Two teachers' pedagogies in teaching problem solving in Singapore lower secondary mathematics classrooms

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Abstract: This paper explores the pedagogical practices of two lower secondary teachers teaching mathematics in an attempt to address the question of the extent emphasis on mathematical problem solving as mandated in the curriculum gets enacted in classrooms. Using a video coding scheme which divides each lesson into phases, namely, problem solving, teaching concepts/skills, going over assigned work and student activities, a series of lessons were coded. Patterns of practices which show similarities and differences between the two teachers were discerned. The paper discusses some findings which suggest that the emphasis leans more on concepts/skills acquisition, going over assigned work than on problem solving.

Keywords: teaching problem solving; video coding; classroom practices

Background

Mathematical Problem Solving (MPS) has been a central focus of the mathematics curriculum framework in Singapore (Ministry of Education, 2000) since the early 90's. The purported aim is to "enable pupils to develop their ability in MPS." Accordingly, it "includes using and applying mathematics in practical tasks, in real life problems and within mathematics itself." It covers "a wide range of situations from routine mathematical problems to problems in unfamiliar contexts and open-ended investigations that make use of the relevant mathematics and thinking processes (p.5)."

In the implementation of the curriculum in the classroom, much rest on teachers (Howson, Keitel and Kirkpatrick, 1981; Webb and Vulliamy, 1995). The teacher has to conceptualize and enact the curriculum in their own scheme of work, and deliver *it* in the classroom (cf. Moyles and Hargreaves (Eds.), 1998). So while the intended curriculum is to focus on MPS, the question remains: to what extent is MPS emphasized in the classroom, and with what degree of success?

Such issues as reformed curricula emphasizing problem solving, investigation, real-life applications and modeling, and how they are implemented or enacted in the classrooms remain pertinent. For instance, Spillane and Zeuli (1999) investigated mathematics instruction in 25 classrooms and found distinct patterns of practice in response to reforms spelt out by the National Council of Teachers of Mathematics (NCTM, 1989, 1991). Only four of the 25 most closely approximated the spirit of the reforms: tasks were set up to help students *do* mathematics as in conjecturing, problem-solving and justifying ideas. The next pattern evident in 10 of the 25 classrooms was not as closely aligned with the intent of the reforms. While appropriate tasks were set, the discourse focused more on getting the procedures and the *answer* right. The third pattern found in remaining 11 classrooms involved tasks and discourse norms that emphasized procedural mathematical knowledge and computational skills. Such findings contribute to the analysis and evaluation of the progress of reforms.

In a two-year study in two Singapore primary schools, Chang, Kaur, Koay and Lee (2001) found that traditional expository teaching predominated – classroom activities were teacher-directed; teacher talk dominated, and student interaction with the teacher was restricted to answering teacher-initiated questions. Students spent time practicing routine exercises either in their textbooks or in worksheets. This suggests that the classroom focus is very much on acquiring skills and concepts, and that students are not exposed to a wider range of situations to include unfamiliar contexts and open-ended investigations as mandated by the syllabus. In other words, it appears that an important component of MPS, i.e. the processes incorporating thinking skills and heuristics are not given due emphasis in classrooms.

This warrants a need for a closer look at teaching practices in classrooms. In particular, the development of a systematic, evidence base that describes current practices in mathematics instruction, in curriculum delivery. This paper reports on a part of the preliminary analysis of a larger project (Hedberg, Wong, Ho, Lioe and Tiong, 2005) undertaken to develop the evidence base. It takes an exploratory look at the classroom practices of two Grade 7 teachers; to better understand classroom practices by identifying main pedagogical features and address the question of the extent MPS is emphasized.

Method

The two teachers' involvement in the larger project came about on an opportunistic basis through a meeting with their schools' superintendent. In order to identify pedagogical features particularly those related to MPS that exist in the two Grade 7 mathematics classrooms, arrangements were made for a series of lessons to be video recorded. A video coding scheme was developed to code them.

The two teachers

Teacher *X* is from an upper band (i.e. above national average in academic terms) secondary school. She has been teaching for 21 years. She taught accountancy for many years before the subject was phased out and she was retrained to teach mathematics. She was not the form teacher of the classes observed. Her class was considered below the average of the Grade 7 level of the school.

Teacher *Y* is from a middle-band neighborhood school. *Y*'s class was considered an average class within the Grade 7 level of the school. He had just started his teaching career less than a year before the observation began.

The video coding scheme

The scheme used is taken from Ho, Hedberg, and Lioe (2005). It was developed using the Grounded Theory approach (Glaser & Strauss, 1967) where ideas of pedagogical phases emerged as the videos were reviewed. It divides each lesson to five phases comprising *problem solving*, *teaching of concepts or skills*, *going over assigned work*, *student activities* and *other events*. The problem solving phase happens when the teacher presents a problem to teach problem solving. It typically involves teacher's explication or teacher initiated question-and-answer. This phase is further broken into Polya (1988)'s four stages of problem solving namely understanding, planning, executing and reflecting. Teaching of skills occur when the teacher uses an example usually from the textbook to teach *specifically* skills such as arithmetic or algebraic manipulation, estimation/approximation, mental calculation, communication, use of mathematical tools, handling data, etc. For the teaching of concepts, time is spent showing, demonstrating, defining, explaining numerical, geometrical, algebraic or statistical concepts. The *Going Over Assigned Work* phase refers to a time when the teacher goes over work that had been assigned. Students would have had spent some time on the assigned work. Common examples include teachers reworking given exercises or worksheets, going over procedures to get answers, checking correctness of answers, etc. The *Student Activites* phase refers to students spending class time doing assigned tasks. This phase is further divided into: Presentation, Group Work and Seat Work. Presentation is when student(s) are tasked to present their solutions of assigned problems to their classmates/teacher using the white board or overhead projector, with or without their oral presentation. Group work happens when groups of students are given tasks to do as a group. Seat work refers to students doing assigned tasks individually (sometimes with short consultation with the teacher). The *Other Events* is a catch-all category to include all other events not listed in the previous phases. The scheme also notes the type of word problems used in class and the range of heuristics that are used in the class to solve those problems. In identifying types of problems, an adaptation from Foong (2002) and Frobisher (1994)'s classification of problem types is used. There are four types of problems, namely closed-routine, closed non-routine, open-ended with goals known and open-ended investigations/projects. The range of heuristics to be coded in the scheme is taken directly from the official syllabus (Ministry of Education, 2000, p. 6).

Considering that phases do change quickly, it was decided that the duration of any one phase be rounded up or down to the nearest half minute. Short phases lasting 14 seconds or less are not coded into the scheme. At this early stage of development, the reliability of the coding scheme was estimated using percentage agreement, which is defined as the proportion of the number of agreements to the number of agreements and disagreements. Based on an earlier study (Ho, Hedberg and Lioe, 2005) which used the same scheme, the percentage agreement was about 78%.

Results

At the time of the scheduled observation, the topic both teachers were teaching was algebra and the solving of algebraic equations. For *X*, seven lessons were observed with a total time of 5 hours 26 minutes; for *Y*, six lessons with a total time of 5 hours and 8 minutes.

The coded videos

Using the coding scheme, the series of lessons for each teacher were coded and the total time spent on each phase is summarized in the following table:

Table 1: Amount of Time Each Teacher Spent on Each Phase

Teacher	Problem Solving (as % of total)	Teaching Concepts/Skills (as % of total)	Going Over Assigned Work (as % of total)	Student activities (as % of total)	Others (as % of total)	Total Time Observed
X	0:17:00 (5%)	1:28:00 (27%)	0:49:00 (15%)	1:45:30 (32%)	1:06:30 (20%)	5:26:00 (100%)
Y	0:28:00 (9%)	1:01:30 (20%)	0:35:00 (11%)	2:05:00 (41%)	0:58:30 (19%)	5:08:00 (100%)

The two teachers spent a fairly similar amount of time in each of the phases. Using the percentage of time in each phase as an unit of analysis, the co-relational factor between the two teachers was 0.87, noting that 1.00 would indicate equal proportion of time for each phase for the two teachers, and zero, very different proportions.

Figure 1 illustrates the amount of time each teacher spent on each phase per lesson over the series of lessons:

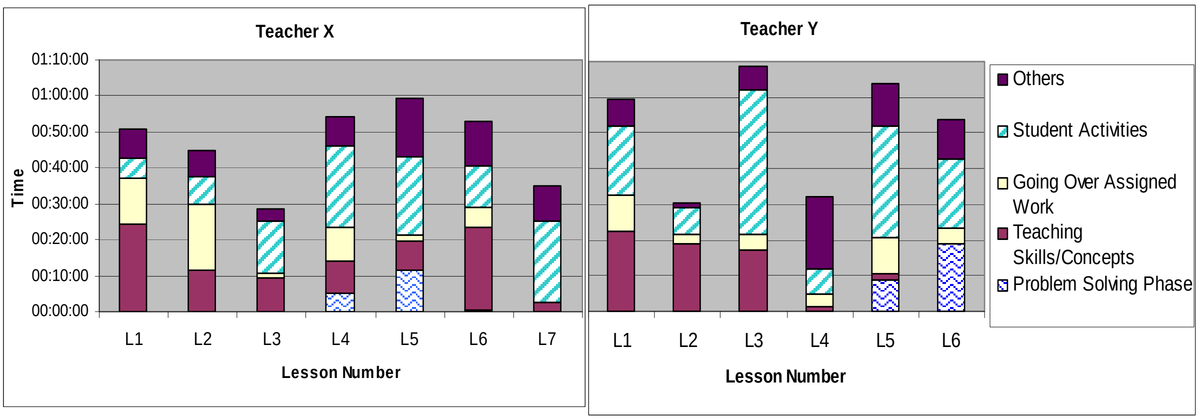

Figure 1 Amount of time X and Y spent on the phases through a series of lessons

From *Figure 1*, a similar pattern of approach to the teaching of solving algebraic equations can be discerned between the teachers. Both spent a fair amount of time teaching the skills and concepts related to algebra and the solving of equations – lesson numbers L1 to L3 for *X*, and L1 to L4 for *Y*. Each component skill involved in balancing and solving equations was decomposed and decontextualized for purposes of instruction, and taught systematically. This was done before engaging with word problems and using the newly taught skills and concepts in solving them – L4 and L5 for *X*, and L 5 and L6 for *Y*. *X* continued to reinforce the skills acquisition of solving equations in L6, and finished the topic with some group problem solving work in L7. *Y* finished the topic on algebra by L6.

The proportion of time devoted to teaching skills and concepts is much higher than the time spent on problem solving. In *X*'s case it is 27% to 5% while *Y*'s is 20% to 9%. This suggests that the emphasis is more on acquiring procedural skills and concepts than on problem solving per se. In fact both teachers spent more time going over assigned work than problem solving in the Polya's sense (see Table 1), i.e. more time is spent going over the procedural steps for getting the solutions, checking the answers to given assignments or assessment than on guiding students through the four stages of problem solving as recommended by Polya (1988).

Overall, the proportion of time *X* and *Y* spent on the problem solving phase is low at 5% and 9% respectively. Breaking this problem solving phase further into *Understanding*, *Planning*, *Executing* and *Reflecting*, the following Table 2 shows the time each teacher spent on each of them:

Table 2: Time Spent on Each of the Four Stages of Problem Solving

Teacher	Understanding (% of total)	Planning (% of total)	Executing (% of total)	Reflecting (% of total)	Total Time Spent on Problem Solving Phase
X	0:03:00 (18%)	0:00:30 (3%)	0:06:30 (38%)	0:07:00 (41%)	0:17:00 (100%)
Y	0:14:00 (50%)	0:01:30 (5%)	0:10:00 (36%)	0:02:30 (9%)	0:28:00 (100%)

While the proportion of time for each phase differs, one clear point that emerges from the table is that the two teachers spent very little time on the *planning* stage when giving instructions on problem solving. This observation reinforces the findings of an earlier study (Ho, Teong & Hedberg, 2004). While *X* spent relatively more time looking back at solutions, *Y* dwelled more on reading and understanding the problems.

The types of word problem used

Teacher *X* covered a total of 8 word problems in her problem solving phase while *Y* did 9. All 17 problems used were taken directly from their textbooks. All of them can be considered as *routine* in that the students more or less knew the procedural steps to take by following the textbook.

The range of heuristics

The heuristic employed in tackling all 17 word problems was '*use equations*'. This is expected as it relates closely with the algebra topic, and is listed as something to be taught explicitly. The teachers did not attempt to employ *other* heuristics in solving the problems, i.e. there was no emphasis on looking for alternative solution pathways. On one occasion, students in *X*'s class raised the possibility of using a *draw a diagram/model* heuristic they had learned in their earlier grades for one of the problems. *X* refocused the discussion back to '*using equations*', citing its importance in examinations.

Types of student activities

Student activities took up 32% and 41% of *X*'s and *Y*'s class time respectively. This phase is further divided into *presentation*, *small group work* and *individual seat work*. The following *Figure 2* shows the distribution of time for each of the categories:

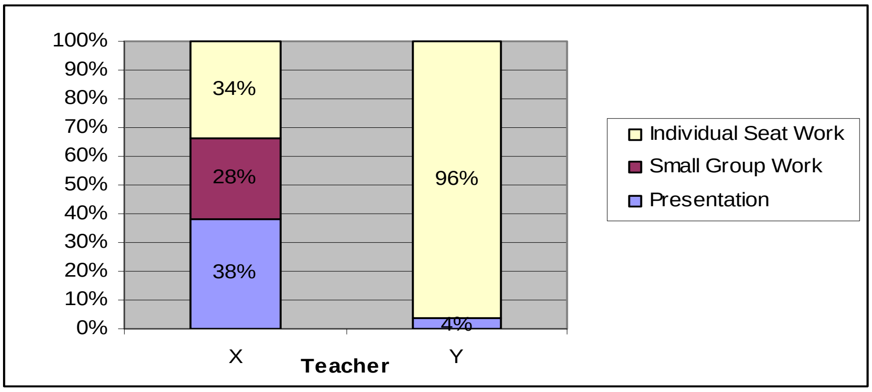

Figure 2 Distribution of Student Activities time

The individual seat work in both classes was mainly devoted to exercises and practicing of skills. *X* used small group work for problem solving and spent about two thirds of the student activities time on it together with students presenting their solutions to assigned problems to their classmates/teacher either using the white board or overhead projector. *Y* did not use group work. Instead much of his student activities were limited to students doing assigned exercises or corrections, and copying solutions written on the white board.

Discussion

This paper sets out to explore two teachers' classroom practices in the teaching and learning of mathematics with a particular focus on problem solving. The video coding scheme enabled the identification of five phases, namely, problem solving, the teaching of concepts/skills, going over assigned work, student activities and other class events.

The amount of time each teacher spent on the phases describes their typical classroom practices. Traditional patterns of teaching a topic, explaining concepts and giving students exercise to practice related skills prevailed. The amount of time spent giving problem solving instruction was very limited, and the emphasis centered on reading and doing the problems. The observed sequence was mainly: reading of the problem, followed by naming of the unknown, forming an equation, solving it and checking if the answer was correct. There was very little *planning* in terms of *understanding* the given information in the problem *further*, relating it to what was known, and exploring the possible strategic approaches that could be used within the *using an equation* heuristic, and possibly other heuristics listed in the syllabus as well. There is a need for teachers, after reading the problem, to spend more time and be more explicit about planning before carrying out the plan. For the *reflection* part, the focus was limited to checking the correctness of the answer. There was no attempt to derive the result differently as in reflecting to see if it was possible to use another heuristic, and to use the result for some other problem as recommended by Polya (1988, p. xvii).

These observations highlighted the main areas of pedagogical focus of the two teachers. The discerned pattern for the series of lessons for both teachers seemed to be: start with the teaching and learning of concepts, followed by skills acquisition and exercises, then word problems and practice. This could be because both teachers were using the same textbook and following the sequence therein closely. Comparing the amount of time spent on each of the phases, there is indication that the emphasis is more on getting the concepts and skills acquisition than on solving problems per se. While the proportion of time spent on the phases were similar for the two teachers, the main difference between them lies in their *student activities* – with *X* having a wider spread of time over student presentation, group and individual seat work, while *Y* spent most of the time on individual seat work.

The range of heuristics and the type of problem were also important aspects in exploring classroom practices and MPS. In the observed lessons only the *using an equation* heuristic was seen. Granted that the topic was algebra and that the textbook used did not extend problem solving to include other heuristics, the observations nevertheless suggest a limited approach to MPS. Instead of exploring heuristics, extending and connecting them, the teachers seemed to approach problem solving as part of the topic of algebra. So, problem solving was seen mainly in terms of using and solving equations correctly.

The type of problem used was also limited to those in the textbooks. As such the students knew more or less the steps to take to solve them, and were not pushed to discuss their problem solving solutions in terms other than procedural steps. Perhaps provisions can be made to include a wider variety of problems where appropriate heuristics are not limited to using equations but include others as well.

These issues point to the need to address these areas if the curriculum mandate of enabling pupils "to develop their ability in MPS", and to cover "a wide range of situations from routine mathematical problems to problems in unfamiliar contexts and open-ended investigations that make use of the relevant mathematics and thinking processes (MOE, p.5)" is to be fulfilled.

Conclusion

This exploratory study offers a glimpse into the classrooms of two Grade 7 teachers teaching algebra. Clearly the findings cannot be generalized beyond what was observed, but nonetheless, it provides some ideas about the main pedagogical features, particularly those that relate to the teaching of MPS. It gives a picture that hopefully in some ways address the question of the extent emphasis on MPS in the curriculum gets enacted in the classrooms. While the initial findings about the two teachers point to a limited focus on MPS, it is hoped such findings about the extant practices would contribute to the development of a systematic evidence base, and inform work towards further teacher development along MPS.

Acknowledgement

This paper is drawn from a funded project CRP 01/04 JH, "Developing the Repertoire of Heuristics for Mathematical Problem Solving", Centre for Research in Pedagogy and Practice, National Institute of Education, Nanyang Technological University, Singapore. Thanks to Luis T. Lioe and John Tiong for assisting in part of the coding.

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