Title	Fraction division in the Singapore mathematics curriculum
Author(s)	Koay Phong Lee and Ho Siew Yin
Source	5th East Asia Regional Conference on Mathematics Education, Tokyo, 18-22 August 2010

Fraction Division in the Singapore Mathematics Curriculum

Koay Phong Lee <phonglee.koay@nie.edu.sg>
Ho Siew Yin < siewyin.ho@nie.edu.sg>
National Institute of Education,
Nanyang Technological University
Singapore

Abstract

The teaching and learning of fractions span across the Singapore primary mathematics curriculum from Primary One to Primary Six. Currently, the teaching and learning of fraction division take place at Primary Five and Primary Six. The topic of fraction division has a chequered appearance in the Singapore mathematics curriculum, reflecting the curriculum changes that have taken placed periodically over the years. The periodically reviewed syllabuses are reflected in the school textbooks as well. In the *Third International Mathematics and Science Study* (TIMSS), textbooks are viewed as mediators between general intentions stated in the syllabus and classroom instruction, they provide the links between aims and reality (Valverde, Bianchi, Wolfe, Schmidt & Houang, 2002). Textbooks and the accompany teachers' manuals play a crucial role in helping teachers to teach the concept with relational understanding (Ma, 1999). In this paper, we examine the history of fraction division in the Singapore mathematics curriculum and how the topic on Fraction Division is presented in the mathematics textbooks.

Introduction

Fraction division is the most abstract and complicated operation in arithmetic. There are various fraction-division situations. When is the fraction division algorithm taught in the primary schools in Singapore? How is the concept presented in the textbooks? In Singapore, the primary and secondary schools and the junior colleges follow the national curriculum and syllabuses developed by the Curriculum Planning and Development Division (CPDD) of the Ministry of Education (MOE). The curriculum is periodically reviewed, usually in 10-year cycles with two revised versions within the period. The reviews take into consideration the initiatives introduced by the Ministry of Education at that time. The general aims of mathematics education, suggested content and instructional strategies and assessment practices are delineated in the syllabus documents issued by the MOE and made accessible to all educators. These periodically reviewed syllabuses provide guidelines for teachers to plan and implement mathematics programs in their schools and to prepare their students for the high stake national assessment at the end of primary, secondary school and junior college. The reviewed syllabuses are reflected in the school textbooks. In the *Third International Mathematics and Science Study* (TIMSS), textbooks are viewed as mediators between general intentions stated in the syllabus and classroom instruction, they provide the links between aims and reality. (Valverde, Bianchi, Wolfe, Schmidt & Houang, 2002). This paper attempts to answer the following two questions:

- What is the place of fraction division in the mathematics curriculum in Singapore over the years?
- How is fraction division represented in various textbooks used in Singapore?

Singapore Mathematics Syllabuses

Singapore gained its independence in 1965. Since then, its educational system has evolved through four stages of development, reflecting the changing needs of the nation. The education system in the 60s and 70s was survival-driven, attempting to cater for the needs of the newly independent nation. It gave way to the efficiency-driven system of education in the 80s and 90s so as to reduce educational wastage. This, in turn was replaced by the present ability-driven system of education (Lee, 2004) Consequently, the mathematics curriculum in Singapore has changed over the years in tandem to the changing educational system. Although MOE reviews the mathematics curriculum regularly to reflect the changing educational needs, the general aim of mathematics education in Singapore remains reflectively unchanged since 1990. Since then, the primary aim of mathematics education in Singapore is to enable pupils develop ability in mathematical problem solving. This is achieved through the attainment of five interrelated components: concepts, skills, attitudes, metacognition, and processes as illustrated in the pentagonal framework for the mathematics curriculum. Figure 1 show the pentagonal framework found in the current mathematics curriculum.

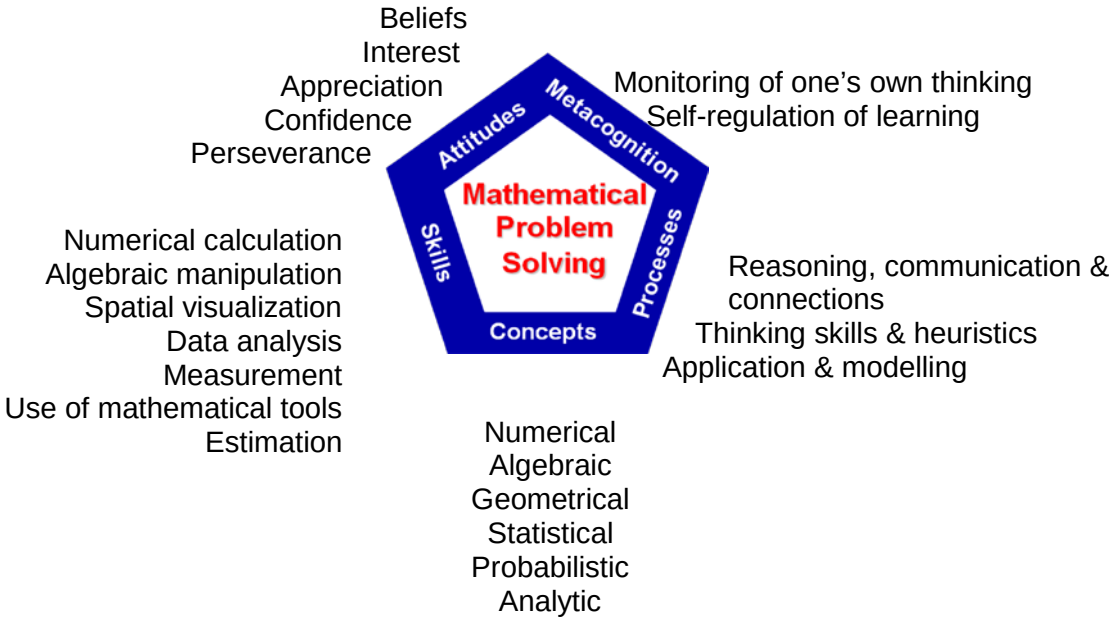


Figure 1. The framework for the Mathematics curriculum in Singapore (2001)

MOE launched three initiatives in 1997 and announced the intention to reduce the content of the curriculum to make more instructional time available for teachers to infuse thinking skills, integrate ICT in their lessons and deliver the National Education messages. The immediate measures taken resulted in about 10% - 30% content deduction for most subjects, including mathematics. Those concepts and skills removed from the curriculum included concepts and skills:

- that were deemed not fundamental to the essence of the subject studied or which relied on plain recall;
- that overlapped with that taught at other levels in the same subject or with what was taught in other subjects;
- that focused on technical details rather than conceptual understanding and was no longer relevant in the Singapore context or in real world practice;
- that were too difficult or abstract for the intended level'. (MOE 1998 press release)

Fraction Division in Singapore Primary Mathematics Syllabuses

This section describes the change in the Singapore Primary Mathematics syllabus with regard to the topic on the division of fractions. For mathematics, the topic division by fractions was one of the concepts and skills that were affected as a result of the reduction of content in 1997. Pupils were found to rely on the recall of 'invert and multiply' rule to perform the division correctly and the teachers tend to focus on the procedural skills rather than conceptual understanding of the algorithm. The division of proper fraction / whole numbers by a fraction was also considered too abstract for the primary school students to understand. Hence, from 1999, the teaching of division of fractions was limited to Division of a Proper Fraction by a Whole Number. Meanwhile, the second stage of the review of the curriculum involving the pedagogies, learning approaches and assessment mode was carried out, resulting in the publication of a new syllabus document in 2000 and its implementation from 2001. There was no change regarding the topic division by fractions.

The last curriculum review was completed in 2004 with the new syllabuses being phased in Primary 1 to Primary 4 in 2007, Primary 5 in 2008 and Primary 6 in 2009. Some topics were added, deleted or moved to other levels. The major change was the introduction of the use of calculator in the teaching and learning of mathematics at the upper primary level to provide a better balance between the emphasis on computational skills and processes such as problem solving skill. The 2007 syllabus places a greater emphasis on the process skills. In addition to thinking skills & heuristics, other process skills such as reasoning, communication and connections, applications and modeling are also included.

Among the topics added is division of whole number / proper fraction by proper fraction because division of fractions is deemed as "fundamental to the learning of algebra in Secondary 1". (Kho, T. H., 2009, personal communication). Table 1 shows the presence of Division of Fractions in the mathematics syllabus over the years.

Table 1
Division of Fractions as Found in the Mathematics Syllabuses (Standard Level)

Subtopics	1990	1999	2001	2007
Proper fraction ÷ Whole number	P5	P5	P5	P5
Whole number ÷ Proper fraction	P6	-	-	P6
Proper fraction ÷ Proper fraction	P6	-	-	P6

The flexibility of the Singapore mathematics curriculum caters for the needs of individual students and allows less mathematically able students to follow simpler Foundation Mathematics syllabus. These students are not expected to learn as many mathematics topics and in the same way as the students following the standard mathematics syllabus. With regard to division by fractions, the standard of achievement to which these students are held is also different. Unlike the students following the standard mathematics syllabus, they are allowed to use calculator to divide fractions (see Table 2).

Table 2
Current Mathematics Syllabus (2007) with regard to Division of Fractions

Primary mathematics		
Year	Standard level	Foundation level
5	• division of a proper fraction by a whole number **without** using calculators.	-
6	• division of a whole number / proper fraction by a proper fraction **without** using calculators. Exclude: • division of an improper fraction / mixed number by a proper fraction • division by an improper fraction / mixed number.	Calculator is allowed. • Association of a fraction with division • Division of a whole number/proper fraction by a whole number / proper fractions

Fraction Division in Singapore Mathematics Textbooks

As pointed out by Robitaille et al. (1993), the potentially implemented curriculum as represented by textbooks and other organized resource materials, is the link between the specified aims of mathematics education stated in the syllabus and the implemented curriculum found in the classroom activity. So how are the changes in the mathematics curriculum concerning division of fractions reflected in the textbooks in Singapore?

Before 2001, textbook development was undertaken by MOE. All schools adopted the same set of textbooks developed by MOE who also provided related resource materials including teachers' guides to all schools. The teachers and school administrators did not have a choice of textbooks. Since 2001, the MOE has outsourced the publication of textbooks to commercial publishers, providing teachers and school administrators a wider choice of textbooks to choose from.

In Singapore, all different textbook series cover the same topics as dictated in the syllabus. Both the textbooks and the accompanying activity /practice books are vetted by MOE officers. The textbook reviewers at the MOE make sure that there is a balance between practice of computational skills, conceptual understanding and applications in the content of these books, and concepts and skills beyond the syllabus are excluded. They also make sure that the textbooks reflect the initiatives launched by the ministry, and the appropriate pedagogical approach is adopted. Those textbooks that have passed examination would have an approval seal from the Ministry in the first page of the books. Hence, the content reliability and validity are achieved.

Teachers follow the national curriculum closely. Hence, the content and organization of their lessons are very much dictated by the textbooks and the teachers' guides. However, teachers may choose to teach it differently. The different primary mathematics textbook series cover the same topics in each grade level as dictated in the syllabus, albeit in different order. Division of fraction is taught in Primary 5 and Primary 6. The standard algorithm seems to be 'invert and multiply'.

The following describes how the topic division of fractions was presented in the textbook series used in the pre 1999 era and the two popular textbook series currently used in primary schools in Singapore. All these textbooks attempt to focus on conceptual understanding, using the 'partition' concept to explain the division of a

proper fraction by a whole number in Grade 5 and 'measurement' concept to explain the division of a whole number/proper fraction by a fraction in Grade 6.

The Primary Mathematics Series (2nd edition,1994,1995)

A 'real life' scenario of sharing two-third of pie equally among 4 boys was used to introduce division of a fraction by a whole number in 5A. It was stated that students could solve the story problem by $\frac{2}{3} \div 4 = \frac{1}{4} \text{ of } \frac{2}{3} = \frac{1}{4} \times \frac{2}{3}$ or $\frac{2}{3} \div 4 = \frac{2}{3} \times \frac{1}{4}$. Except that the quotient are the same, there was no explanation of why the two number statements are equivalent.

In 6A of the textbook, again only one 'real life' situation was given to introduce $3 \div \frac{1}{2}$. It was followed by examples (no context) on

- divide a whole number by a unit fraction
- divide a proper fraction by a whole number (1-digit)
- divide a fraction by a fraction

The examples were interspersed with thought bubbles and statements such as *Dividing by* $\frac{1}{2}$ *is the same as multiplying by 4* and *Dividing by* $\frac{3}{4}$ *is the same as multiplying by* $\frac{4}{3}$. There was a great emphasis on procedural skills. Story problems involving 'measurement concept' were given as practice exercise after students had learnt the 'invert and multiply' rule.

Shaping Maths Series (2nd edition, 2005, 2006)

The topics 'Multiplication of fractions' and 'Expression of fractions as decimals (and vice versa)' precede the division of a proper fraction by a whole number in the Primary 5 textbook. A 'daily life' example of sharing a 4/5 of pancake equally among 4 children provides the context (Figure 1). A circular representation of $\frac{4}{5}$ is used to illustrate the situation. Two ways of written representations are provide, one interpreting $\frac{4}{5}$ divided by 4 as a quarter of $\frac{4}{5}$ and the other illustrate the 'invert and multiply'. The first representation is an attempt to explain the second. Students are reminded of the commutative property. Here the partition interpretation of the concept is highlighted.

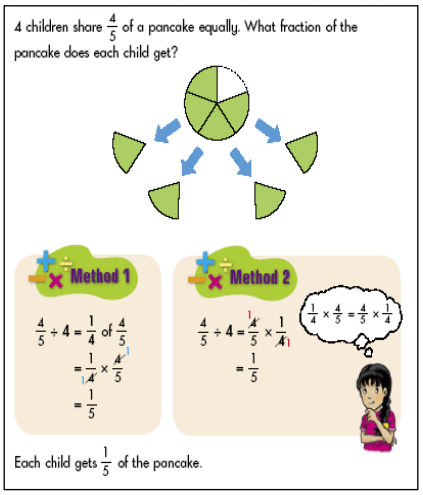


Fig. 1 Division by a whole number

In Primary 6, an attempt was made to introduce the 'measurement concept' of division using manipulatives such as the pattern blocks. For division by a fraction, the chapter starts with comparison of different pieces of the pattern blocks (see figure 2)

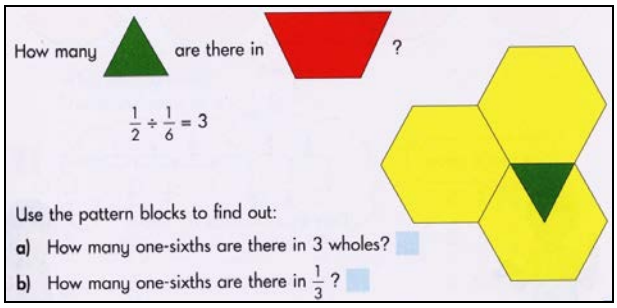
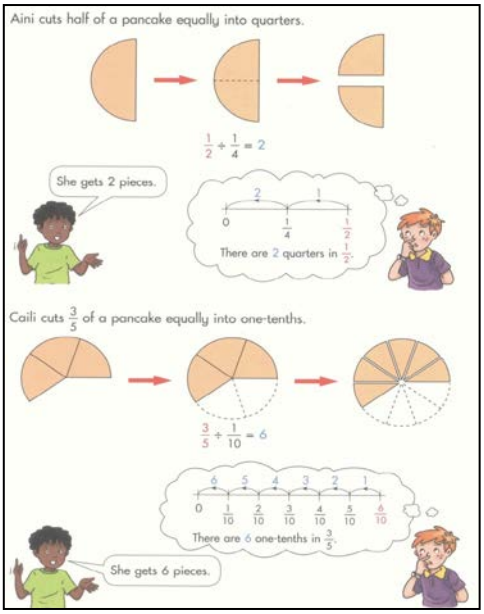
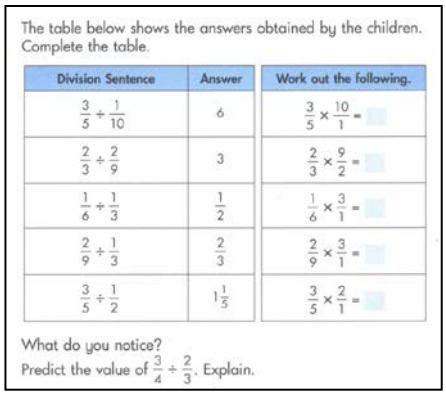


Fig. 2 Example of activity using pattern blocks

Formal algorithm is introduced later through inductive approach, helping pupils make connection between division by fractions and multiplication of proper fractions. The geometrical representations include both fraction discs and number lines (See Figure 3). Howver, teachers' knowledge of the use of number lines may be limited as number line is not used extensively in the primary mathematics classrooms in Singapore.

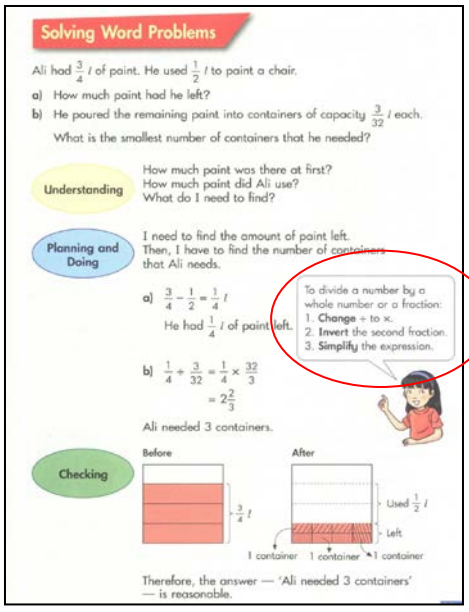



The table below shows the answers obtained by the children. Complete the table.

Division Sentence	Answer	Work out the following.
$\frac{3}{5} \div \frac{1}{10}$	6	$\frac{3}{5} \times \frac{10}{1} =$
$\frac{2}{3} \div \frac{2}{9}$	3	$\frac{2}{3} \times \frac{9}{2} =$
$\frac{1}{6} \div \frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6} \times \frac{3}{1} =$
$\frac{2}{9} \div \frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{9} \times \frac{3}{1} =$
$\frac{3}{5} \div \frac{1}{2}$	$1\frac{1}{5}$	$\frac{3}{5} \times \frac{2}{1} =$

What do you notice?
Predict the value of $\frac{3}{4} \div \frac{2}{3}$. Explain.

Figure 3. Division by a fraction (Shaping Maths 2006)



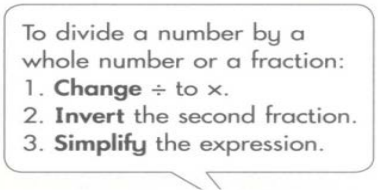
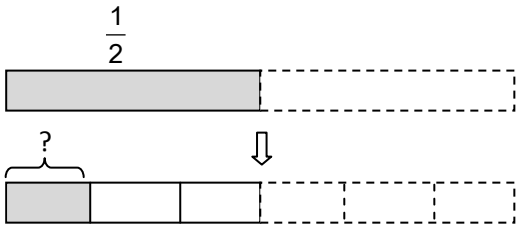


Figure 4. Solving word problem (Shaping Maths 2006)

In the section on solving word problems, pupils are guided through the 4-step process of problem solving: Understanding, Planning, Doing and Checking. They are reminded of the 'invert and multiply' rule learnt earlier. Drawings are used to help pupils check the reasonableness of their answer.

My Pals are Here Series

The approach found in theis series are quite similar to that found in the Primary Mathematics Series. However, the representation used in the series is different. A bar is used instead. The drawing of such bar is familiar to both teachers and students as it is the common problem solving heuristics in Singapore. For example, for $\frac{1}{2} \div 3$, the following bar-model is drawn:



In addition to the above approach, two other methods similar to that found in the Primary Mathematics Series are given. That is, to interpret $\frac{1}{2} \div 3$ as $\frac{1}{3}$ of $\frac{1}{2}$ and represent it as $\frac{1}{3} \times \frac{1}{2}$ and the other is to write $\frac{1}{2} \div 3$ as $\frac{1}{2} \times \frac{1}{3}$. For division by a fraction, the familiar bar-model is used. However, only examples with whole numbers as quotients are selected as examples to lead readers to see that dividins a fraction $^{a}/_{b}$ is the same as multiplying by $^{b}/_{a.}$ Unlike the Primary Mathematics Series, there are more story problems given as guided examples. In addition, alternate algorithm using common denominator as shown below

$$\frac{3}{5} \div \frac{1}{3} = \frac{9}{15} \div \frac{5}{15} = \frac{9 \div 5}{1} = \frac{9}{5} = 1\frac{4}{5}$$

is mentioned under 'Home Maths' which teachers may ignore in the mathematics classroom.

Discussion

All the textbook series use the 'partition concept' to introduce fraction division in Primary 5 and the 'measurement' concept in fraction division in Primary 6. There is no evidence that all teachers are aware of these two concepts and communicate the difference between the two concepts to their students. What knowledge the pre-service teachers have concerning these two concepts of division by fractions ?

Textbook series published after 2001 also try to promote the process skills and other initiatives of the MOE. Concepts are introduced through 'real life experience' and built upon students 'informal knowledge' that they bring to the classroom. The emphasis is placed on learning with relational understanding rather than learning with instrumental understanding. As illustrated in division of fractions, textbook writers now attempt to use different representations as response to the Concrete- Pictorial-Abstract approach in mathematics education in Singapore. However, teachers would probably use approaches that they are familiar with in their mathematics classroom, like those that they were exposed to when they were in schools. That is, 'invert and multiply' for division of fractions.

The MOE reviews all the mathematics textbooks used in the primary schools to ensure that they follow the guidelines of the intended curriculum and to maintain a uniform quality of mathematics education in Singapore. The textbooks and accompany teacher's guides play a crucial role in delineating pedagogical approaches that promote discourse and relational understanding (Ma, 1999). However, not all teachers would use the textbooks as they are intended. The main objective of the teachers is to help their students perform well in the examination. A way to achieve this is to teach the procedural skills and provide much practice. Hence, there are teachers who prefer to teach the procedural skills instead of discussing mathematical ideas and issues that promote relational understanding of the concept. Is this because teachers do not know how to use the textbooks? If textbooks are to be a means for reform in mathematics education in Singapore, then both teachers and trainee teachers must be given opportunities to reflect on the features of the textbooks and learn how to use them properly.

References

Collars, C.; Koay, P.L.; Lee, N.H.; Ong, B.L.; Tan, C.S. (2005). *Shaping Maths 5A coursebook,* (2nd Edition). Singapore: Marshall Cavendish Education

Collars, C.; Koay, P.L.; Lee, N.H.; Ong, B.L.; Tan, C.S. (2006). *Shaping Maths 6A coursebook*, (2nd Edition). Singapore: Marshall Cavendish Education

Fong, H.K.; Gan, K. S.; & Ramakrishnan, C. (2005) *My Pals Are Here! Maths* 5A (2nd Edition). Singapore: Marshall Cavendish Education

Fong, H.K.; Gan, K. S.; & Ramakrishnan, C. (2006) *My Pals Are Here! Maths 6A* (2nd Edition). Singapore: Marshall Cavendish Education

Lee, N.H. (2004): *Nation Building Initiatives: Impact on School Mathematics Curriculum*. Paper presented at ICME-10, Denmark: Copenhagen

Ma, L. (1999). *Knowing and teaching elementary mathematics: teacher's understanding of fundamental mathematics in China and the United* States. Mahwah, N.J.: Lawrence Erlbaum Associates.

Ministry of Education (1998): Content Reduction in the Curriculum Retrieved 8th Dec 2009 http://www.moe.gov.sg/media/press/1998/980716.htm

Ministry of Education (1990): *Mathematics Syllabus (Primary),* Curriculum Planning Division. Singapore.

Ministry of Education (2000): *Mathematics Syllabus (Primary)* Curriculum Planning Division. Singapore.

Ministry of Education (2006): *Mathematics Syllabus (Primary)* Curriculum Planning Division. Singapore.

Primary Mathematics Series 5 A and 6A (1994, 1995) Curriculum Planning & Development Division, Ministry of Education. Singapore.

Robitaille, D.F., Schmidt, W.H., Raizen, S.A., McKnight, C.C., Britton, E.D., & Nicol, C. (1993). *Curriculum frameworks for mathematics and science* (Vol. TIMSS Monograph No. 1) Vancouver: Pacific Education Press.

Valverde, G.A., Bianchi,L.J., Wolfe, R.G., Schmidt, W.H. & Houang, R.T. (2002). *According to the Book, Using TIMSS to investigate the translation of Policy into Practice Through the World of Textbooks*. Dordrecht: Kluwer Academic Publisher.