1993). Challenging Mathematical Problems: The Model Approach)

#### ***Solution***

##### *Algebraic Method*

$$r + 2p = 1.40 \quad \text{--- (1)}$$

$$r - p = 0.20 \quad \text{--- (2)}$$

(1) – (2),

$$3p = 1.20$$

$$p = 0.40$$

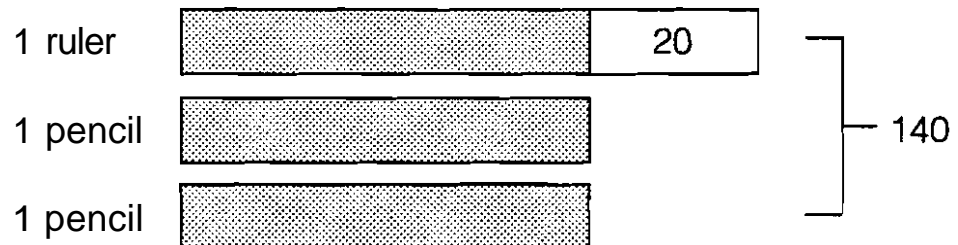
$$\begin{aligned} r &= p + 0.2 \\ &= 0.6 \end{aligned}$$

Cost of a ruler = 60 cents

*Model Approach*

Let the cost of a pencil be 1 part.

Then the cost of a ruler = 1 part + 20



Reading from the diagram,

$$\begin{aligned} 3 \text{ parts} &= 140 - 20 \\ &= 120 \end{aligned}$$

$$\begin{aligned} 1 \text{ part} &= \frac{120}{3} \\ &= 40 \end{aligned}$$

The cost of a ruler

$$\begin{aligned} &= 1 \text{ part} + 20 \\ &= 40 + 20 \\ &= 60 \text{ cents} \end{aligned}$$

**(b) The Multiple Problems*****Problem***

Ann's age is twice the age of Bill. Bill's age is three times the age of Caihe. If their total age is 70, what is the age of Bill. (Extracted from Fong, H.K. (1993). Challenging Mathematical Problems: The Model Approach)

***Solution****Algebraic Method*

Let the age of Bill be  $x$ .

Ann's age =  $2x$

$$\text{Caihe's age} = \frac{1}{3}x$$

$$\text{Total} = 70$$

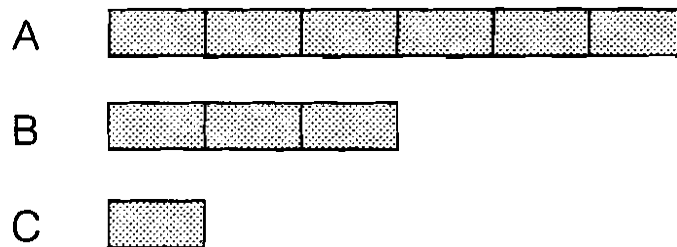
$$x + 2x + \frac{1}{3}x = 70$$

$$3x + 6x + x = 210$$

$$10x = 210$$

$$x = 21$$

*Model Approach*



Let Caihe's age be 1 part.  
Then Bill's age is 3 parts and Ann's age is 6 parts.

Reading from the diagram,  
 Total number of parts = 10  
 10 parts = 70  
 1 part = 7  
 3 parts = 21

Bill's age is 21.

### (c) Two-variable Problem

#### **Problem**

Two apples and one orange cost 140 cents. One apple and two oranges cost 160 cents. What is the cost of 1 apple and 1 orange?

#### **Solution**

##### *Algebraic Method*

Let the cost of an apple be  $x$  and the cost of an orange be  $y$ .

$$2x + y = 140 \quad \text{--- (1)}$$

$$x + 2y = 160 \quad \text{--- (2)}$$

$$2 \times (2), \quad 2x + 4y = 320 \quad \text{--- (3)}$$

$$(3) - (1), \quad 3y = 180$$

$$y = 60$$

$$x = 160 - 2 \times 60$$

$$= 40$$

$$x + y = 100 \text{ cents}$$

Or,

$$(1) + (2) \quad 3x + 3y = 300$$

$$3(x + y) = 300$$

$$x + y = 100$$

The cost of an apple and an orange = 100 cents

*Model Approach*

Let  be the cost of an apple and  be the cost of an orange.

Apple

Orange

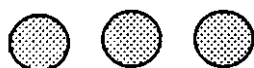


$$= 140$$



$$= 160$$

Add the number of apples, the number of oranges and the numerical values on the right hand side.



$$= 300$$

Regroup them so that each group has an apple and an orange. There are altogether 3 groups.



$$= 300$$



$$= 100$$

$$\begin{aligned}\text{Each group} &= \frac{300}{3} \\ &= 100\end{aligned}$$

The cost of 1 apple and 1 orange is 100 cents.

**(d) Constant-value Problem**

***Problem***

Ali's age is 42 and his son's age is 10. In how many year's time will Ali's age be twice the age of his son?

***Solution***

*Algebraic Method*

Let  $x$  be the number of years in which Ali's age is twice his son.

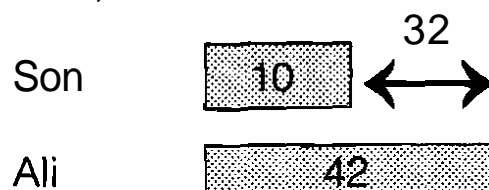
	<i>Now</i>	<i><math>x</math> years later</i>
Ali	42	$42 + x$
Son	10	$10 + x$

$$\begin{aligned}42 + x &= 2(10 + x) \\ &= 20 + 2x \\ 42 - 20 &= 2x - x \\ x &= 22\end{aligned}$$

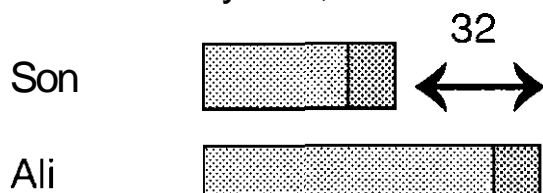
In 22 years time, Ali's age is twice his son.

*Model Approach*

Now,



After some years,



The difference in ages (32) between them remains the same even after some years. Notice that the difference is exactly half of Ali's age.

Reading from the diagram,

$$1 \text{ part} = 32$$

$$\text{Son's age} = 32$$

(after some years)

$$\begin{aligned} \text{Difference in son's age} \\ &= 32 - 10 \\ &= 22 \end{aligned}$$

In 22 year's time, Ali's age will be twice his son's age.

### (e) Fraction Problem

#### **Problem**

John and Michael had 128 marbles altogether, John gave  $\frac{1}{4}$  of his marble to Nordin but Michael gave 37 of his to Nordin. Then they had the same number of marbles left. How many marbles had Michael at first?

#### **Solution**

#### *Algebraic Method*

Let John have J marbles and Michael M marbles.

$$J + M = 128$$

$$J - \frac{1}{4}J = M - 37$$

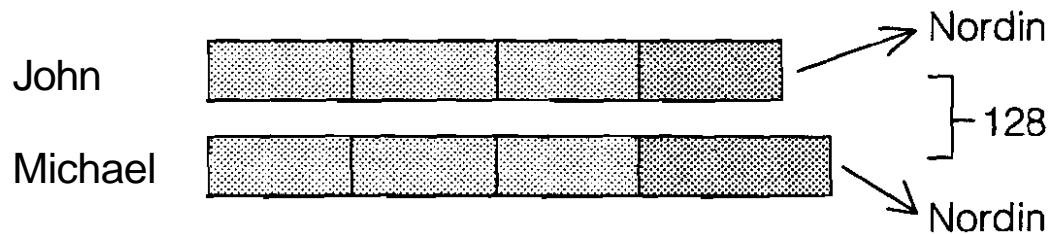


Solving for J and M,

$$J = 42$$

$$M = 39 \text{ marbles}$$

*Model Approach*



Reading from the diagram,

John had 4 parts.

Michael had 3 parts and 37.

Total number of marbles = 128.

$$\begin{array}{rcl}
 4 \text{ parts} + 3 \text{ parts} & = & 128 - 37 \\
 7 \text{ parts} & = & 91 \\
 & & 91 \\
 1 \text{ part} & = & \frac{\quad}{7} \\
 & = & 13 \\
 3 \text{ parts} & = & 3 \times 13 \\
 & = & 39
 \end{array}$$

$$\text{Michael had } 39 + 37 = 76 \text{ marbles.}$$

### Strategies For Bridging The Gap In Problem Solving

The previous paragraphs illustrate the two approaches of solving mathematics problems: the semi-concrete model approach and the algebraic method. Experience by teachers have shown that there is tendency for pupils to fall back on their original approach when similar problems are given to them. Being aware of this problems, secondary teachers should initially equip themselves with the pupils' approaches of solving the problems. Next, they should see how they can incorporate their methods of solving the problems

by linking them to their pupils' approach of solving the same problems.

(a) **Linking the model approach with the algebraic method**

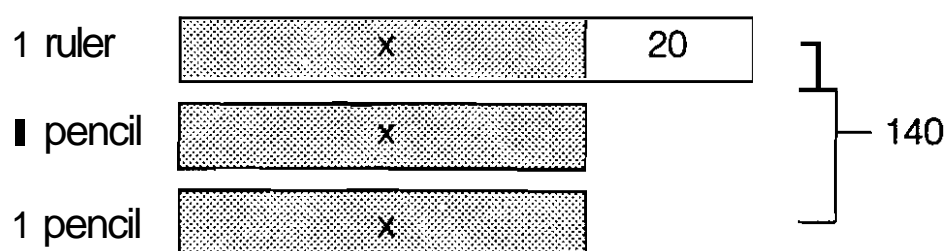
The main reason for pupils' inability to apply the new method of solving mathematical problems is their reluctance to forgo the methods they have learned at the primary level. This could be because they have already mastered the methods. Thus they are required to make a great effort to shift to a new method which they are not familiar with. Hence the strategy here is to introduce a method of problem solving which is not too drastic a change from the method the pupils are familiar with. A combination of the two methods is needed so that pupils will find it comfortable to shift to the new method. This is done by retaining some of the features of the model approach and at the same time introducing some new features of the current method. The following two examples illustrate this innovative strategy for solving algebraic problems.

With reference to the 'difference' problem (see (a)), the modified solution is presented as follows:

*Modified solution*

Let the cost of a pencil be  $x$ .

Then the cost of a ruler is  $x + 20$ .



Reading from the diagram,

$$\begin{aligned}
 x + 20 + x + x &= 140 \\
 3x + 20 &= 140 \\
 3x &= 140 - 20 \\
 &= 120 \\
 x &= 40
 \end{aligned}$$

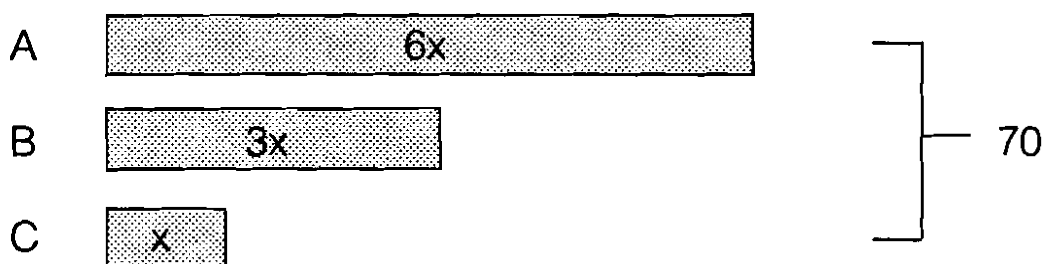
The cost of a ruler is  $x + 20 = 60$  cents.

Notice from the solution that some features from the model's approach have been retained. The diagram and the equating statement, sum of parts = 140 have not been altered. The new features which have been introduced are the value of  $x$  which replaces part from the proportion concept. In fact using a combination of features help lower ability pupils to conceptualise algebraic concept in solving mathematical problems.

Referring to the 'multiple' problem (see (b)), the modified solution is presented as follows:

### *Modified Solution*

Let the age of Caihe be  $x$ .



Then the age of Bill is  $3x$  and Ann's age is  $6x$ .

Reading from the diagram,

$$\begin{array}{rcl}
 6x + 3x + x & = & 70 \\
 10x & = & 70 \\
 x & = & 7 \\
 3x & = & 3 \times 7 \\
 & = & 21
 \end{array}$$

Bill's age is 21.

Similarly, some features from the model approach have been retained. The diagram and the equating statement, sum of parts = 70 have not been altered. The new features which have been introduced are the value of  $x$  which replaces part from the proportion concept.

The two examples illustrated above show the new technique of problem solving with a combination of the model's features and the algebraic features. Once the pupils have become familiar with the algebraic method, the diagram may be left out in the solution.

**(b) Investigation of strategies for problem solving**

One strategy to help pupils apply the algebraic method is to create more opportunities for them to practice the methods of solving problems. This could be done by getting pupils to find the various methods of solving a particular problem. The following activity is a typical example which allows pupils to practice the new method of problem solving which they have just learned.

**Activity**

Work out the following problem using the following methods of solving:

- (i) algebraic method,
- (ii) model approach, and
- (iii) any other method(s) you can think of.

**Problem:** The sum of two numbers is 100.  
The difference of the two numbers is 16.  
Find the two numbers.

There are two purposes in this activity. Besides getting pupils to practice the algebraic method they have just learned, they are also given the opportunity to investigate all possible methods of solving the problem.

## Summary

This paper discusses the current issue in mathematics education arising from the revision of mathematics syllabus. The main issue is the problem solving approaches adopted by pupils at the lower secondary level. They tend to carry along with them the model approach in solving problems which have been over emphasised at the primary level. Five examples are illustrated to show differences in the problem solving approaches used in the primary and secondary schools for solving the same problem. The author introduces a modified solution which incorporates both features taken from the two approaches. This helps pupils to move smoothly from using the model's approach to the algebraic method of solving the problem.

## Reference

Fong, H.K. (1993). *Challenging mathematical problems: The model approach*. Sydney: Kingsford Educational Services.