

# Designing web-based cognitive tools to enhance teaching and learning of mathematical problem solving

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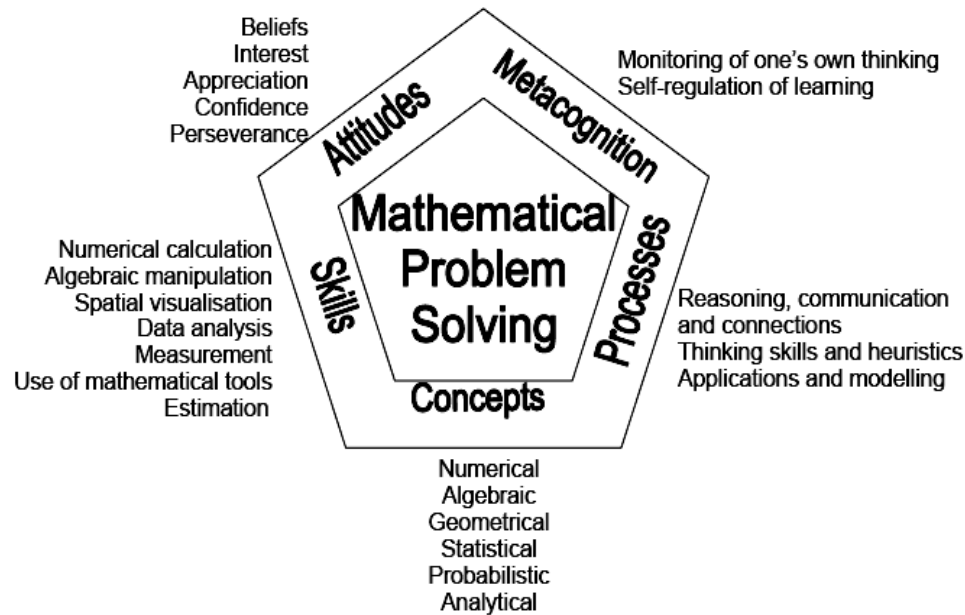
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# CRPP Project: Development Repertoire of Heuristics for Mathematical Problem Solving – Project 1 and 2

Singapore Mathematics Curriculum 2007

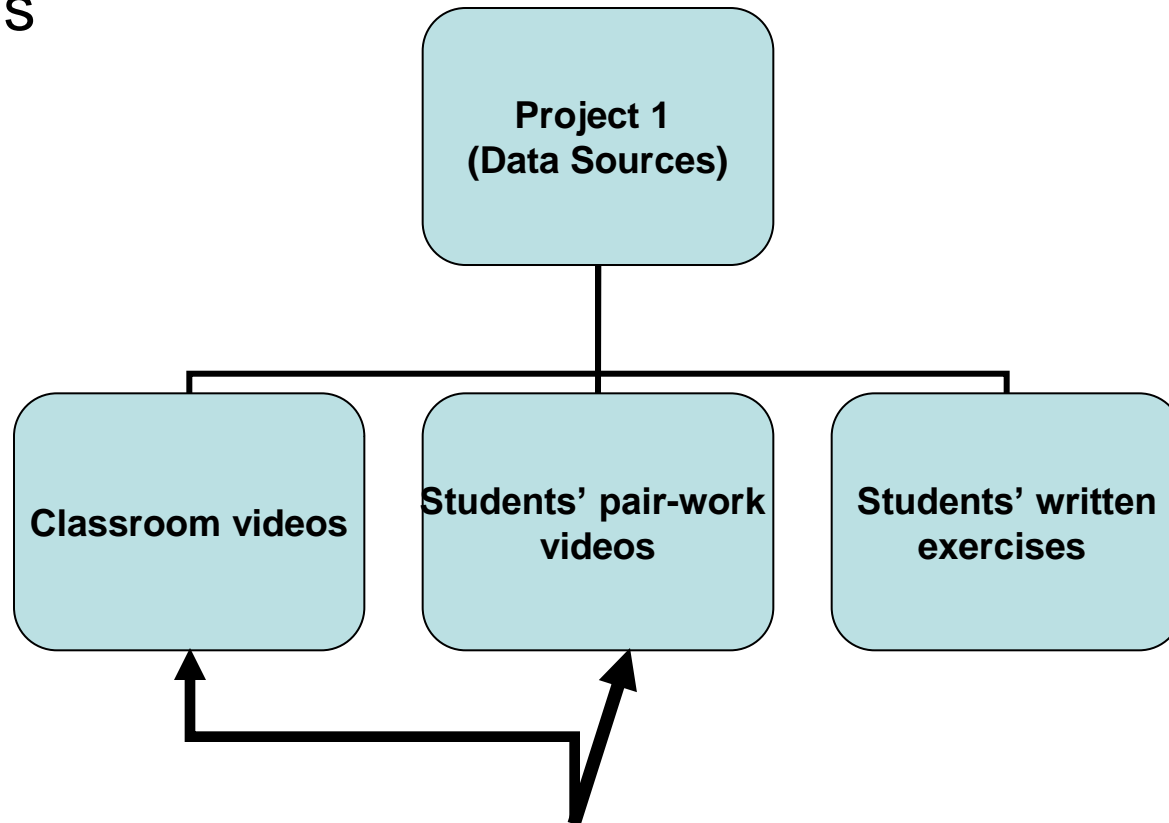


Project 1: Baseline data of implementing MPS curriculum in schools

Project 2: Designing web-based video cases to support teaching and learning of MPS in schools

# CRPP Project: Development Repertoire of Heuristics for Mathematical Problem Solving – Project 1 and 2

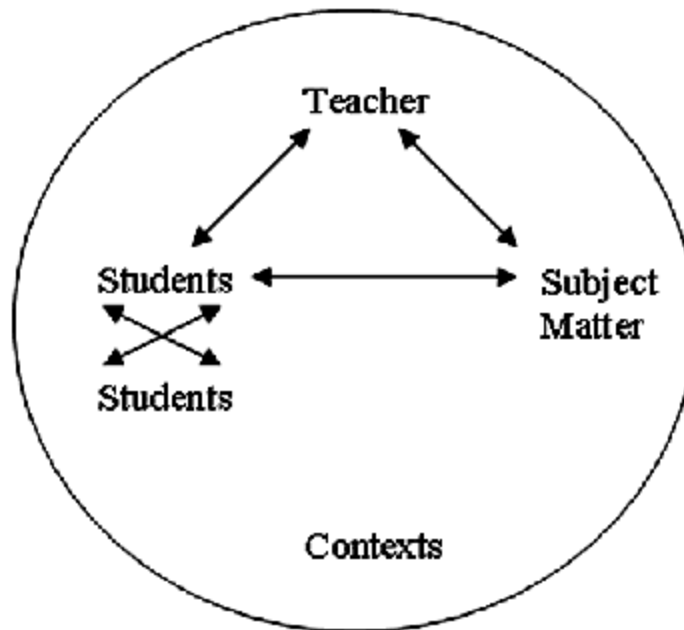
Project 1: Baseline data of implementing MPS curriculum in schools



Project 2: Designing **web-based video cases** to support teaching and learning of MPS in schools

# Conceptualisation

Ball and Cohen



# Video Cases: Initial Design

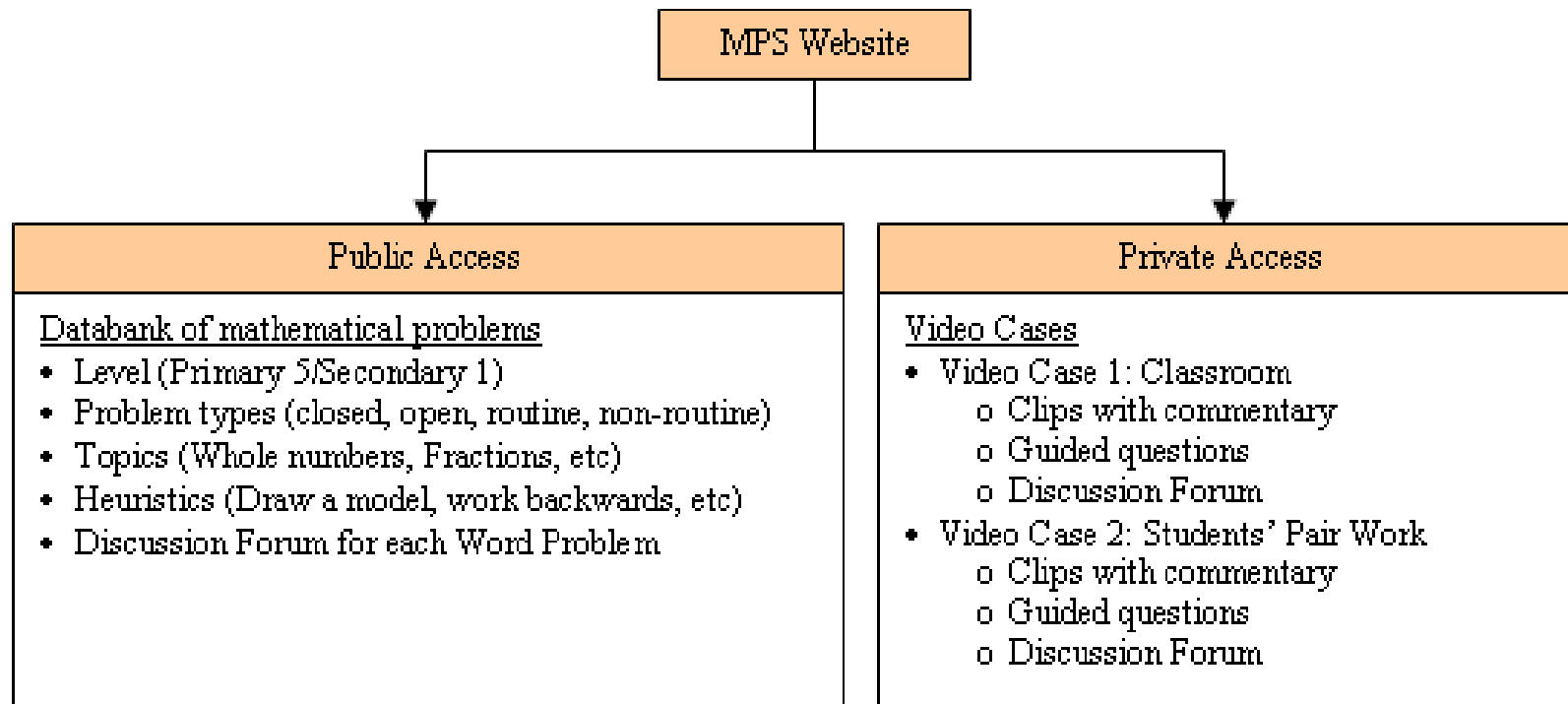
- 2 Cases
- Theme: Teacher-Student and Student-Student Interactions in Problem Solving Activity
- Framework: Ball and Cohen (1999)

# Devt of the Web-Based Cognitive Tool


The tools comprise two main parts:

- a **public-accessible** part comprising a searchable databank of word problems, and
- a **restricted-access** part comprising video cases of classroom practices and students' pair work

# The map of the website



# The main page: Databank



Navigation

- Main Page
- Community portal
- Current events
- Recent changes
- Random page
- Help
- Donations

Search

Go Search

Box

- What links here
- Related changes
- Upload file
- Special pages
- Printable version

article discussion edit history

## Main Page

**Mathematical Problem Solving (MPS)** website provides a database of mathematical problems that are organised along related topics and various heuristics approach. The classification of problems follow a range of routine type, non-routine type, open-ended type, and investigative type. The database aims to expand repertoire of heuristics for mathematics educators, researchers, students, and anyone who have an interest in mathematics especially in the teaching and learning of mathematical problem solving area.

### School Levels

The current orientation of MPS project is at Primary 5 and Secondary 1 levels. Navigate the data bank of mathematical problems according to the school levels.

Enter [Mathematical Problem Levels](#)

### Types of Mathematical Problems

Navigate the the data bank of mathematical problems according to four different problem types: Closed & Routine, Closed & Non-Routine, Open-Ended with Known Goal, and Open-Ended with Investigation

Enter [Types of Mathematical Problems](#)

### Topics related to the Mathematical Problems

Navigate the the data bank of mathematical problems according to specific topics that are related to each problem. Some problems have multiple topic relevance.

Enter [Mathematical Problem Topic](#)

### Heuristics for Mathematical Problems

Navigate the the data bank of mathematical problems according to various heuristics to approach the problem. Some problems have multiple heuristics approach.

Enter [Mathematical Problem Heuristics](#)

### Samples of Students' Work

Find out samples of how students approach mathematical problems

Enter [Students Work Sample](#).

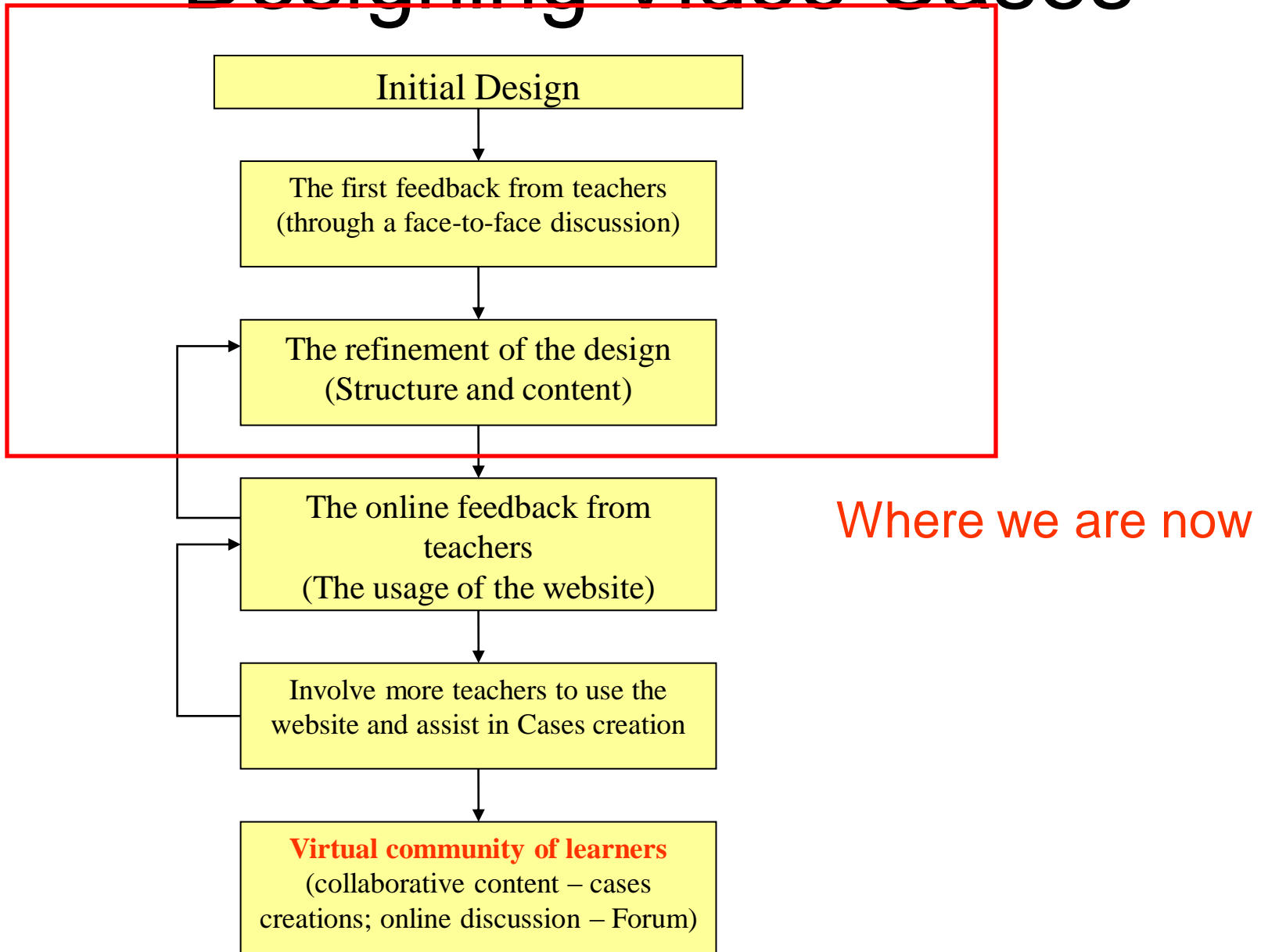
### MPS project Activities

Find out activities that the MPS team have conducted

Enter [MPS Activities](#).



# Designing Video Cases



# Video Cases: Initial Design

- Data and Analysis from MPS 1 Project – Classroom data and Pair Work data.
- Stages of Initial Design:
  - Choosing cases from the findings
  - Extracting video clips
  - Drafting commentaries and questions for users to analyse the clips
  - Generating issue for discussion (Forum)

# Video Cases: Choosing cases

## Video Case 1:

### Begin with: 4 teachers' Pedagogy

- Whole class, rich content knowledge
- Whole class, mainly discussing procedural steps
- Group work & presentation, rich student interaction
- Group work & presentation, low student interaction

## Video Case 2:

### Begin with: Students' Pair Work

- Different challenges and struggles
  - Idea of fractions
  - Ignoring crucial quantitative relationships
  - Conceiving inconsistent relationships
  - Failing to identify a quantity evaluated by a calculation

# Video Cases: Choosing Cases

- Issue on using 4 teacher pedagogy
  - Strong sense of comparison and contrast
  - The need to lead teachers (users) to analyse interactions in two problem-solving-activity settings (**teacher-led** and **group work**) and not to compare them.
- Issue on Pair Work
  - No issue in comparing
  - The need to lead teachers (users) to focus on specific area of **students' challenges**.

# Video Cases: Choosing Cases

- First Development
  - Reduce cases from 4 teachers to 2 teachers
    - Whole class setting, rich content knowledge
    - Group work, rich interaction
  - Include introduction: Background of lesson and activity
    - Lesson series
    - Timeline of the lesson in the clip
  - Student Case follow Teacher Case format

# Video Cases: Extracting video clips

- Software used:
  - *Studiocode 2.5.45*: Timeline, Clips and subtitles.
  - *Final Cut Pro*: Video editing, cases commentaries and animation.
  - *Adobe Video Converter*: Stream the video into the website.

# Video Cases: Questions for Video Analysis

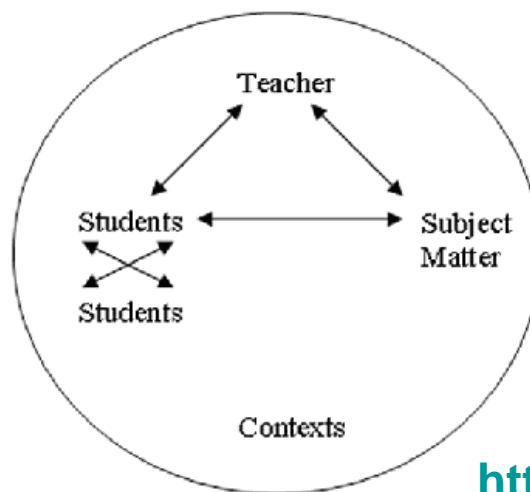
- Purpose: To guide teachers (users) in looking at particular aspect in video clips (interaction, challenges, etc)
- Starting from research questions
- Translating research questions to practitioners language (several review-and-refine iteration)
- Group into several sections and link each section to particular segment of the clip
- Used to generate possible discussion issue

# The main page: Video Cases (top part)

## Mathematical Problem Solving (MPS) Video Cases

Problem solving is a dynamic activity that requires higher-order thinking to approach problems in various ways, to choose effective solution strategies, and to constantly monitor and regulate the solving processes. For teachers, developing students' problem solving capacity, especially in mathematics, is essential though at the same time is not an easy task to fulfill. The dynamics nature of problem solving makes it impossible to have a fixed-and-single recipe to teach problem solving. Different student might have different learning experience and often they do not make their thinking overt that make it difficult for teachers to assess them and help them overcome their difficulties in learning. Furthermore, the standard to evaluate the development of students' capacity, both as a mean and an ends of a problem solving activity, also varies. The context of learning also depends on what the teachers believe in "what problem solving is", "what constitutes learning in a problem solving activity", "what are the tools to assess this learning outcome", etc.

Such dynamics of a problem-solving instruction in a classroom can therefore be seen as the ongoing interactions among teachers and students around educational materials such as word problems, questions posed to students by teachers, questions posed by fellow students, or questions posed by students to teachers. The dynamics of this instruction can be illustrated by the following model that is adapted from Ball & Cohen (1999).



<http://video.maths.crpp.nie.edu.sg>

Knowing and learning the interactions that occur in a dynamic of an instruction is an essential step for teachers to understand the effectiveness



# The main page: Video Cases (bottom part)

Knowing and learning the interactions that occur in a dynamic of an instruction is an essential step for teachers to understand the effectiveness of their teaching instructions. Based on this understanding, teachers might then think of how to modify their teaching strategies to enhance the effectiveness of the instruction. Nevertheless, teachers' opportunity to reflect on the interactions that are happening in their classroom is rare. The heavy pace demand to rush the syllabus and the deep engagement in the subject matter often make it difficult for teachers to take a third-party perspectives to analyse the teachers-students or students-students interactions around the materials discussed.

To address this issue, two video cases were designed to offer opportunity for teachers to learn how to understand and analyse the teachers-students and students-students interactions in the classroom. The first video case (referred to as Case I) consists of two teachers' lessons when they conducted a problem-solving activity in their normal classroom, and the second video case (referred to as Case II) consists of two pairs of students when they worked collaboratively solving a mathematical problem. By understanding the interactions that occurred in Case 1 and Case 2, it is hoped that teachers will have some idea on how to reflect on the interactions in their classroom and facilitate better understanding on how to enhance and improve their own practices.

[Video Cases - Case 1](#) || [Video Cases - Case 2](#)

[Feedback survey](#) | [Tutorials](#)

# Video Case 1: Classroom Interaction - Introduction

[article](#)[discussion](#)[edit](#)[history](#)[unprotect](#)[delete](#)[move](#)[watch](#)

## Video Cases - Case 1

### Contents [\[hide\]](#)

[1 Case 1: Teacher-Student and Student-Student Interaction](#)[2 Timelines and Video Clips](#)[2.1 Teacher 1: Mr Alex Tan](#)[2.1.1 Timeline](#)[2.1.2 Video Clip](#)[2.2 Teacher 2: Ms Chan Li Yin](#)[2.2.1 Timeline](#)[2.2.2 Video Clip](#)[3 Video Analysis](#)[4 See Other Cases](#)

## Case 1: Teacher-Student and Student-Student Interaction

[\[edit\]](#)

Case 1 was derived from a real observation of 1 curriculum unit on "Fractions" by two Primary 5 teachers in their classroom, from two different schools. Both classes were ranked above-average in their schools. For each teacher, we selected a lesson that was dedicated for discussing mathematical word problems in the class.

In a problem-solving activity, there are various ways for teachers to interact with students. Some activities might involve more teacher-led instructions like discussing word problems with the whole class to weave the problem discussed and conceptual understanding of the subject matters. And some activities might involve more students' active roles like students' discussion in groups and students' presentation to their classmates. We selected two lessons that used these two types of settings. The objective is not to say that one setting is better than another, instead for teachers to have an opportunity to examine the interactions that occurred in each setting. The goal is for teachers to reflect moment-to-moment pedagogical reasoning and actions that are usually hard to capture when teachers are engaged in their own practices.

## Timelines and Video Clips

[\[edit\]](#)

# Video Case 1: Classroom Interaction – Timeline and Clip 1

## Timelines and Video Clips

[\[edit\]](#)

### Teacher 1: Mr Alex Tan

[\[edit\]](#)

There were nine consecutive lessons on Fractions (Lesson 1-9) being observed. Prior to lesson 1, the teacher had taught basic concepts of fractions and assigned them revision worksheets to be marked. Lessons 1 to 3 were dedicated to thoroughly go over the worksheets, alternated with teaching concepts of fractions. Mid-way of Lesson 3, after teacher finished going over the worksheets, he started to discuss 'fresh' problems from the workbook. The word problems discussed in this lesson were taken from the students' workbook. Lesson 4 and Lesson 5 were dedicated to discussing 'fresh' problems from the workbook. The clip was extracted from Lesson 5 when they discussed 'fresh' problem in the class. From mid-way of Lesson 5 and the whole Lesson 6, the teacher involved students to present their working on the board followed by going over their working. At the end of Lesson 6, teacher assigned a worksheet to the students and went over them in Lesson 7. Lesson 8 and 9 were mainly students' individual seat work working on the worksheet and gave the closure to the unit of Fractions.

Lesson 1 Teaching concepts of fractions through going over revision worksheets

Lesson 2 Checking students' answers to assigned work; Quick check students' solution on the board, teaching skills, ends with seatwork.

Lesson 3 Teaching concepts of fractions through going over revision worksheets, followed by discussing 'fresh' word problems in the workbook

Lesson 4 Extra class – Discussing 'fresh' word problems in the workbook, end with students' seatwork.

**Lesson 5 Teaching skills and discussing 'fresh' word problems in the workbook. Seatwork and going over students' solution on the board.**

Lesson 6 Students writing solutions on the board, followed by teacher going over the solutions and teaching some skills. Ends with a quick check of students' answers.

Lesson 7 Going over worksheets thoroughly

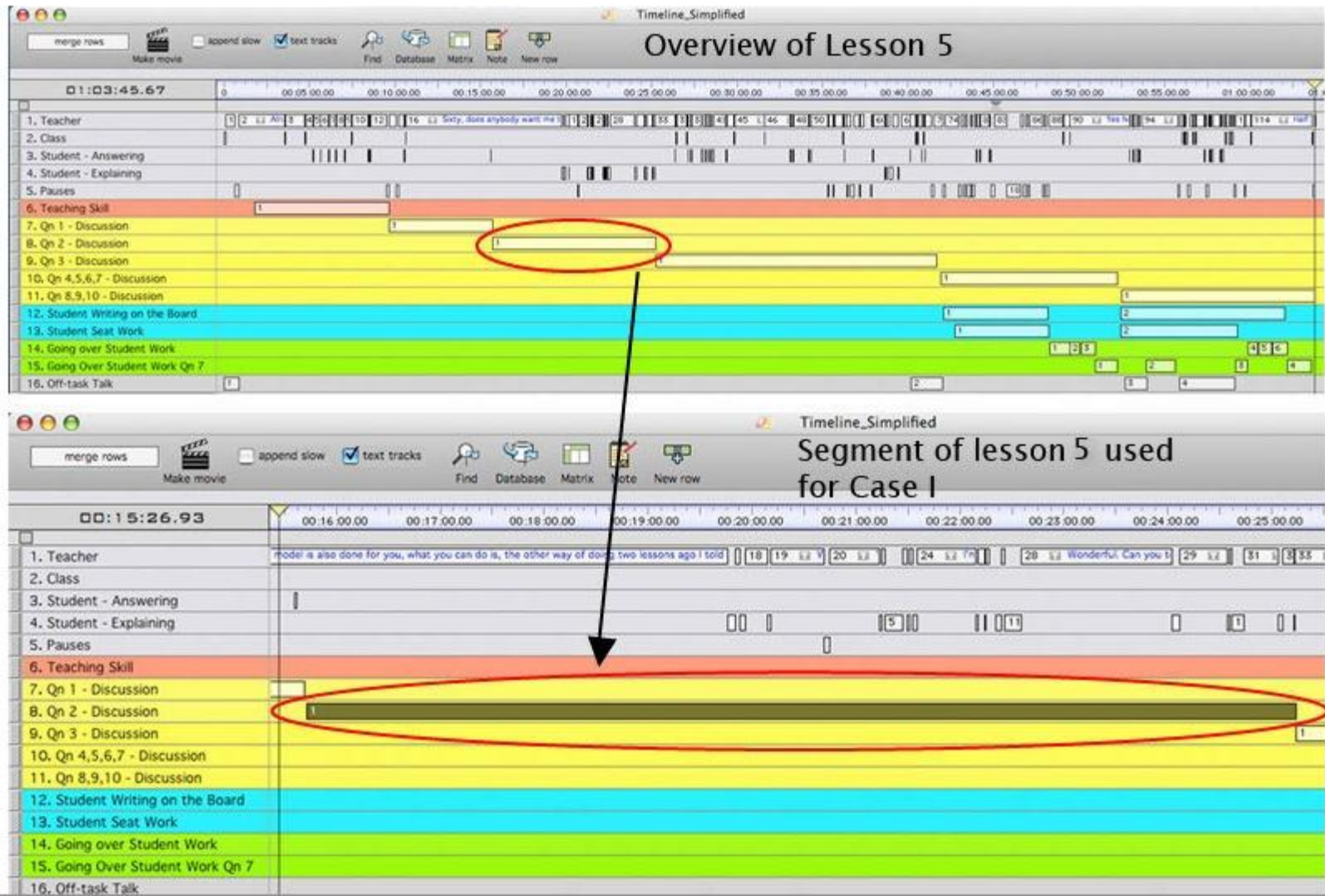
Lesson 8 Individual seatwork, followed by going over the assigned work.

Lesson 9 Individual seatwork with short spell of teaching of concepts.

# Video Case 1: Classroom Interaction – Timeline and Clip 1

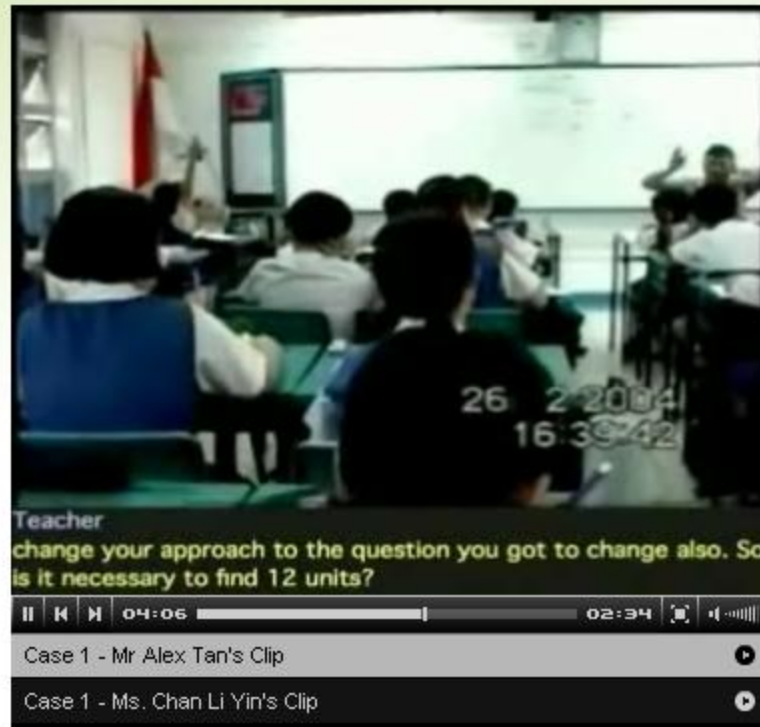
## Timeline

[e]



# Video Case 1: Classroom Interaction – Video Clip

## Case 1: Teacher-Student and Student-Student Interaction



[Guiding Questions to Mr Alex Tan's Video Analysis](#)  
[Guiding Questions to Ms. Chan Li Yin's Video Analysis](#)

# Video Case 1: Classroom Interaction – Questions for Analysis

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## Mr Alex Tan - Part 2: Teacher interaction with one student - Part 1

Q3. What might be Lily's reasoning by "1 unit equals to 4"? Why might have led Mr. Tan to clarify with Lily: "Now what was my question?"

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## Mr Alex Tan - Part 3: Teacher interaction with one student - Part 2

Q4. Why do you think Lily said "12 units"?

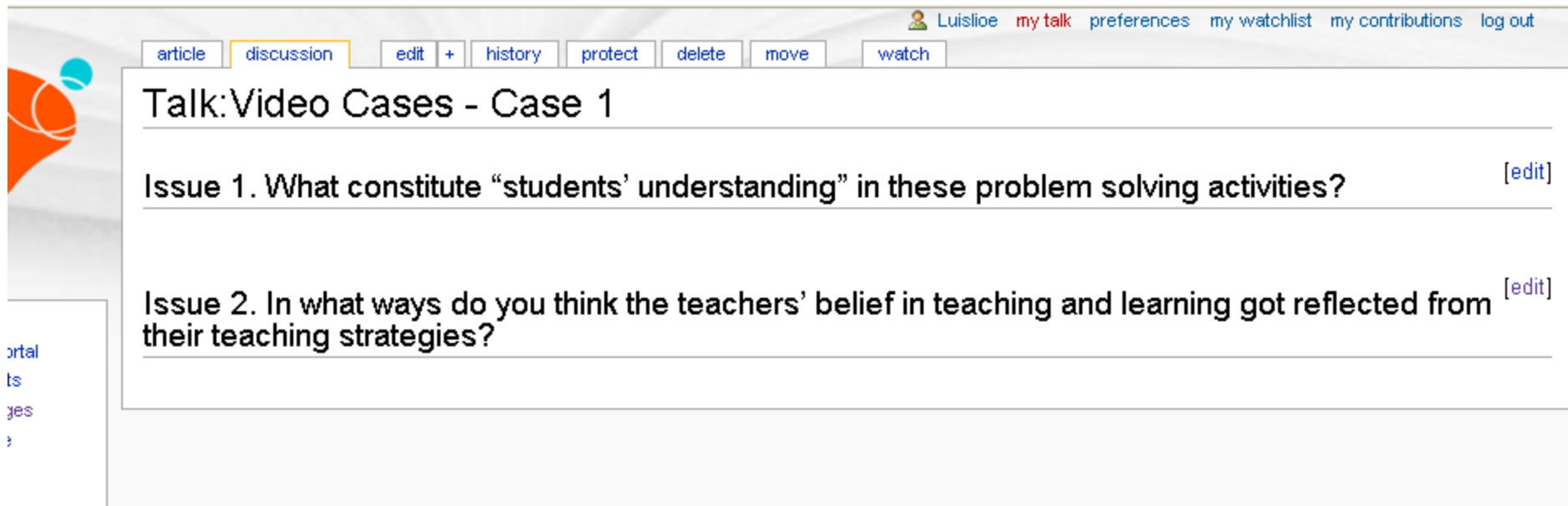
Q5. Mr. Tan interrupted Lily. What do you think may happen if Mr. Tan let Lily continue her explanation?

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## Mr Alex Tan - Part 4: Teacher interaction with one student - Part 3

Q6. Why did Mr. Tan suggest Lily to use "any method of your choice"?

# Video Case 1: Classroom Interaction – Discussion Forum



The screenshot shows a web-based discussion forum interface. At the top, there is a navigation bar with a user profile icon and the name "Luislioe", followed by links for "my talk", "preferences", "my watchlist", "my contributions", and "log out". Below this is a row of action buttons: "article", "discussion" (highlighted with a yellow border), "edit", "+", "history", "protect", "delete", "move", and "watch". The main content area is titled "Talk:Video Cases - Case 1". It contains two discussion issues. Issue 1 is "What constitute 'students' understanding' in these problem solving activities?" with an "[edit]" link to its right. Issue 2 is "In what ways do you think the teachers' belief in teaching and learning got reflected from their teaching strategies?" with an "[edit]" link to its right. On the left side of the interface, there is a vertical sidebar with a logo consisting of an orange circle and a blue circle, and a list of links: "portal", "ts", "ges", and a closing bracket "}". The background of the interface is light gray.

article discussion edit + history protect delete move watch

Luislioe my talk preferences my watchlist my contributions log out

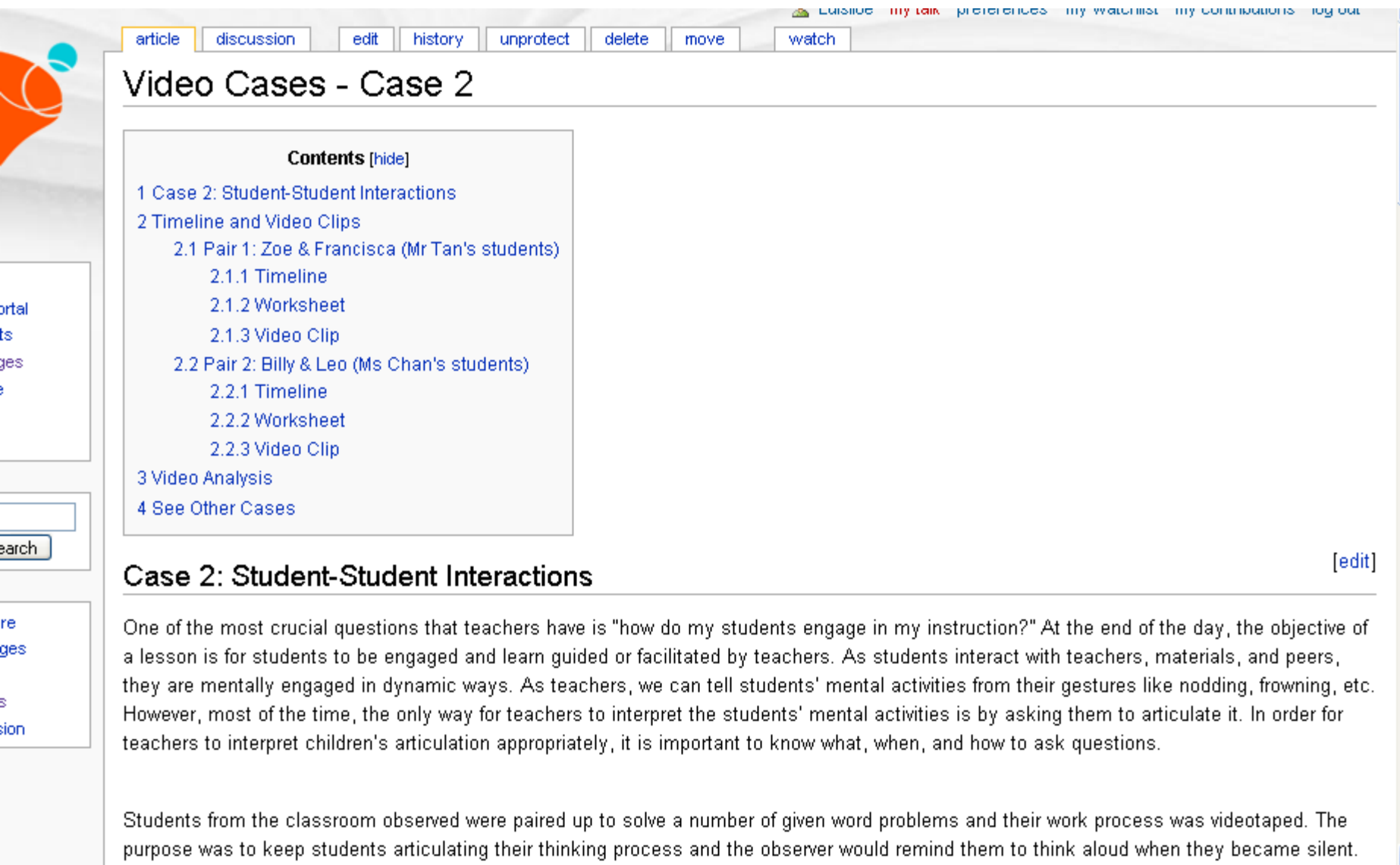
## Talk:Video Cases - Case 1

Issue 1. What constitute "students' understanding" in these problem solving activities? [edit]

Issue 2. In what ways do you think the teachers' belief in teaching and learning got reflected from their teaching strategies? [edit]

portal  
ts  
ges  
}

# Video Case 2: Pair Work - Introduction



The screenshot shows a wiki page interface. At the top, there is a navigation bar with links: [article](#), [discussion](#), [edit](#), [history](#), [unprotect](#), [delete](#), [move](#), and [watch](#). Below this is the page title "Video Cases - Case 2". On the left side, there is a sidebar with a search box and a list of links: [portal](#), [ts](#), [ges](#), [e](#), [re](#), [ges](#), [s](#), and [sion](#). The main content area is titled "Contents [hide]" and lists the following sections: [1 Case 2: Student-Student Interactions](#), [2 Timeline and Video Clips](#), [2.1 Pair 1: Zoe & Francisca \(Mr Tan's students\)](#), [2.1.1 Timeline](#), [2.1.2 Worksheet](#), [2.1.3 Video Clip](#), [2.2 Pair 2: Billy & Leo \(Ms Chan's students\)](#), [2.2.1 Timeline](#), [2.2.2 Worksheet](#), [2.2.3 Video Clip](#), [3 Video Analysis](#), and [4 See Other Cases](#). The page content is titled "Case 2: Student-Student Interactions" and includes a paragraph about student engagement and a section about videotaping student work.

[article](#) [discussion](#) [edit](#) [history](#) [unprotect](#) [delete](#) [move](#) [watch](#)

## Video Cases - Case 2

**Contents** [hide]

- [1 Case 2: Student-Student Interactions](#)
- [2 Timeline and Video Clips](#)
  - [2.1 Pair 1: Zoe & Francisca \(Mr Tan's students\)](#)
    - [2.1.1 Timeline](#)
    - [2.1.2 Worksheet](#)
    - [2.1.3 Video Clip](#)
  - [2.2 Pair 2: Billy & Leo \(Ms Chan's students\)](#)
    - [2.2.1 Timeline](#)
    - [2.2.2 Worksheet](#)
    - [2.2.3 Video Clip](#)
- [3 Video Analysis](#)
- [4 See Other Cases](#)

### Case 2: Student-Student Interactions

One of the most crucial questions that teachers have is "how do my students engage in my instruction?" At the end of the day, the objective of a lesson is for students to be engaged and learn guided or facilitated by teachers. As students interact with teachers, materials, and peers, they are mentally engaged in dynamic ways. As teachers, we can tell students' mental activities from their gestures like nodding, frowning, etc. However, most of the time, the only way for teachers to interpret the students' mental activities is by asking them to articulate it. In order for teachers to interpret children's articulation appropriately, it is important to know what, when, and how to ask questions.

Students from the classroom observed were paired up to solve a number of given word problems and their work process was videotaped. The purpose was to keep students articulating their thinking process and the observer would remind them to think aloud when they became silent.



# Video Case 2: Pair Work – Timeline and Worksheet

## Timeline and Video Clips

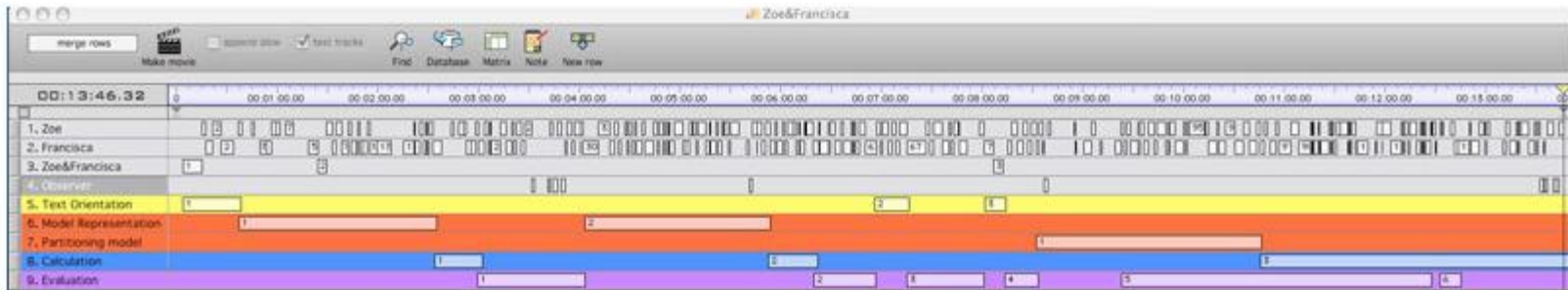
[edit]

### Pair 1: Zoe & Francisca (Mr Tan's students)

[edit]

#### Timeline

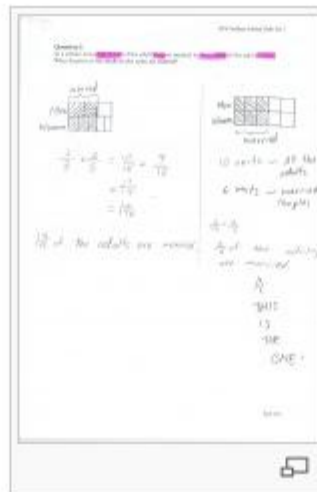
[edit]



#### Worksheet

[edit]

Right-click and open it in new window to enlarge the image.



# Video Case 2: Pair Work – Worksheet (enlarge)

Image

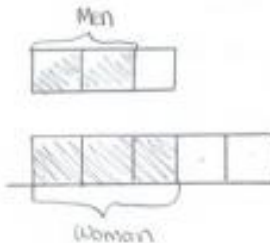
File history

Links

MPS Problem Solving Tasks Set 1

Question 1:

In a certain town, two thirds of the adult men are married to three fifths of the adult women.  
What fraction of the adults in the town are married?

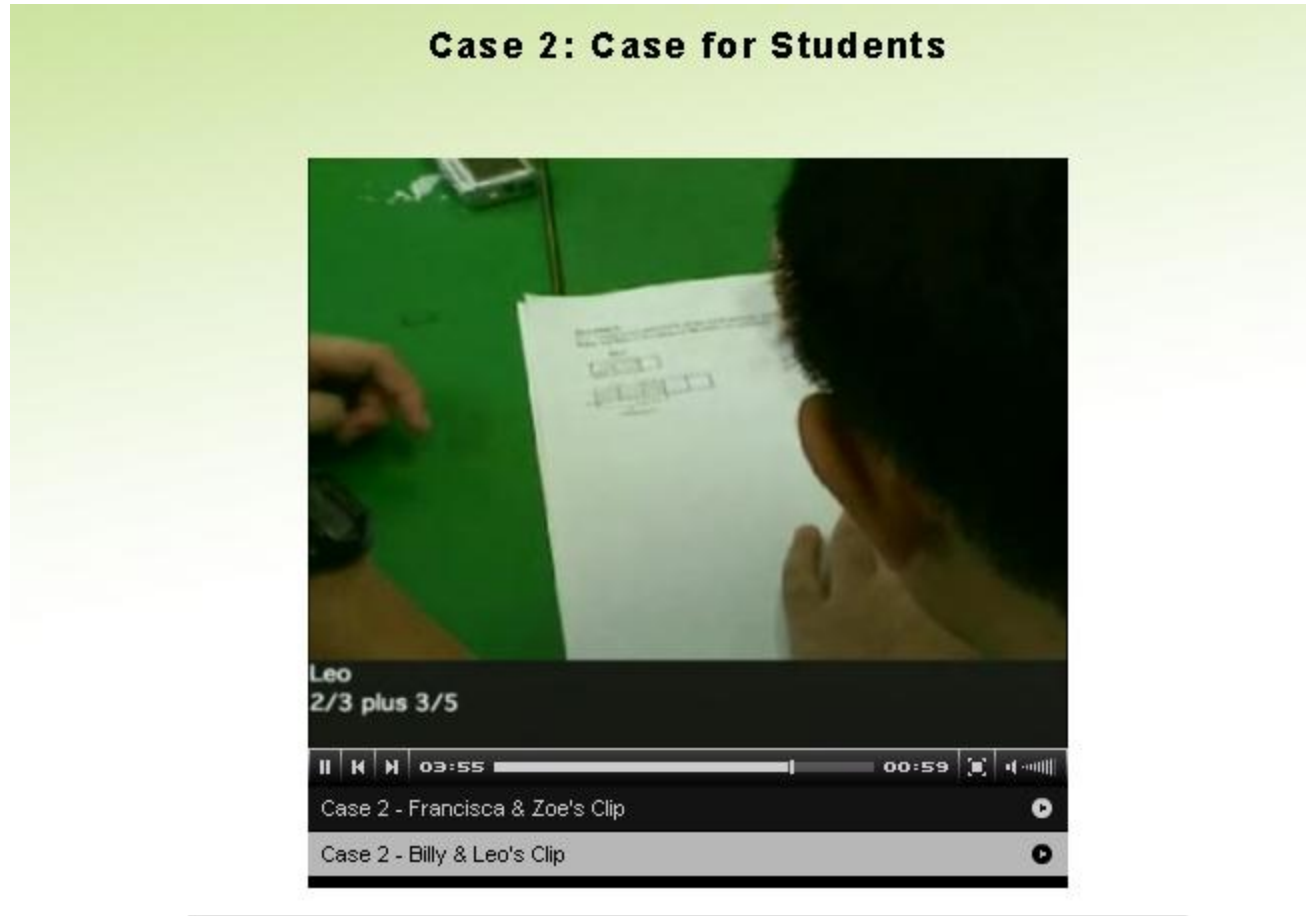


$\frac{2}{3}$  is the same as  $\frac{3}{5}$

$$\begin{aligned}\frac{2}{3} + \frac{3}{5} &= \frac{10}{15} + \frac{9}{15} \\ &= \frac{19}{15} \\ &= 1\frac{4}{15}\end{aligned}$$

$1\frac{4}{15}$  of the adults in the town are married.

# Video Case 2: Pair Work – Video Clips



[Guiding Questions to Zoe & Francisca's Video Analysis](#)

[Guiding Questions to Billy & Leo's Video Analysis](#)

# Video Case 2: Pair Work – Questions for Analysis

Q2. Leo stated that "that means there were concubines". What did he mean?

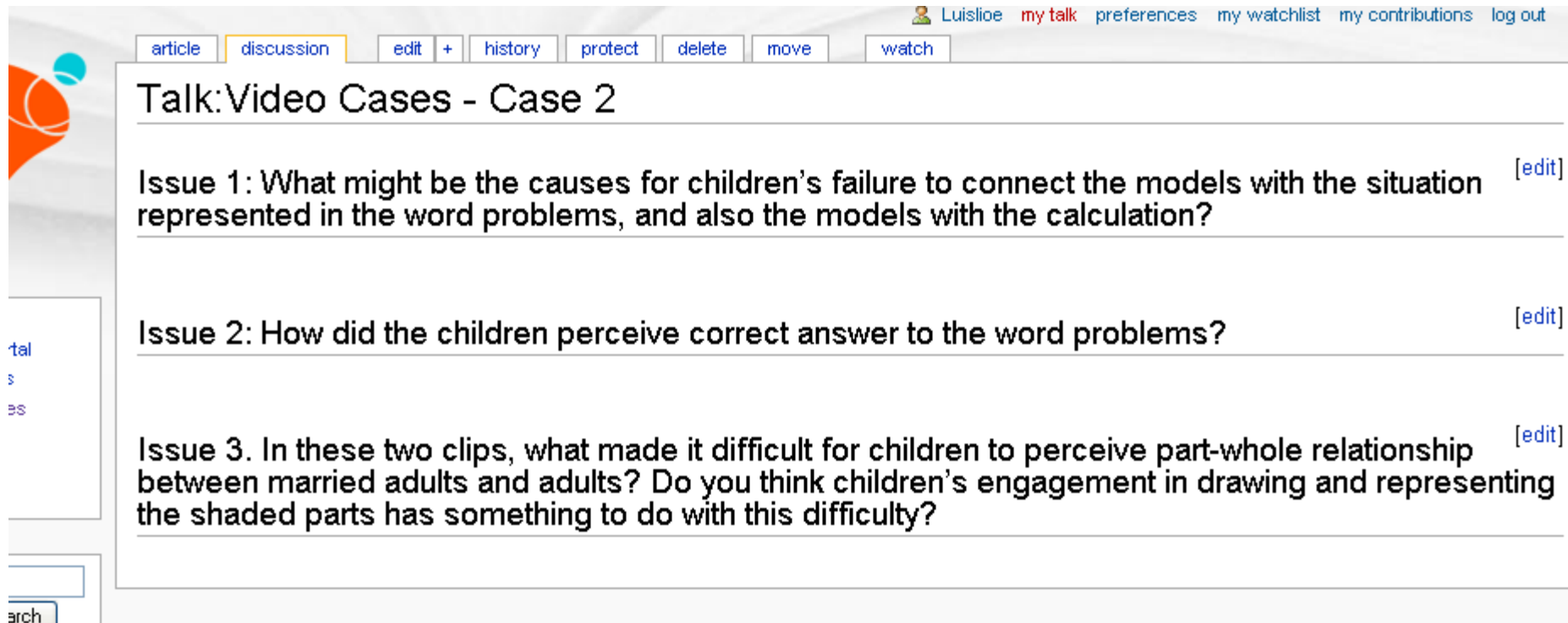
Q3. Why did Billy want Leo to change the model but Leo did not do so? What did Leo do instead? And why did Billy did not insist the change of the model?

Q4. Was the numerical expression " $\frac{2}{3} + \frac{3}{5}$ " connected with the model that they drew?

Q5. How do we make sense of Billy's suggestion to add up the two fractions to make the total of the married adults as a fraction of adults in the town?

Q6. Why did Billy and Leo accept the result of the calculation as the final answer without checking whether that makes sense in the context of the given word problem?

# Video Case 2: Pair Work – Discussion Forum



The screenshot shows a web-based discussion forum. At the top, there is a navigation bar with a user profile icon and the name 'Luislloe', followed by links for 'my talk', 'preferences', 'my watchlist', 'my contributions', and 'log out'. Below this is a row of buttons: 'article', 'discussion' (highlighted), 'edit', '+', 'history', 'protect', 'delete', 'move', and 'watch'. The main heading is 'Talk:Video Cases - Case 2'. There are three discussion issues listed, each with an '[edit]' link to its right. The first issue asks about causes for children's failure to connect models with situations. The second issue asks about children's perception of correct answers. The third issue asks about difficulties in perceiving part-whole relationships. On the left side, there is a vertical sidebar with a logo and some partially visible text.

Luislloe my talk preferences my watchlist my contributions log out

article discussion edit + history protect delete move watch

## Talk:Video Cases - Case 2

Issue 1: What might be the causes for children's failure to connect the models with the situation represented in the word problems, and also the models with the calculation? [\[edit\]](#)

Issue 2: How did the children perceive correct answer to the word problems? [\[edit\]](#)

Issue 3. In these two clips, what made it difficult for children to perceive part-whole relationship between married adults and adults? Do you think children's engagement in drawing and representing the shaded parts has something to do with this difficulty? [\[edit\]](#)

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# The First Feedback

## Meeting with five teachers

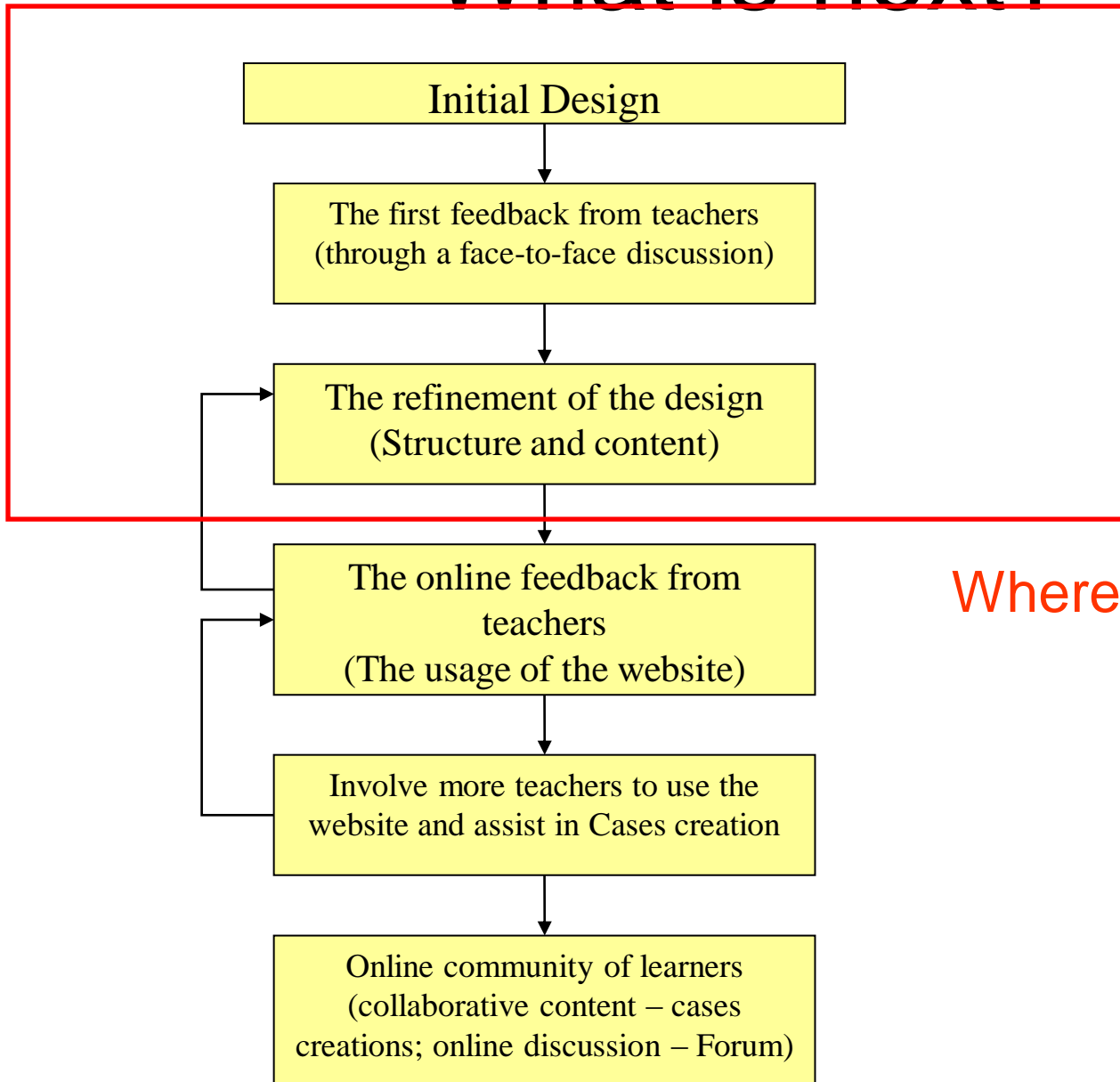
- Teachers are familiar with Wiki format (from Wikipedia)
- The number of questions are just nice; questions are clear and concise.
- Teachers are eager to verify their answers to the analysis questions
- Teachers acknowledge multiple perspectives and the need to discuss their perspectives.
- Highlighting specific theme might be needed (e.g. how decision made by teacher gives impact on the students)

# The First Feedback

## Meeting with five teachers

- Questions on Video Case 2 (Pair Work) are more difficult than those on Video Case 1 (Classroom)
- Teachers express difficulties to interpret students' mental activity and the challenges that the students faced.
- Some questions have repeated answers.
- Emerging issues: how to effectively pair up students, the impact of having one student more dominant than the other.
- Put students' worksheet on the website
- Potential learning tools: to promote discussion among students in the class.

# What is next?



Where we are now



# What is next?

- Subsequent feedback: online usage
- Invite more participation
- Collaborate with teachers to create cases
- Towards building a community of learners  
(Gee, 2003)

# Thank You!

Contact us:

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