
Title	Assessing for understanding in mathematics learning
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Assessing for Understanding in Mathematics Learning

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Abstract

In order that learning and teaching can benefit from the use of assessment strategies which enhance the understanding aspect in the learning of mathematics, it is inevitable the teacher would be required to write items in a consistent way to improve and judge students' understanding in mathematics. This workshop outlines major considerations in assessing the levels of understanding in mathematics learning, developing and implementation of a multi-dimensional assessment for learning strategy. Participants will be inducted into the dimensions of mathematics understanding, and be introduced to a series of procedures to modify items corresponding to these different dimensions. The workshop will include sorting of items according to the dimensions of understanding, item development-practice which is fundamental to understanding the principles behind assessment for mathematics understanding and to devise rules for modifying conventional classroom items to strengthen learning and teaching of mathematics understanding.

The 90-minute workshop begins with an introduction to the five dimensions of understanding, namely, Skills, Properties, Uses and Representation (SPUR). Participants will be provided with sample items under each of the categories. To be acquainted with the key features of each dimension of understanding, participants will be immersed in a hands-on sorting activity. This would allow participants to clarify their understanding of SPUR before the workshop demonstrates how textbook exercise problems used in daily classroom teaching could be transformed to assess different levels of understanding. Opportunities for participants to share their modified items will be part of the item revision process so that participants could receive feedback on how to sharpen their revised items. The workshop will consolidate the learning of the group and participants can hope to leave with a collection of items classified under the five dimensions of understanding.

To ensure a productive session, participants will be expected to bring five mathematics problems from a classroom exercise on a specific topic, to the workshop

Objectives

- The workshop seeks to immerse participants into learning about mathematics understanding by doing.

Participants will

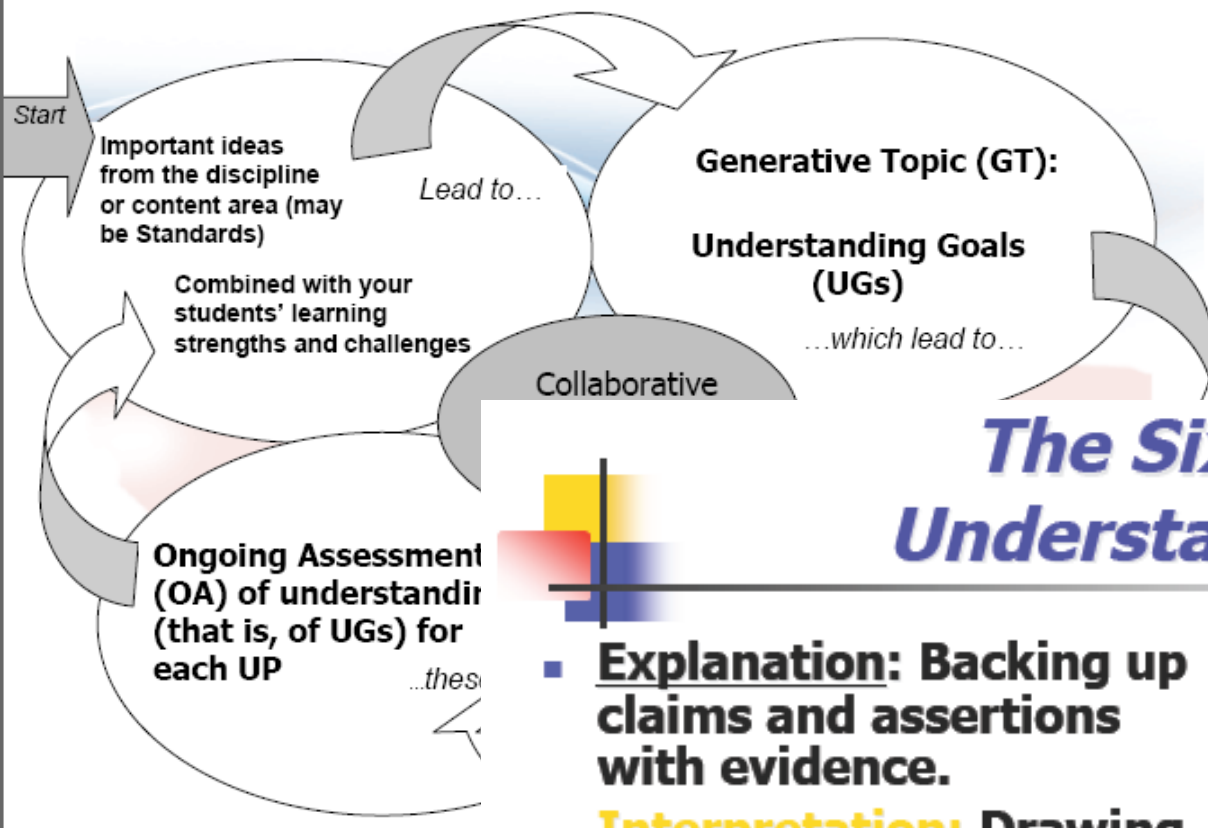
- Code given maths items according to the dimensions of understanding
- Revise standard textbook items for assessing different understanding

Outline

1. Understanding in maths learning
2. Coding practice
3. Extending a maths item to assess SPUR
4. Clarification of the SPUR descriptors
5. Item revision

What are others saying about
understanding?

A Thinking Path for Planning with the Teaching for Understanding Framework



The Six Facets of Understanding (P. 155)

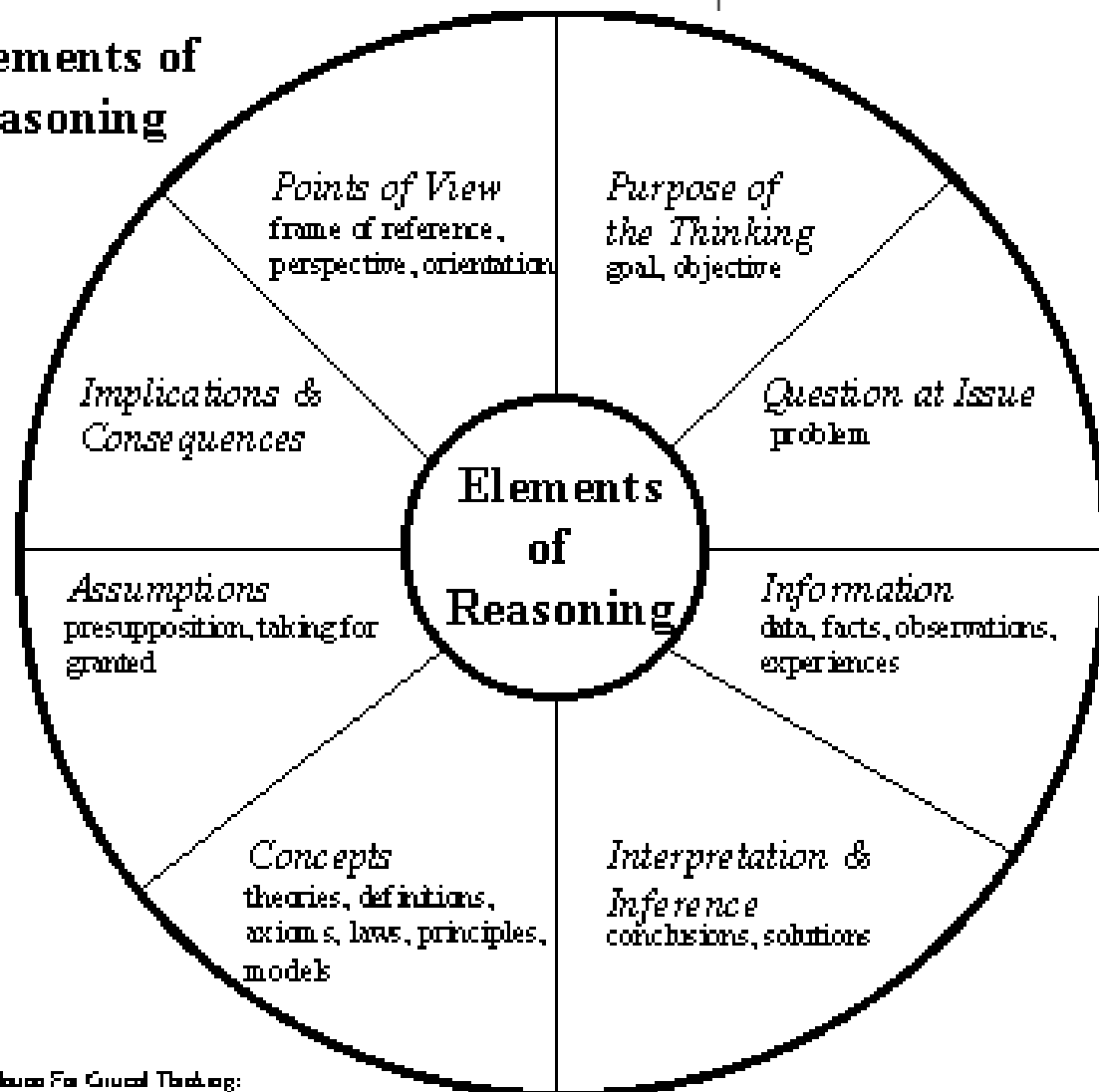
- **Explanation:** Backing up claims and assertions with evidence.
- **Interpretation:** Drawing inferences and generating something new from them.
- **Application:** Using knowledge and skills in a new or unanticipated setting or situation.
- **Perspective:** Analyzing differing points of view about a topic or issue.
- **Empathy:** Demonstrating the ability to walk in another's shoes.
- **Self-Knowledge:** Assessing and evaluating one's own thinking and learning: revising, rethinking, revisiting, refining.

Organize by W. H. E. R. E. T. O.



W Where are we headed? (the *student's Q!*)
H How will the student be helped?
E What opportunities will be provided?
R What opportunities will be experienced, and how?
E What will provide the evidence to refine and revise?
T How will student thinking be tracked?
O How will the work be organized?
O How will the work be organized?
O How will the work be organized?

Elements of Reasoning



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Scenarios for Authentic Tasks T

Build assessments anchored in authentic tasks using GRASPS:

G ■ What is the Goal in the scenario?

A MULTI-DIMENSIONAL APPROACH TO UNDERSTANDING IN
MATHEMATICS TEXTBOOKS DEVELOPED BY UCSMP

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ABSTRACT

© This paper describes two features of the textbooks written by the University of Chicago School Mathematics Project. The first, called the SPUR perspective, is a multi-dimensional approach to understanding that guides the development of lessons and chapters. The second, called the CARE perspective, guides the development of the problem sets to ensure that students engage with the essential concepts, have an opportunity to apply concepts to more

Understanding in maths learning

- Background
- What is SPUR?

Examples of SPUR

Definitions

Skills

- Procedures that students should master with fluency e.g.
 - Application of standard algorithms
 - Selection of algorithms
 - Comparison of algorithms
 - Discovery / invention of algorithms
- deal with procedures such as factoring expressions or solving equations to get answers

$$\text{Solve } 3(n - 7) = 4 - n$$

Properties

- represent the reasoning expectations within mathematics
- deal with the principles behind the mathematics, e.g.
 - Justifying statements (naming of properties to justify)
 - Using definitions,
 - Derivations, or
 - proving theorems;

Which point is on the graph of $y = \log_5 x$?

(a)

(b)

(c)

(d)

Uses

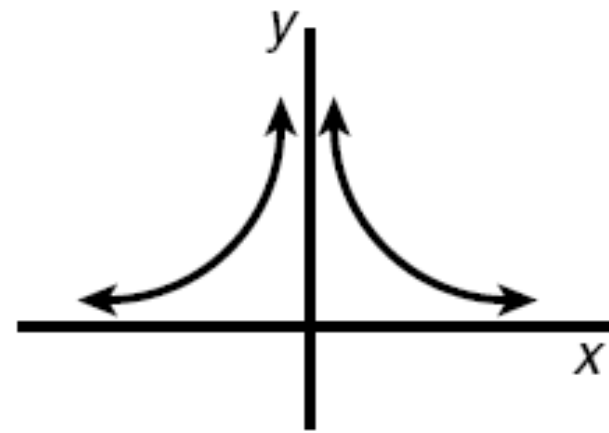
- deal with the contextual aspect of mathematics
- applications of mathematics to real situations or other concepts in maths
 - Word problems
 - Development & use of math models

If you invest \$300 for 4 years at 5% annual yield, then how many dollars will you have at the end of this time?

Representations

- deal with pictures, graphs, or objects that
 - illustrate concepts or relations and
 - connect to the graphical interpretations of mathematics
- Graphs, pictures, other visual depictions of the concepts
 - Standard representations
 - Discovery of new ways to represent concepts

Which equation below could
generate the graph at the right?
($k > 0$)



Coding Hands-on

Group work

- Generate a set of SPUR for each of the given items
- Prepare an answer key
- Find a group to exchange items
- Classify
- Present
- Feedback from designer group

Item Coding

Present & Feedback I

Extending a maths item to assess SPUR

An Illustration

Extending a maths item to assess SPUR

$$\textbf{S} \text{ Solve } 3(n - 7) = 4 - n$$

U Mother divides some pies into equal slices to share among her 3 children. Abel ate 7 slices less than Barry. Calvin has three times more slices than Alice. If Calvin and Barry has a total of 4 slices, how much pies did Barry eat?

Extending a maths item to assess SPUR

S Solve $3(n - 7) = 4 - n$

P Write a line parallel to $3(x - 7) = 4 - y$

R Draw two line graphs to solve for x in

$$3(x - 7) = 4 - x$$

Item Extension

Present & Feedback II

Comments & Clarifications

Thank you