Thinker-Listener Pair Interactions to Develop Students' Metacognitive Strategies for Mathematical Problem Solving

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Abstract

This paper describes a strand of a larger project examining student mathematical problem solving (Hedberg, Wong, Ho, Lioe, & Tiong, 2005). The initial parts of the project studied the effectiveness of students' metacognitive strategies as they solved problems in pairs. This follow-up study employed the Whimbey & Lochhead (1999) Thinker-Listener pair approach and one of the same pairs was selected to solve problems within this context. The protocols when the pair solved a similar problem in pre- and post-tests were transcribed and analysed using modified Artzt & Armour-Thomas's (1992) framework classifying their cognitive and metacognitive thinking, and their performances were compared. The analysis revealed that within the Thinker-Listener context, they were more engaged in each other's ideas, spent more time on metacognitive behaviours, and had higher regulatory behaviour. The results suggest that Thinker-Listener approach can be effectively extended on to a larger scale of pairs with longer periods of time to observe the effectiveness in developing students' metacognitive strategies

Keywords: mathematical problem solving, metacognitive strategies, Thinker-Listener pair problem solving.

Metacognition and Problem Solving in Singapore

Since 1990, Mathematical Problem Solving (MPS) has been a central focus of the mathematical curriculum framework in Singapore (Ministry of Education, 2000).

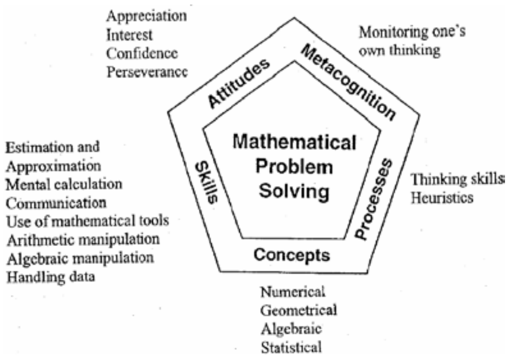

Figure 1 Singapore Mathematics Curriculum Framework

In this framework (Figure 1), metacognition is deemed one of the main components of mathematical problem solving. "Monitoring one's own thinking" is in line with Flavell's (1976) definition of metacognition that refers to students' awareness of their own cognitive processes and students' regulation of these processes to achieve a specific goal. Garofalo & Lester (1985) have also described regulation of cognitive processes as involving various activities such as planning an overall approach to problems, selecting appropriate strategies, monitoring problem solving progression, assessing local and global results, and revising plans and strategies if necessary. The explicit

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inclusion of metacognition in the framework underscores its importance in the teaching and learning of MPS within the curriculum.

Emphases in the syllabus aside, the question of how much metacognition is actively considered in school's mathematics instructions remains elusive. Hedberg, Wong, Ho, Lioe, & Tiong (2005) suggest that teachers' guidance using metacognition is not made explicit in the classroom, and students are seldom given opportunities to raise their metacognitive awareness during the teaching and learning of mathematical problem solving. Studies on students' mathematical performances (Foong, 1994; Yeap & Menon, 1996) revealed that students generally exhibited metacognitive behaviours when solving non-routine problems; however their metacognitive strategies were not always efficient and successful. Both studies highlighted the need for mathematical instruction that supports students' development of metacognitive strategies and systematic thinking.

Collaborative Work and Metacognition

Collaborative work, as a form of instructional practice, may help students develop their metacognitive strategies. Artzt & Armour-Thomas (1992) found that a small-group setting was likely to promote students' spontaneous verbalisation which allowed them to offer ideas for critical examination. However, peer interaction may also hinder students' metacognitive decisions. Stacey (1992) found the diminishment of Grade 9 students' problem solving performance when they worked in groups. Goos, Galbraith, & Renshaw (2002) observed that such diminished performance was caused by students' failure to create Vygotsky's (1978) notion of *zone of proximal development* (ZPD) through peer interaction (termed as *socially mediated ZPD*). They further highlighted that the key elements that create socially mediated ZPD were the interplay between *transactive* challenges and metacognitive decisions. The notion of *transactive* reasoning was defined by Kruger (1993) as the clarification, elaboration, justification, and critique of one's own or a peer's reasoning.

To promote socially mediated ZPD in pair work, Whimbey and Lochhead (1999) introduced a format where well-defined and distinct roles were assigned to each student. One pair member was assigned the role of *Thinker* to solve the problem entirely and to think aloud for every single step. The other member was assigned the role of *Listener*, to check the Thinker's accuracy and to require the Thinker's constant vocalisation. Whimbey and Lochhead argued that Thinker's thinking aloud would make the thinking processes not only more observable to the Listener, but also help the Thinker to reflect on his or her own thinking activities. The Listener must not work independently from the Thinker, rather, the Listener must actively work alongside the Thinker, never letting the Thinker get ahead and asking the Thinker to wait when necessary. Furthermore, while catching an error, the Listener must only point it out to the Thinker and never provide the correct answer. In short, the Listener must always *transactively* challenge the Thinker.

The roles of the Thinker and Listener must be interchanged between the pair members over a wide variety of problems. Eventually, each student is expected to act as both Thinker and Listener, and then adopt this mode in their learning, including classroom learning when they listen to teachers teaching. In particular, when solving problems on their own, they are expected to internalise both the Thinker's and Listener's roles. The Thinker's roles would help them generate their awareness and regulation of problem solving progression, and the Listener's roles would help them *transactively* challenge their own ideas and strategies to better regulate their cognitive processes. In other words, they would internalise a habit of "monitoring their own thinking" during problem solving. To reiterate, this proposed Thinker-Listener format by Whimbey and Lochhead offers a way to work with pairs of students to promote socially mediated ZPD and metacognitive behaviours.

In line with the emphases on metacognition as an essential component of mathematical problem solving, this paper examines how the Thinker-Listener approach affects the pairs' cooperative level and their regulatory behaviours. This study follows up on a strand of a larger project (Hedberg et al., 2005; Lioe, Ho, & Hedberg, 2005), where pairs' metacognitive strategies were observed when they solved mathematical problems. Preliminary findings suggest that working in pairs involves two possible cooperative levels (*high* and *low*) and certain regulatory problem solving progression. The *low* cooperative levels would likely affect pairs' metacognitive strategies that determine their regulatory behaviours.

Methodology

The *pre-test* to mathematical problem solving set was administered to Primary 5 pairs in Hedberg et al. (2005). In this follow-up study, one of the same pairs was selected to solve *post-test* problems using Thinker-Listener pair setting. The post-test consisted of six questions and the first five questions were designed for the pair to practice Thinker's and Listener's roles in solving mathematical problems. The last question in the post-test was similar to the last question in the pre-test (see Table 1). A short briefing was conducted before they started solving the post-test. Guidance was provided in doing the first four questions and was not in doing the fifth question.

Table 1: *Word problems used in the pre- and post-tests.*

Selected word problem in the pre-test	Selected word problem in the post-test
(a) With a full tank of petrol, we drove for 352 km up Malaysia and had to fill up at a petrol station. The tank took in 42 litres. How many km per litre did we get?	(a) It takes 2 hours to charge Lynn's handphone battery from empty to full. With the battery fully charged, Lynn brought her handphone to go to Thailand. She did not make nor receive any phone calls, and did not turn it off. At the end of the second day, she recharged her battery for half an hour. How much of her battery was used up in an hour?
(b) On the way back, after driving 2/3 of the way, the tank was a quarter full. Do we have a problem if we don't stop by a petrol station to fill up?	(b) On the way back home, after travelling 1/3 of the way, her battery was 1/4 left. Would her battery run out before she reached home?

The think aloud protocols in the pre- and post-tests were transcribed and analysed using a modified Artzt & Armour-Thomas's (1992) framework. This framework parsed the protocols into eleven episodes: *reading* (cognitive), *analysing* (metacognitive), *exploring* (cognitive), *exploring* (metacognitive), *planning* (metacognitive), *implementing* (cognitive), *implementing* (metacognitive), *verifying* (cognitive), *verifying* (metacognitive), *watching and listening* (unassigned cognitive level), and *others* (unassigned cognitive level). For a more detailed explanation of the modified framework, see Hedberg et al. (2005) and Lioe et al. (2005).

Analysis and Results

Table 2 demonstrates the time and percentages of each student's cognitive-metacognitive behaviours while solving the selected problem in pre- and post-test, as well as the status of solution paths and answers. It was shown that within Thinker-Listener context, the Thinker spent more time in metacognitive behaviours (53%) compared to the pair performance in the pre-test (35% for S1 and 38% for S2). No time was used for *unassigned cognitive level* category that involved *watching and listening* and other *off-task* behaviours. These figures suggest the positive effects of Thinker-Listener pair collaboration, which are students' tendency to devote more time to metacognitive behaviours and to be on-task. However, they still obtained wrong answers for both parts of the question. Detailed of their progressions in pre-test and post-tests are discussed in the next sub-sections.

Table 2: *Amount of time in mm:ss and percentage devoted by each student to metacognitive, cognitive, and unassigned cognitive level of behaviours, and their solution status during pre-test and post-test.*

	Pre-test			Post-test
	S1	S2		S2 (Thinker)
Metacognitive (% of total)	04:45 (35%)	05:15 (38%)		04:22 (53%)
Cognitive (% of total)	05:40 (41%)	02:14 (16%)		03:49 (47%)
Unassigned (% of total)	03:22 (24%)	06:18 (46%)		00:00 (0%)

Total Time Spent	13:47 (100%)	13:47 (100%)		08:39 (100%)
Part (a)	incomplete could not settle solution path			complete correct path, wrong answer
Part (b)	complete wrong path, wrong answer			complete correct path, wrong answer

S1 and S2's Progression of Problem Solving Activity in Pre- and Post-tests

Figure 2 and Figure 3 illustrate sequences of pair's problem solving episodes in the pre- and post-tests. In Figure 2, S1's behaviours are indicated in blue/darker shade and S2's in pink/lighter shade. Figure 3 only shows S2's since the behaviours coded in this context are the Thinker's.

S1 and S2's protocol in pre-test

In the earlier part of solving part (a), S1 and S2 did not share the same activity. While S1 started *reading* the problem, S2 was still checking their earlier problem's results in the same problem-set. It was then followed by the discussion about the checking that drew them away from the task. They returned to the task after 2mins 30secs, and the subsequent protocol could be summarised as *short reading* → *planning/implementing* (C, M) with little *analysing* involved. Within this activity, each of them persisted with their own approaches. Most time was spent on arguing for own strategy, abandoning peer's strategy, and taking over implementation work. As such, they did not achieve solution path agreement.

Therefore, their problem solving activity was predominantly spent on *planning* and *implementing*. In terms of cooperative level of interaction, they were more inclined to the competitive mode of "whose strategy was correct" discourse.

S2: (*taking over worksheet from S1 and cancelling S1' working*) Don't need to write this one, just do thing one! Just do the thing! (*referring to S2's own strategy, which is "42 divided by 352"*)

S1: 352 divide by…

S2: No! 352? Because we need to find how many litres. So, you must do this first.

S1: 352, lor!

S2: How to divide? If 352 divide by 42, you will get how many…. Aiyah never mind lah! It's still the same. Divide 352… (*"starting to calculate 352 divided by 42"*)

S1: No! (*cancelling S2's working*) That one cannot, lah! (*pinching S2*)

After 10mins, they had not settled on which formula to be used and the researcher stopped them due to time contingency and asked them to proceed to part (b). The sequence of activity in solving part (b) was *reading* → *analysing* → *planning* → *implementing* (C, M) → *verifying* (M) completing within 4 minutes. They could settle with one solution path, though their plan was incorrect and reflected their limited resources in dealing with this problem. They seemed to infer directly a mathematical operation to combine two numerical data given without considering the situations described in the problem statement. However, such inference may also be caused due to the time contingency. It was possible that within the time limit, they tried to write something presentable in order to complete solving this problem.

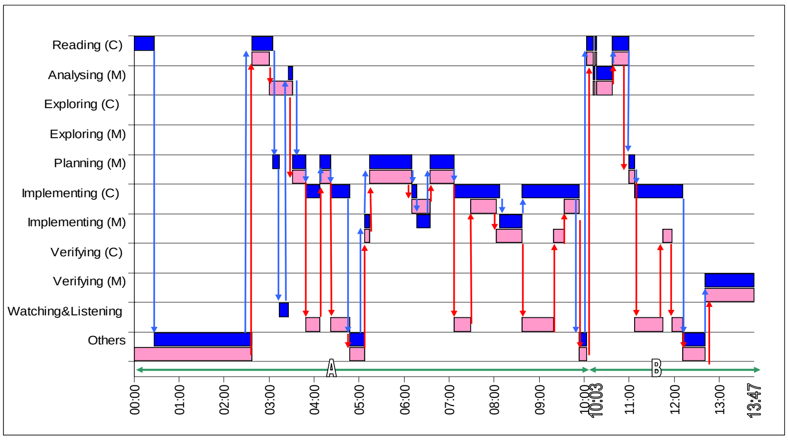

Figure 2 A timeline representation of S1 and S2 solving word problem in the pre-test

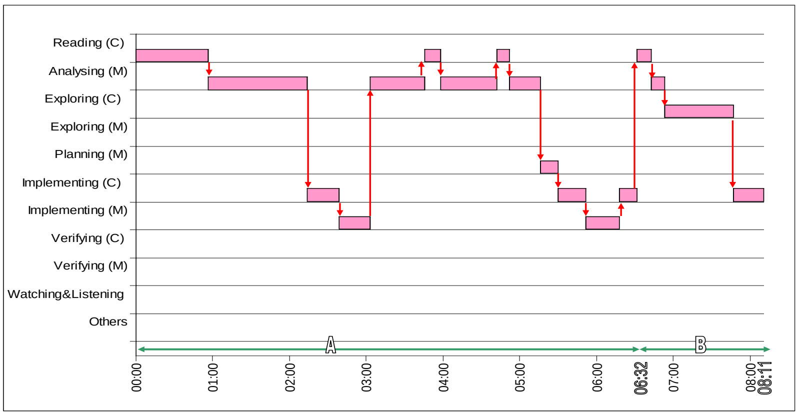

Figure 3 A timeline representation of S2 (Thinker) solving word problem in the post-test

S1 and S2's protocol in post-test

In this protocol, S2 was assigned as the Thinker and S1 was the Listener. Both students adopted their roles well. To reiterate, in this context, the cognitive-metacognitive behaviours coded were the Thinker's behaviours. Figure 3 illustrates Thinker's well-regulated activities when they solved part (a): *reading* → *analysing* → *implementing* (C, M) → *analysing* (with some *reading*) → *planning* → *implementing* (C, M). It was observed that when the Thinker spotted the incorrect result of the first implementation (coded as *implementing* (M)), he returned to another well-regulated activities after abandoning his previous work. The Listener's roles were also important in affecting the regulation.

S2: So… when they say only 1 hour… which mean errm they need… they want , the…

S1: Uhm, please refer to the question again, ya… I think you did something wrong.

S2: How much of her battery [was used up in an hour (*reading part of the question*)

S1: [No, I mean…. before that.

S2: At the end of the second day … (*reading part that was pointed out by S1*)

The Listener was also able to spot and prevent the Thinker from making computational mistakes

S2: (*calculating 3/2 times 1/48*)1/48… 3/96. Make into the simplest form equals 1/13.

S1: Uhm… I think you make a mistake there.

S2: Because.. erm this one, 48 hour… 48 hour… should be the numerator.

S1: Check with the answer again… I don't think you did it correctly.

S2: Yea, 1/32 (*noticing his mistake, correcting his work*).

S1: Okay, I agree.

Despite these well-regulated activities, their result for part (a) was incorrect. This was caused by inappropriate use of the information given in the problem that laid them to use incorrect procedures.

In part (b), after short *reading* and short *analysing*, the Thinker involved in *exploring* relationship between "battery capacity" and "the journey travelled". The Listener was able to keep track on this exploration; however, they did not assess it after the Thinker had obtained an answer. He proceeded to writing the final statement which was coded as *implementing* (C) and stop the problem solving session. In the exploration, the Thinker used the given info inappropriately resulted in getting incorrect answer. The Thinker's solving process was quick and the Listener appeared to focus on catching up with the Thinker's pace that made it difficult for the Thinker to check the Thinker's accuracies.

Discussion

The analysis on pre- and post-test items showed significant differences between this pair's performances, especially in terms of their cooperative level, their regulatory behaviours, and their approaches to the problems. In the pre-test, they were engaged at a *low* cooperative level. They appeared to persist in their own solution approaches and had difficulties engaging in each other's ideas. They tended to speak for their own arguments instead of listen and challenge their peer's arguments. In the post-test performance, Thinker's and Listener's roles had placed them into a highly cooperative context. As the Thinker, S2 appeared to explain his problem solving progression to the Listener in order to make his solving progression be understood by the Listener. Similarly, the Listener also appeared to paying attention to the Thinker's processes and explanation. The Listener seemed able to engage in the Thinker's processes and to prevent the Thinker from making careless computational mistakes. Overall, in the Thinker-Listener context, they had high levels of engagement with each other's ideas.

During the pre-test, their problem solving progression was mostly dedicated to the interplay between *planning* and *implementing* (see Figure 2) as a result of arguing solution approaches and abandoning each other's working. Little time spent on *reading* and *analysing* the problem itself. However, during the post-test, they spent most time on *reading* and *analysing* the problem before implementing any solution strategies. When they abandoned an implementation strategy, their metacognitive strategy involved returning to analyse the problem before proceeding to other implementation. The Thinker's predominant *analysing* behaviours were rephrasing problem statements in his own words and representing the relationships between the given information. There was evidence of a more systematic presentation in their post-test worksheet. This suggests that thinking aloud and the Listener's attentions to the Thinker's thoughts would likely direct the Thinker to better regulate his progression.

Although the Thinker-Listener context resulted in the pairs' better regulation, higher cooperative level, and clearer path towards a solution, students' misinterpretations of the problem and inappropriate use of the information still led them to use incorrect formula. As Stillman & Galbraith (1998) suggest that metacognitive training focuses on developing students' repertoire of metacognitive strategies over an extended period of time. Thus, one possible factor that affects these inaccuracies is the short period of training and it only occurs once that making it difficult for students to fully internalise the Thinker and Listener's metacognitive habits. Another factor is that the improvement of the final solution might still require more scaffolding and support. This might be provided through a more formal checklist of solution strategies and a structuring of problem solving heuristics. This suggests that a conceptualisation about what strategy the problem might require will assist the student in finding a path, a representation or some other form of clearer strategy through the embedded complexities. As suggested by Kramarski, Mevarech, & Arami (2002), incorporating metacognitive instruction in the pair setting may result in higher effectiveness of problem solving performance. The results suggest that the Thinker-Listener approach can be effectively extended on to a larger scale of pairs with longer periods of time to observe the effectiveness in developing students' metacognitive strategies.

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