The Contribution of Working Memory to Structural an Procedural Errors in Algebraic Word Problem Solving

Introduction

Primary pupils in Singapore are exposed to a variety of problem solving heuristics. Of those listed in the syllabus (CPDD, 2000), the model method is the most popular. Children can use the model method to represent all the information presented in a given problem as a cohesive whole, and not as distinct parts. This cohesive whole called the model is a structure made up of rectangles. These rectangles which represent unknowns are known locally as units. In formal algebra these units could be replaced by letters. The value of the unknown is solved by solving for the value of one unit. The following example (Figure 1) illustrates how the model method can be used to solve complex word problems which are common in the Singapore Primary mathematics syllabus. The procedures on the right are those recommended by the local teacher's guide.

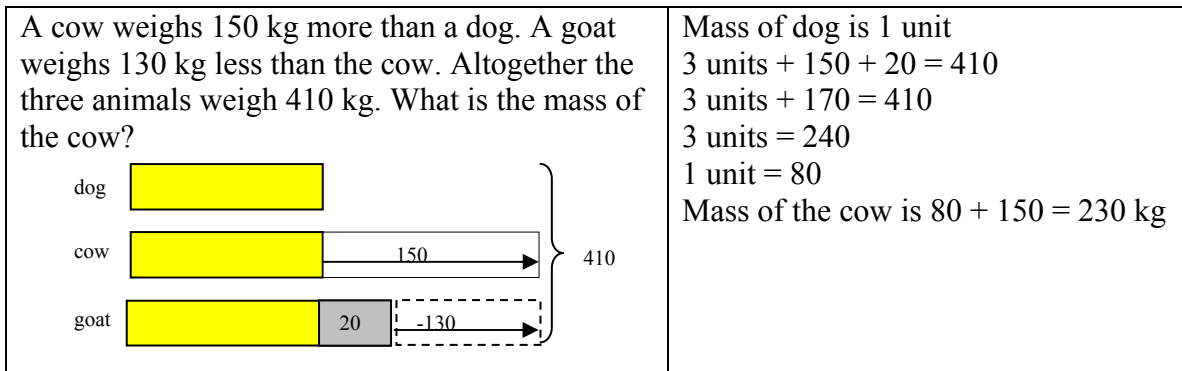| A cow weighs 150 kg more than a dog. A goat weighs 130 kg less than the cow. Altogether the three animals weigh 410 kg. What is the mass of the cow?	Mass of dog is 1 unit 3 units + 150 + 20 = 410 3 units + 170 = 410 3 units = 240 1 unit = 80 Mass of the cow is 80 + 150 = 230 kg

Figure 1

In the above example the mass of the cow is found by constructing a possible model using the dog as the 'generator' (Berdnarz and Janvier, 1996). The mass of the cow is found by relating the additional mass of 150 kg to the value of one unit. However should the mass of the cow be taken as the 'generator', than the problem can be solved by treating the mass of the cow as one unit. Where the mass of the cow was taken as the generator, one could infer that these children were able to relate that the mass of one unit was the answer they were looking for. Then the following equations are the product of this particular perspective.

Mass of cow as 1 unit.
3 units = 410 + 150 + 130
3 units = 690
1 unit = 230
The mass of the cow is 230 kg.

With the model method, children are presented with three modes of representation of a given problem - text mode (T), structural mode (S) and procedural mode (P). To use the model method successfully to solve word problems, children have to be proficient in interpreting the three modes of presentation –text, structural and procedural (TSP). Also children must necessarily be competent at five different levels and we discuss these below.

Level 1 - Reading the Text

To engage with the problem, children must be able to read the text.

Level 2 - Comprehending the Text

Children have to comprehend what they read. Even if they are competent at Level 1, non-comprehension of the text may mean that children could either construct erroneous models or carry out erroneous procedures.

Level 3 - Translating Text into the Structural mode (TS)

Once they have comprehended the information provided in the text, children then transform the information into the structural mode, a very demanding task. It is important to toggle between T and S to confirm that the S mode accurately depicts the T mode.

Level 4 - Processing the Information

Although they may have constructed the correct model children still need to process the information encapsulated in the model into a correct set of procedures. Children who use models to solve a given word problem could be competent at Levels 1 – 3 but may still fail at competency Level 4. This does not mean that such children were not competent up to Level 3. Rather this shows that another level of understanding is needed to transform information captured by the model into the appropriate set of procedures. Also we propose that at this level, children need to toggle ($\leftrightarrow$) back and forth between the information presented in the structural mode with the translated information in the procedural mode ($S\leftrightarrow P$) If children fail to do this, although they may have constructed the correct model, the procedures may not be correct as these procedures may not match the information provided in the structural mode (See Figure 6).

Level 5 – Relating the Answer to the Question

Once they have found the solution, children relate the solution to the question in text mode and check if they have answered the question appropriately.

In summary, because they are taught to use the model method to solve word problems, primary children in Singapore are engaged with processing information presented in three different modes. Children who used the model method successfully to solve word problems could be said to have facility in the three modes – text, structural and

procedural. However it is also possible that children who could solve word problems using the TP route and offer the correct model for the solution even though these children may not need to draw the model. Thus these children are competent in the T, S and P modes although the S mode could be redundant. However regardless of whether they have drawn the model, correct solution to a given problem contains a logical structure.

We offer the following scheme (Figure 2) to explain how children, who are taught to use the model method to solve word problems, engage with the different modes of presenting information.
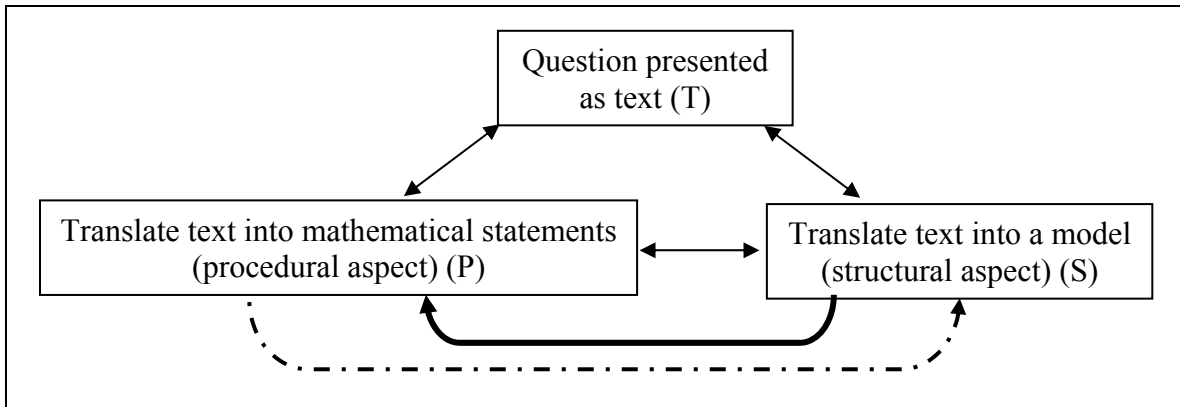

Figure 2: Modes of representation and possible routes to solve word problems when the model method is used.

The Study

In August 2003, one hundred and fifty-one primary five children (77 boys, with an average age of 10.7 years, $SD = .65$) were recruited from five schools located in the western zone of Singapore and participated with parental consent. Most of the children in these schools were drawn from middle to lower middle class areas. In Singapore, Primary 5 classes are streamed according to children's' performances in the languages and mathematics. The sample was roughly representative of the national distribution where 21% were from the top stream and the remainder were from the middle stream. Children from the lowest stream were not included in the sample because they follow a different mathematics syllabus.

These children were given one hour to answer ten questions and they were specifically asked to use the model method to solve the problems. The aim was to ascertain how children use the model method to solve these word problems.

The Test Instrument

Research (e.g. Lewis & Mayer, 1987; Riley, Greeno & Heller, 1983) showed that children faced difficulties working with relational statements. The Singapore primary mathematics syllabus stipulates that primary 5 children should be able to solve up to 3-step word problems involving whole numbers. Children are also required to solve word problems involving fractions. Thus the test items were constructed based on research

findings as well as the expectations of the syllabus with two problems reflecting the primary four syllabus, six primary five and the remaining two, primary six. The latter were included to stretch the more able children. The ten questions, one arithmetic and nine algebraic word problems were of increasing complexity. These items tested children's understanding of relational concepts such as more than, less than, as many as, older than and concepts involving proportional reasoning.

	The ten test items
1	Dunearn Primary school has 280 pupils. Sunshine Primary school has 89 pupils more than Dunearn Primary. Excellent Primary has 62 pupils more than Dunearn Primary. How many children are there altogether?
2	A school bought some mathematics books and four times as many science books. The cost of a mathematics book was $12 while a science book cost $8. Altogether the school spent $528. How many science books did the school buy?
3	A tank of water with 171 litres of water is divided into three containers, A, B and C. Container B has three times as much water as container A. Container C has as much water as container B. How much water is there in container B?
4	A cow weighs 150 kg more than a dog. A goat weighs 130 kg less than the cow. Altogether the three animals weigh 410 kg. What is the mass of the cow?
5	At a sale, Mrs Tan spent $530 on a table, a chair and an iron. The chair cost $60 more than the iron. The table cost $80 more than the chair. How much did the chair cost?
6	Mae Ling bought a new box of cat biscuits. In the first week she gave the cat half the biscuits and three more. In the second week she gave the cat half of the remaining biscuits and 3 more. In the third week, she gave the cat half of the remaining biscuits and 3 more. There was only 1 piece of biscuit left. How many pieces of biscuits were there in the new box?
7	Mr Raman is 45 years older than his son, Muthu. In 6 years time, Muthu will be his father's age. How old is Muthu now?
8	Vincent bought a total of 62 pieces of $3 and $5 stamps. Altogether Vincent spent $254. How many $3 stamps were there?
9	Each week Yah Hui gets $6 more pocket money than Philip. Each week Yah Hui and Philip each spend $19 on books. After some weeks, Yah Hui saved $98 while Philip saved $56. How much pocket money does Philip get each week?
10	During a class game, Peter threw a ball to 4 times as many boys as girls. Mei Lin threw the ball to 5 times as many boys as girls. If Peter and Mei Lin threw the ball to every pupil in the class, how many boys were there in the class?

These questions test concepts of whole numbers and fractions which children were taught from primary three (9+) to early primary five (11+).

Table 1 – Five Test Items

	Item	Success
1	Dunearn Primary school has 280 pupils. Sunshine Primary school has 89 pupils more than Dunearn Primary. Excellent Primary has 62 pupils more than Dunearn Primary. How many pupils are there altogether?	95 (62.9%)
2	At a sale, Mrs Tan spent $530 on a table, a chair and an iron. The chair cost $60 more than the iron. The table cost $80 more than the chair. How much did the chair cost?	67 (44.4%)
3	A cow weighs 150 kg more than a dog. A goat weighs 130 kg less than the cow. Altogether the three animals weigh 410 kg. What is the mass of the cow?	56 (37.1%)
4	A tank of water with 171 litres of water is divided into three containers, A, B and C. Container B has three times as much water as container A. Container C has $\frac{1}{4}$ as much water as container B. How much water is there in container B?	30 (19.9%)
5	A school bought some mathematics books and four times as many science books. The cost of a mathematics book was $12 while a science book cost $8. Altogether the school spent $528.How many science books did the school buy?	22 (14.6%)