Concept Maps as a Potential Assessment Tool in Mathematics

Jin Haiyue
<haiyue.j@gmail.com>
Wong Khoon Yoong
<khoonyoong.wong@nie.edu.sg>
Mathematics and Mathematics Education Academic Group
National Institute of Education
Nanyang Technological University, Singapore

Abstract

Concept maps are a direct method of looking into the organization of an individual's knowledge within a particular domain. Research has shown that concept maps are effective instruments for assessing conceptual understanding in science education. Comparatively little use of concept mapping has been made in mathematics. It is worthwhile to explore the use of concept maps in mathematics, especially in the light of current emphasis on conceptual understanding in mathematics curriculum. This paper provides a brief overview of the use of concept maps as a tool for assessing secondary students' conceptual understanding of a mathematics topic, quadrilateral. The way how concept maps are introduced, constructed, and assessed are described. Students' performance of concept mapping is discussed, together with selected examples of their concept maps. Suggestions for future implication are also provided.

1 Introduction

Cognitive psychologists use the construct *structural knowledge* to describe the ways that human beings construct and store knowledge (Jonassen, Beissner & Yacci, 1993). Specifically, structural knowledge portrays how concepts within a domain are interrelated (Diekhoff, 1983) and the interrelations substantiate the meaning of each concept. Researchers have taken different representational approaches, such as word association, tree construction, and relational diagram, to capture the organizational properties of structural knowledge (e.g., Goldsmith, Johnson & Acton, 1991; Novak & Gowin, 1984; White & Ginstone, 1992). Among these approaches, *concept map,* a graphic representation of structural knowledge, has been proposed as a more direct approach to capture the interrelatedness among concepts in a domain (Ruiz-Primo, 2000; Ruiz-Primo & Shavelson, 1996).

Concept map was developed by Novak and his colleagues in the 1970s on the basis of Ausubel's (1968) theory of meaningful learning (Novak & Musonda, 1991). It is generally defined as a two-dimensional map using nodes representing concepts and labeled lines denoting the relations between pairs of nodes (Baroody & Bartels, 2000; Novak & Gowin, 1984; Ruiz-Primo & Shavelson, 1996). However, this definition has been used differently by different researchers and with different interests. For example, Novak and Gowin (1984) required that maps be hierarchical, while researchers from the semantic-network tradition prefer “spider maps” (e.g., Harnisch, Sato, Zheng, Yamagi & Connell, 1994). Besides, Nesbit and Adesope (2005) claimed that the lines in a concept map may be labeled or unlabeled, directional or non-directional. In this study, the term *concept map* refers only to graphic representations using nodes representing concepts and labeled arrows denoting connections among nodes. These nodes can be mathematical concepts, examples and non-examples of the concepts, diagrams, symbols, and formulas; and the labels, also named *linking phrases*, can be verbs or adjective phases. The linked two or more concepts, together with the labeled lines, form a meaningful statement. This statement is named a *proposition*. Although, in light of Skemp’s (1986) *Schema Theory*, a hierarchical-structured concept map might be a better representation of the essential interrelatedness among mathematical concepts, young learners may have difficulty distinguishing or expressing the hierarchy of abstract mathematical concepts (e.g., Novak & Gowin, 1984; Schau & Mattern, 1997). Hence, this study encourages, but does not demand, the hierarchical structure of students-constructed concept maps.

In general, concept map can serve as the “window into the mind” (Shavelson, Ruiz-Primo & Wiley, 2005, p.1). The underlying idea is that the information stored in an individual’s memory can be externalized in the form of concept maps. Once the internal knowledge is represented externally, it can be assessed by others. Over the last three decades, concept map has been used extensively as an assessment tool for assessing conceptual understanding, especially in science education. The justification for this use is based on the idea that to understand a subject domain is to build rich relations among important concepts in the domain (Novak, 1998; Novak & Gowin, 1984) and concept maps can be used to capture the important aspects of conceptual understanding, specifically, the structure and the interrelatedness among the concepts.

In mathematics, educational studies have firmly established the importance of conceptual understanding in the knowledge and activity of proficient people who have the ability to use knowledge flexibly, and apply what is learned in different settings appropriately (Bransford, Brown & Cocking, 1999; NCTM, 2000). This importance has recently received increasing attention from educators, researchers, and curriculum designers (Afamasaga-Fuata’I, 2008; Edwards & Fraser, 1983; Kilpatrick, Swafford & Findell, 2001; Ministry of Education (MOE), Singapore, 2007; NCTM, 2000). Much effort has been devoted to measure levels of conceptual understanding (e.g., Niemi, 2001; Webb & Romberg, 1992; White & Gunstone, 1992; Wilson, 1992). However, research has found that commonly used achievement measurements, such as multiple-choice questions and short answer questions, provide at best only indirect and highly limited information on students’ conceptual understanding in mathematics (Niemi, 2001). Although concept map has its usefulness in science education, its use as an assessment in mathematics has not been well examined. It is worthwhile to further explore this use in mathematics, especially in the light of current advocate for conceptual understanding. This paper describes some initial experience of using concept map as an assessment of secondary students’ conceptual understanding of quadrilaterals.

2 Methods and Procedures

This exploration is a preliminary part of a PhD study. It is not designed to produce conclusive results, but to provide a better understanding of the procedures of using concept maps as a tool for assessing secondary students' mathematical conceptual understanding in Singapore, where almost no study about concept map in mathematics has been reported. The experience reported in this paper focuses on the training in concept mapping and the interpretations of students' concept maps.

2.1 Participants

Eight Secondary Express stream students were randomly selected from a Government school in Singapore. Two of them (one female student A and one male student B) were Secondary 1 (S1) students and the other six (male, student C, D, E, F, G, and H) were Secondary 2 (S2) students. In addition to these eight students, students I and J were specifically selected. Student I, a S1 male student, had some difficulty in learning mathematics and knew little about concept map before this exploration; while Student J, a Secondary 3 (S3) female student, was a top student in her class and she mentioned she had used concept map for her own study since Primary 4 when she was first introduced to concept map by her science teacher.

2.2 Training in Concept Mapping

Students' ability with concept mapping is an important factor that would influence their performance in the mapping products (concept maps). Studies into concept map as an assessment technique usually start with an introduction on what a concept map is and training on how to construct a concept map (e.g., Ruiz-Primo, Schultz, Li & Shavelson, 2001; Williams, 1994). The process of drawing a concept map is named *concept mapping* accordingly. Training on concept mapping is necessary, just as to conduct a computer-based test teachers need to make sure students first know how to operate the software for the test.

There are many different ways of using concept maps; thus, the training method may different accordingly. For example, the training for fill-in-a-map and construct-a-map technique is different (Ruiz-Primo, Schultz, Li, and Shavelson, 2001). In the fill-in-a-map task, students are provided with the skeleton of a concept map where some of the concepts and/or linking phrases have been left out. Students only need to know how to read the information in the skeleton and fill in the blanks. However, the construct-a-map technique requires students to construct a concept map with only a concept list provided. Students need to know how to build connections, add linking phrases, and structure the propositions proposed. More training efforts are needed for this technique. Since researchers have found construct-a-map can better discriminate different degrees of understanding (Liu & Hinchey, 1996; Ruiz-Primo, Schultz, Li & Shavelson, 2001), it is considered in this study as a better choice for assessing students' conceptual understanding.

There is no consistent answer to how much effort should be given to the training section. The training programme differs among studies for different graders. Results across many studies using concept mapping technique suggested that students can be trained to construct concept maps in a short period of time with limited practice (e.g., Freeman, 2004; Ruiz-Primo, 2004; Ruiz-Primo, Schultz, Li, & Shavelson, 2001); however, other researchers claim that concept mapping imposes a high cognitive demand on the students by requiring them to identify important concepts, relationships, and structure within a specified domain of knowledge, and thus it may be difficult for most students to construct them at the very beginning of use (e.g., Novak & Gowin, 1984; Schau & Mattern, 1997). These two positions seem somewhat contrary that how a high-demand task can be easily acquired with only limited training and practice. To better understand this contradiction, this study divided the participants into small groups and expended different efforts in the training for each group, from brief training to elaborated one.

2.3 Students' performance in the training section

All the participants in this study, except Student J, were not familiar with concept maps. Even though some of the students mentioned that they have heard about it or known some similar maps, e.g., mind map, they admitted that they did not know how to use and construct concept maps.

2.3.1 *Group 1: Student A and B*

For the first group, Student A and B, the researcher attempted to introduce concept map in a simple way. This training took 10 minutes and covered Novak's definition of concept map, focusing on the three features, i.e., nodes representing concepts, arrowed links showing existing connections, and linking phrases describing the relationships between linked concepts. After that the researcher used an example on *whole numbers* to show them how she constructed a concept map. This took another 5 minutes. The steps to construct a concept map included going through all the concepts in the list, arranging them on a piece of blank paper, making connections, and adding linking phrases. Later on, the two students were given a piece of paper with five concepts written on stickers. These concepts were related to *triangles*. The two students were asked to follow the researcher's mapping steps and construct a concept map individually. Their maps met Ruiz-Primo, Schultz, Li, and Shavelson's (2001) three standards for effective training, i.e., use of concepts provided in the list, use of labeled links, and valid propositions. However, most of the students' linking phrases were very general, and when asked, both of them could add more links and more detailed information to the linking phrases in their maps. This phenomenon indicated that this 10-minute simple training was not sufficient for the two students to represent their understanding in the form of a concept map. They knew more than what was shown in their concept maps.

2.3.2 *Group 2: Student C and D*

Based on the first experience, to train the second group, Student C and D, the researcher emphasized the accuracy of linking phrases, and encouraged the students to add as many connections and details as they could. This training took around 15 minutes to explain what a concept map is and how to construct a concept map, and another 15 minutes for students' practice. In the practice section, the students were provided with 11 concepts related to triangles and they were asked to construct a concept map collaboratively using the concepts. These two students had different opinions about the organization of the concepts. Both of them wanted to change the positions of some concepts after their partner's arrangement. After they had fixed the proposition of each concept, they took turns to add connections. With the researcher's reminder, they were able to provide detailed linking phrases. However, their cooperation moved slowly. The researcher had to stop them after 15 minutes. Within the 15 minutes, they only built 7 connections among the 11 concepts.

2.3.3 *Group 3: Student E, F, G, and H*

For the third group, the researcher planned a more detailed training. The training took 50 minutes. It covered what a concept map is, how to construct a concept map, examples of good and poor concept map, and students' practice and discussion. Compared with the training for Group 2, an obvious difference was that this training included three map examples: N_1, N_2, and N_3, on the same topic *numbers*. N_1 and N_2 were good examples with different structures, while N_3 was a poorly constructed map with inappropriate linking phrases and few connections. The researcher explained why N_3 was considered a poor map and asked for the students' suggestions to refine it. After a short discussion, the students were given the same concept list on *triangles* as Group 2. Similarly, they were asked to construct one concept map collaboratively. Without the researcher's help, this group could remind each other about the accuracy of the linking phrases. In the mapping process, they could adjust the structure of the propositions created and propose more connections. Their cooperation appeared to be more successful than Group 2's.

2.3.4 *Student I*

The researcher worked with Student I individually. He was trained in the same way as Group 3, but in the practice phase, he worked individually with 7 concepts related to numbers. In the mapping process, he tried to copy the structure and linking phrases from the researcher's examples. Given 10 minutes, he built only three links by himself, two of which were wrong. His performance indicated that he did not understand the purpose and requirements of concept mapping. The training was far from adequate for this boy to acquire the mapping ideas.

2.3.5 *Student J*

Student J used "concept map" very often for her own study. However, she mentioned that the nodes in her "concept maps" were not necessarily to be concepts; they could be phrases or even sentences; and the links were not necessarily labeled. Based on her description, the training focused on making clear the differences between the concept map required in this study and her "concept map." After that, the researcher briefly explained the same construction procedure described for Group 3. This training took 10 minutes. It was then followed by Student J's working on a concept list with 11 concepts on *triangles*. During her mapping practice, the researcher asked her two questions: (1) Here, you make the proposition "right-angled triangle has acute angle," could you tell me how many acute angles does a right-angled triangle have? (2) Is there any relationship between isosceles triangle and acute-angled triangle? Could you make a link between them? The first question reminded the student to express the linking phrases as accurately as she could, while the second question reminded her to add more possible links. Promoted by these two questions, she was able to add more connections to her map and tried to label them with more accurate phrases.

2.4 Summary for Training Strategy

The training experience with the students above showed that:

(1) an introduction about only the attributes of a concept map and the procedures for constructing a concept map is not sufficient for preparing secondary school students with concept mapping skills, with which students can represent their knowledge in concept maps;

(2) examples, practice, and discussion may contribute to students' understanding about the use of concept mapping tasks;

(3) training should put an emphasis on the accuracy and completion of expressing ideas into propositions;

(4) collaborative mapping might hinder students' mapping progress since different students may have different ideas about creating propositions and structuring concepts; but it might also contribute to students' understanding since they could be inspired by the discussion and benefit from the ideas given by their partners; and

(5) with the same training programme provided, students' proficiency with concept mapping may be different; thus, time must be allowed for the training so as not to challenge the fairness of the assessment.

Based on the above consideration, a four-step training programme introduced by Ruiz-Primo, Schultz, Li, and Shavelson (2001) is revised and adapted for secondary school students on construct-a-map as following:

(a) Introduction: introduce what a concept map is, what it is used for, and what its attributes are; besides, introduction should also specify the requirements of concept mapping tasks, particularly, paying attention to the accuracy of linking phrases and the completion of expressing ideas into propositions;

(b) Construction procedure: provide examples to display the steps for constructing a concept map, including identifying the relations between pairs of concepts, creating propositions, structuring the propositions created, and recognizing good and poor map examples;

(c) Practice: students practice the construction procedures either collaboratively or individually with a given concept list (outside the domain to be assessed); and

(d) Discussion: after the mapping practice, students share their maps together. They may make comments on their classmates' concept maps and propose questions for discussion.

After the training, the seven S2 students C, D, E, F, G, H, and J were given a concept list on quadrilaterals and were asked to construct a concept map individually. The three S1 students A, B, and I were not included because the topic *quadrilateral* is taught in the second semester of S1 in Singapore and these three S1 students had not learnt it yet.

2.5 Interpretation of Students' Concept Maps

Once a concept map has been completed, interpreting the information in the map becomes an important task. This section introduces the methods for analyzing concept maps, followed by interpretations of some students' map examples.

2.5.1 *Methods for Interpretation*

Two methods are normally used for interpreting the information in a concept map: quantitative scoring and qualitative description.

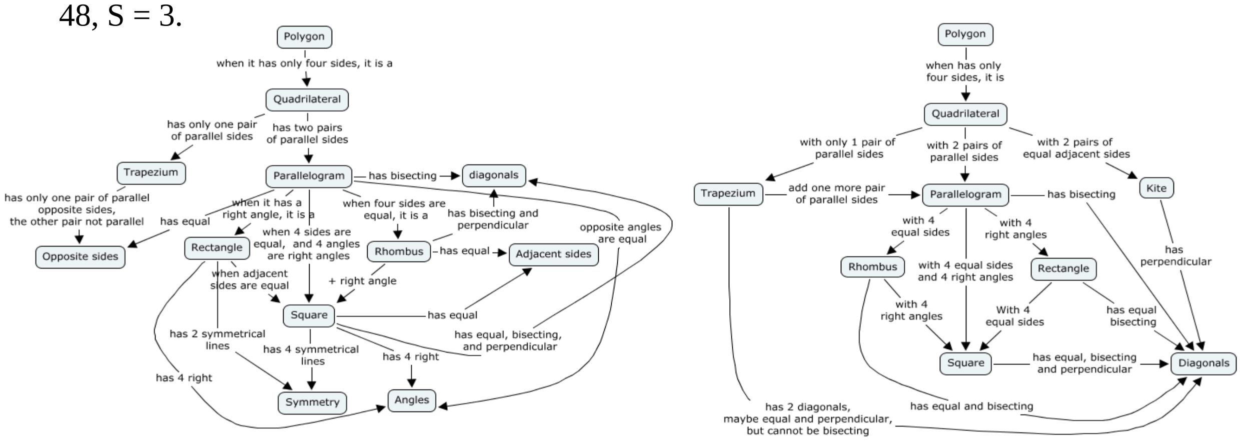
Quantitative scoring builds a scoring rubric and scores the maps on a number of features, e.g., valid nodes, meaningful propositions, and structural complexity. The aggregation of the scoring elements creates an overall score for each concept map. Instead of evaluating the quality of concept map as numbers, qualitative description intends to provide in-depth information of students' concept maps so as to bring benefits to teaching and learning (Kinchin & Hay, 2000). Both qualitative description and the scores are useful for drawing inferences. On one hand, in the context of large-scale assessment, statistical results are ideal if they could differentiate various levels of understanding; on the other hand, in classroom assessment, detailed information about the interrelatedness among concepts is favored since it could help teachers with their lesson planning and decision making to improve teaching.

A scoring rubric is provided in Table 1. This scoring method was used for the quantitative interpretation of the students' concept maps, by comparing with two maps constructed by the researchers. The researcher-constructed maps were called criterion maps in this exploration. The first criterion map was constructed corresponding to a concept list with 12 concepts, while the second criterion map was constructed for another concept list with 9 concepts. Student J was given the concept list with 9 concepts and other students were provided the 12-concept list. These criterion maps were supposed to cover most of the possible propositions among the given concepts, using accurate linking phrases. They would also be scored according to the rubric. The scores of the students' concept maps were then compared with the corresponding criterion map scores. The final grade for a student's map included three parts: (1) the *concept* score according to the rubric (S_c), (2) the proportion of the student's *proposition* score to the *proposition* score of the criterion map using the same concept list (S_p), and (3) the *structure* score (S_s). In a criterion map, the *concept* score and *structure* score equaled to 0 and 3 respectively, but the *proposition* score would differ when different concept list was used. The S_p provided the meaningfulness for comparing among students' concept maps constructed under different concept lists. It compares how much domain knowledge the student has with the corresponding criterion map. On this basis, most students would receive a S_p less than 100%; but it is still possible for some students to earn more than 100% since they might have done better in the concept maps than the criterion maps (Ruiz-Primo, Schultz, Li & Shavelson, 2001), for example, providing creative but valid propositions.

Table 1
Concept Map Scoring Rubric--adapt to the concept mapping tasks with only a concept list provided

Criteria	Performance	Value	Number	Score
Concept (node)	Add one concept to the concept list. This added concept is a domain specific concept related to the concepts in the concept list.	2	*a*	*C=* *2a+b-2c*
	Add one concept to the concept list. This added concept is an example or non-example of one concept in the concept list.	1	*b*	
	Add one concept to the concept list. But the added concept is not a domain specific concept, e.g., mathematics. Or The concept map has exactly the same concepts from the given concept list.	0	/	
	A concept from the concept list is not included in the concept map. Or a concept is isolated from other concepts in the concept map, no connection has been made.	-2	*c*	
Propositions	The proposition is valid and provides a complete description of the relationship between the related concepts (in light of the intended curriculum).	3	*d*	*P=* *3d+2e+f-2g*
	The proposition is valid but the relationship described between the linked concepts is incomplete (in light of the intended curriculum).	2	*e*	
	There is a relationship between the concepts related, but the label is only a common sense or no label is provided.	1	*f*	
	There is no relationship between the concepts connected.	0	/	
	The proposition indicates misconception.	-2	*g*	
Structure	More than 75% concepts in the map are placed hierarchically as in the criterion map.	3	*h*	*S=* *3h+2i+j*
	More than 50% but less than 75% concepts in the map are hierarchically arranged as in the criterion map.	2	*i*	
	Less than 50% concepts in the map are hierarchically arranged as in the criterion map.	1	*j*	
	The entire map is messy and difficult to read.	0	/	

The two criterion maps were shown in Figure 1. According to the scoring rubric in Table 1, they were scored: (a) C = 0, P = 3 × 20 = 60, S = 3; (b) C = 0, P = 3 × 16 = 48, S = 3.



(a) with 12 concepts provided; (b) with 9 concepts provided;

Figure 1. Criterion maps on quadrilateral.

2.5.2 *Interpretation of Students' Map Examples*

The students were given enough time to construct their maps and could submit to the researcher when they felt satisfied with their maps. The researcher provided a concept list with the 12 concepts used in Figure 1(a): *polygon, quadrilateral, parallelogram*, *trapezium*, *rhombus*, *rectangle*, *square*, *diagonals*, *symmetry, adjacent sides*, *opposite sides*, and *angles*, and another concept list with the 9 concepts used in Figure 1(b): *polygon, quadrilateral, parallelogram*, *trapezium*, *rhombus*, *kite*, *rectangle*, *square*, and *diagonals*. Students C, D, E, F, G, and H were asked to construct a map using the 12-concept list, while student J used the 9-concept list.

Student C's and D's maps were given in Figure 2. Each student spent 25 minutes constructing the concept maps shown below. These maps included all the 12 concepts given in the concept list; no extra concept had been added; thus, the *concept* scores were 0. The students tried to arrange the concepts hierarchically. For example, they put the general concepts *Quadrilateral* and *Polygon* on the top of their maps, categorized the objects *Rectangle, Square, Parallelogram, Rhombus*, and *Trapezium* together in a lower but same level. However, the hierarchy of the properties, e.g., sides, angles, and diagonals, was not clearly distinguished; thus, according to the rubric and corresponding criterion map, the S_s of the two maps were valued 1. The maps indicated that, after the training, the students had captured the attributes of a concept map, particularly, connections, linking phrases, and hierarchy in general. However, the links and linking phrases in these two maps only displayed common sense, for example, "rectangle[s] have opposite sides" and "polygon has angles". No detailed information had been provided. When asked, both of them could describe more about the relations among *Rectangle, Square, Parallelogram, Rhombus*, and *Trapezium*. For example, part of the audio-taped discourse was transcribed as bellow:

R (the researcher): What do you mean by this connection "rectangle is similar to square"?

C (student C): Well, they are similar because…they all have right angles.

R: Anything else? Could tell me more details about why these two are similar?

C: Okay…both of them have four sides, they are quadrilaterals. Square has four equal sides…rectangle has two pairs of equal sides…

R: Here, you said "quadrilateral can be split into 5 parts: *Rectangle, Square, Parallelogram, Rhombus*, and *Trapezium*." Could you explain a bit more about this proposition?

D (Student D): These five concepts are all quadrilaterals, but they are different from each other.

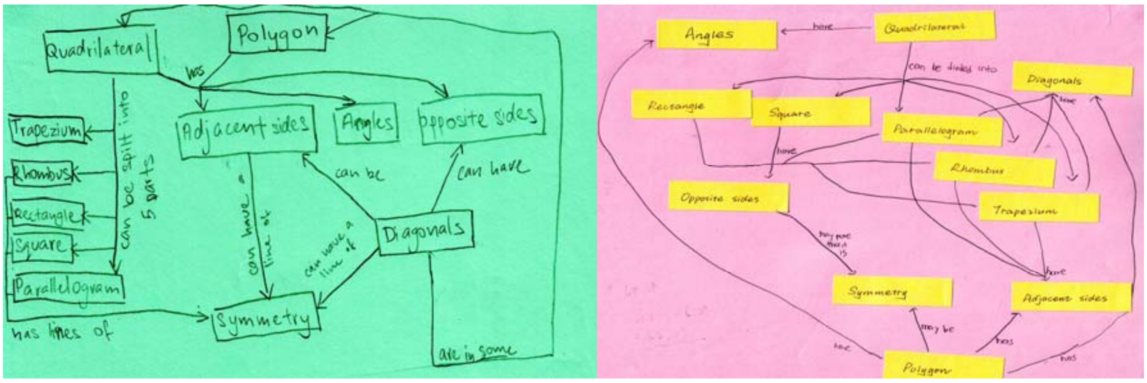
R: Could you give me an example?

D: Sure. See, this is a trapezium (he drew a trapezium)… this is a rhombus (he drew a rhombus beside the trapezium)… they are different.

R: So the difference is…?

D: This one (the trapezium) has one pair of parallel lines, but this one (the rhombus) has two.

The above discourse indicated that the students' concept maps did not provide enough information about what the students actually knew. The propositions constructed were very general; thus, their S_p scores were relatively low. The analysis showed that the two students were not sure about what kind of information was wanted from their concept maps. They treated concept map as a simple drawing task and did not include as much as they knew to the maps. Therefore, training needs to include the explanation about the use and purpose of a concept map, so that students will be clearer about what they are expected to do.



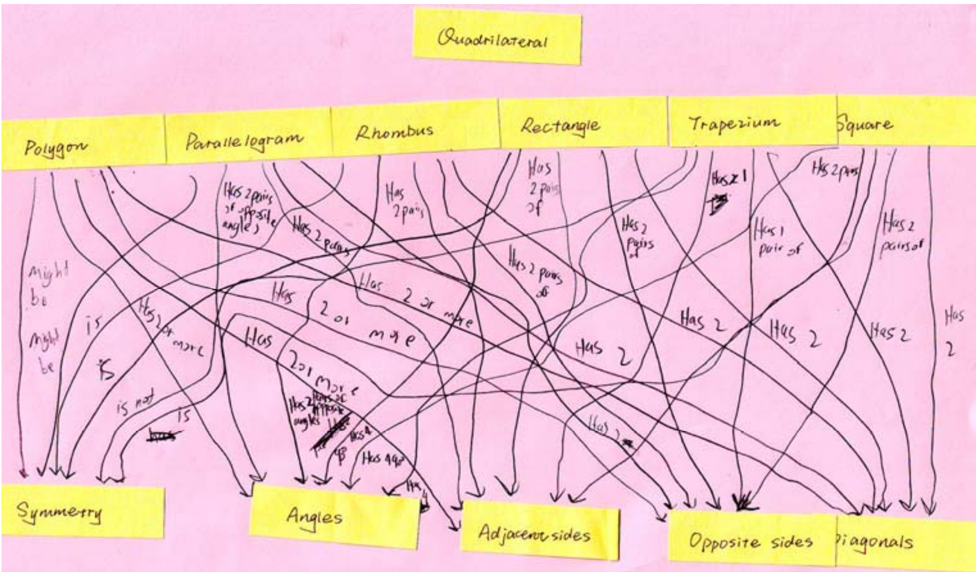
(1) Student C's Map S_c= 0, S_p = 19/60 = 32%, S_s = 1;

(2) Student D's Map S_c= 0, S_p = 16/60 = 27%, S_s = 1;

Figure 2. Student C's and D's concept maps on quadrilaterals.

Students' maps from Group 3 were given in Figure 3, 4, 5, and 6 respectively. Student E spent less than 15 minutes on his mapping; Student F and G spent around 20 minutes, while student E spent 40 minutes.

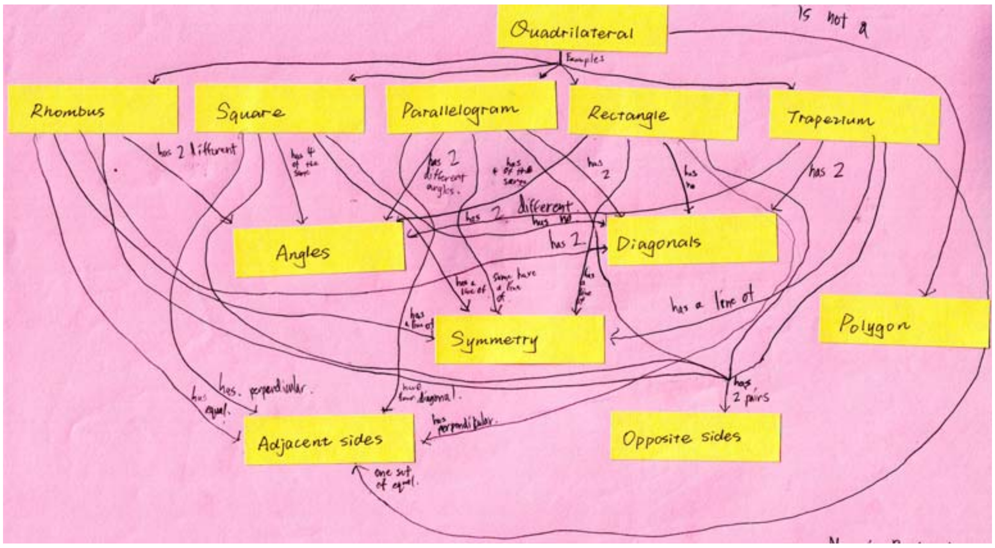
More information could be drawn from these maps than those constructed by Group 2 students.



S_c= -2, S_p = 27/60 = 45%, S_s = 1;

Figure 3. Student E's map on quadrilaterals.

In Student E's map, the 12 concepts were simply divided into three hierarchy levels, with *Quadrilateral* on the top and *symmetry, angles, opposite sides, adjacent* *sides*, and *diagonal* on the bottom. He tried to add as many connections as possible between the concepts in the second and third levels and then add linking phrases to the connections built, but left the concept *quadrilateral* alone on the top. He seemed to draw all possible links between the objects in the middle level and the properties on the bottom. Most of the propositions were quite general and scored 1 point according to the rubric. For example, "parallelogram has two opposite sides" and "trapezium has 4 angles". Although student E could tell the definition of parallelogram well, he did not go deep into the relations and attempt to provide more information. Nevertheless, misconceptions could easily be found from his map. For example, student E proposed the propositions "trapezium is not symmetry" and "trapezium has one pair of opposite sides". This first proposition was inaccurate since an isosceles trapezium is symmetry; and the second proposition should be "trapezium has one pair of parallel opposite sides" or "trapezium has two pairs of opposite sides." He confused parallel sides with opposite sides.

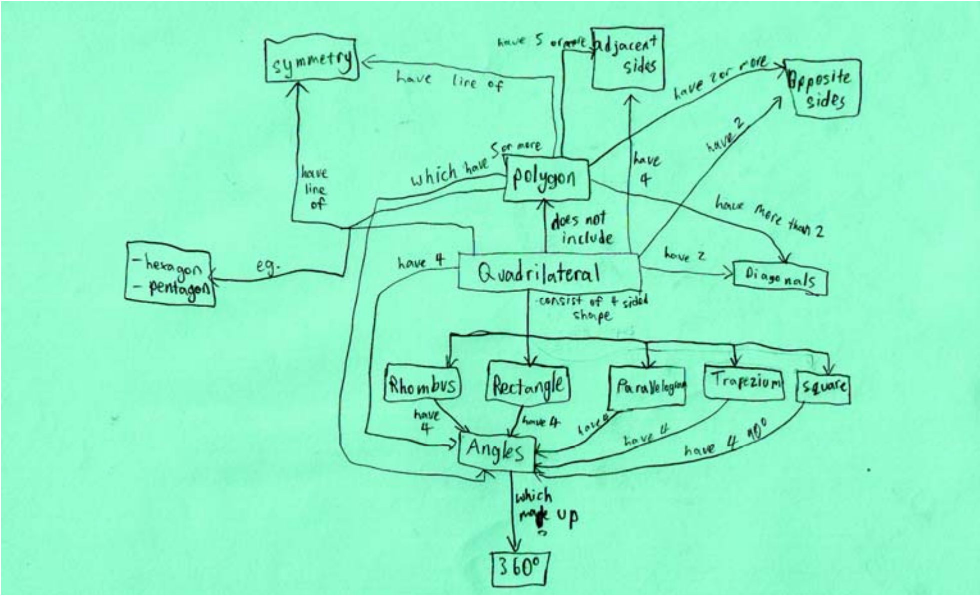


$S_c = 0$, $S_p = 30/60 = 50\%$, $S_s = 1$

Figure 4. Student F's map on quadrilaterals.

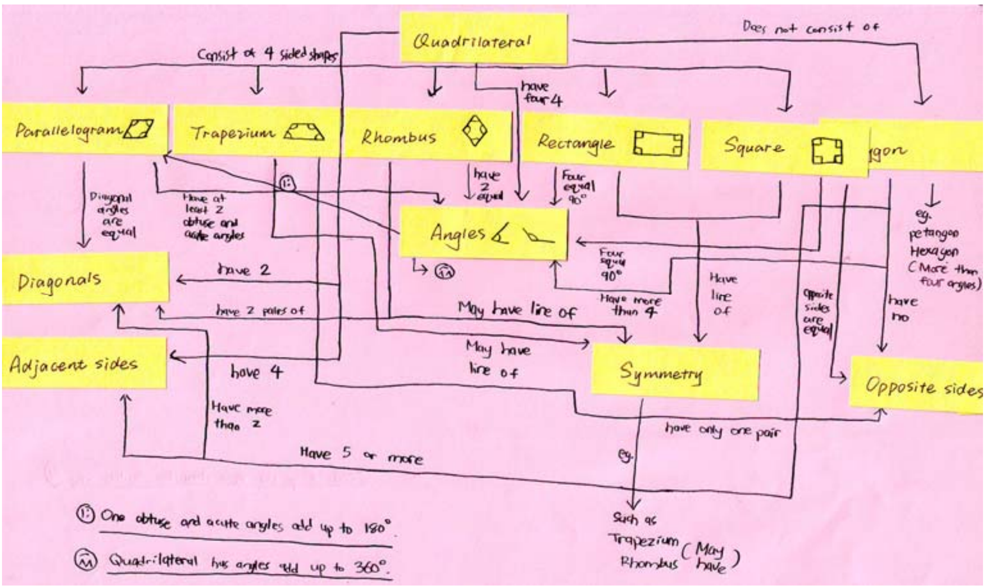
Students F's map was similar to Student E's. There were no links between *trapezium, rhombus, parallelogram, square*, and *rectangle*. Most connections were made between the quadrilaterals and their properties, such as symmetry, angles, sides, and diagonals. Compared with E's map, F included more detailed linking phrases. For example, it added the propositions "rhombus has equal adjacent sides" and "square has perpendicular adjacent sides." As more information was included, more misconceptions had been revealed. For example, student F mentioned the following two propositions "quadrilateral is not a polygon" and "trapezium [has] one set of equal adjacent sides." It seemed the student had confused the property of special cases with general concepts. Quadrilateral is a special polygon with four sides and not all trapeziums have equal adjacent sides.

Student G's map is shown in Figure 5. He included two more nodes, examples for polygon and angle sum of quadrilaterals, in addition to those given in the concept list. This student had captured the attributes of constructing a concept map and tried to include what he knew in the map, though some of the propositions, such as "rhombus has 4 angles", were quite general. Misconceptions were shown in the map, such as "polygon has 5 or more angles" and "quadrilateral have line of symmetry." Once students were trained to represent their understanding in concept maps, more information about their conceptual understanding, especially misconceptions could be drawn directly from the propositions.



S_c= 4, S_p = 19/60 = 32%, S_s = 1

Figure 5. Student G's map on quadrilaterals.



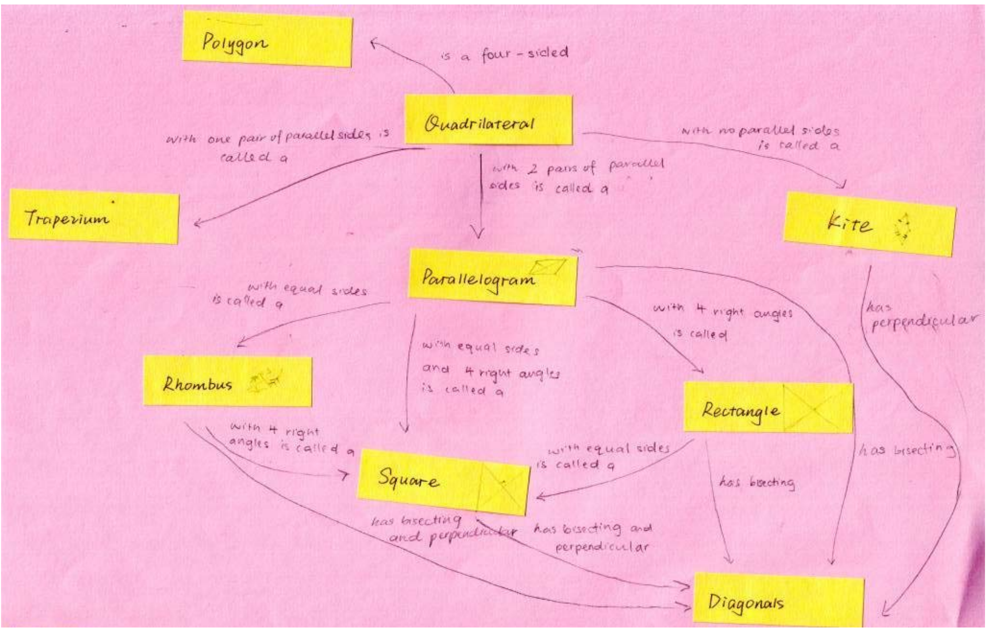
S_c= 9, S_p = 33/60 = 55%, S_s = 1

Figure 6. Student H's map on quadrilaterals.

Student H spent longer time constructing his map than the others. He carefully checked the possible relations among each pairs of concepts. In between, he found that there was no more space for him to add two more propositions neatly. The researcher suggested him to mark the links and write his explanations below the map. He did. From the map, it could be found that student H had confused opposite sides with parallel sides and thought polygon has more than four sides. Besides, he did not understand the concept *symmetry*. These misconceptions are of value for teachers to diagnose student's understanding and provide instructions accordingly.

The scores for the above four concept maps differentiated the quality of the students' maps. Student H earned the highest score for S_p while G earned the lowest score compared with his classmates in the same group. The score S_c indicated that F left one given concept alone in the map, while G and H included more concepts to their maps. The *structure* score S_s showed that, compared with the criterion map, all the four student-constructed maps were only partly hierarchized.

The concept list provided to Student J was different from that for Student C, D, E, F, G, and H. She used the 9-concept list. At the beginning of her mapping, she tried to arrange *trapezium*, *rhombus*, *parallelogram*, *kite*, *square*, and *rectangle* at a same level as Student E and Student F did. However, in the mapping process, she realized that there was also a hierarchy among the six concepts. She erased all the connections and redrew the map. Her final map was shown in Figure 7. Some propositions were not rigorous, for example, "quadrilateral with no parallel sides is called a kite." "With no parallel lines" is a necessary but insufficient condition of kite. Thus, this proposition was scored 2 according to the rubric. Nevertheless, as a whole, this map indicated a very comprehensive understanding of the 9 concepts. Altogether, the first draft and this final map took Student J 15 minutes to complete.



$S_c = 0$, $S_p = 36/48 = 75\%$, $S_s = 3$

Figure 7. Student J's concept map on quadrilateral.

3 Conclusions and Discussion

This study explored the procedure of using concept map as an assessment tool in mathematics, focusing on the training methods and the interpretations of the concept maps.

In the training section, the effects of different training methods on students mapping performance were discussed. The analysis showed that, for secondary school students, a simple training with limited practice on concept mapping has not sufficiently prepared them with mapping skills. With the same training programme provided, students might acquire different proficiency with concept mapping. Thus, comprehensive training must be prepared so as to produce reliable and fair conclusions of assessment.

The interpretations on students' concept maps have shown that concept maps may be a valuable source of information about students' conceptual understanding in mathematics. Students' misconceptions can be captured from their concept maps. When students included more detailed information in their concept maps, more insights would be inferred about their conceptual understanding. The scoring method, with separate scores for *concept*, *proposition,* and *structure,* reduces the risk of Kinchin and Hay's (2000) argument that "the aggregation of scoring elements creates a blurring of what the overall score actually reveals" (p.46). This preliminary study had shown that the rubric could distinguish among different levels of mapping performance. However, since the students in this exploration were trained differently and no evaluation of their mapping proficiency was administered, it is difficult to tell whether the levels of mapping performance were due to students' differences of mapping proficiency or conceptual understanding. The next phase of the PhD research by the first author will take students' mapping proficiency into account before using concept map as an assessment of conceptual understanding in mathematics.

References

Afamasaga-Fuata'I, K. (2008). Students' conceptual understanding and critical thinking: A case for concept maps and vee-diagrams in mathematics problem solving. *The Australian Mathematics Teacher, 64*(2), 8-17.

Ausubel, D. P. (1968). *Educational psychology: A cognitive view.* New York: Holt, Rinehart and Stratton.

Baroody, A.J., & Bartels, B.H. (2000). Using concept maps to link mathematical ideas. *Mathematics Teachers in the Middle School, 5*(9), 604-409.

Bransford, J.D., Brawnn A.L., & Cocking, R. R (Eds.) (1999). *How people learn: Brain, mind, experience, and school*. Washington, D.C.: National Academy Press.

Diekhoff, G.M. (1983). Relationship judgments in the evaluation of structural understanding. *Journal of Educational Psychology*, 75, 227-233.

Edwards, J. & Fraser, K. (1983). Concept maps as reflectors of conceptual understanding. *Research in Science Education, 13,* 19-26.

Freeman, L. A. (2004). The power and benefits of concept mapping: Measuring use, usefulness, ease of use, and satisfaction. In A.J. Canas, J.D. Novak & F.M. Gonzalez (Eds.), *Concept maps: Theory, methodology, technology.* Proceedings of the First International Conference on Concept Mapping. Retrieved on March 25, 2008, from http://cmc.ihmc.us/CMC2004Programa.html

Goldsmith, T.E., Johnson, P.J., & Acton, W.H. (1991). Assessing structural knowledge. *Journal of Educational Psychology, 83,* 88-96.

Harnisch, D. L., Sato, T., Zheng, P., Yamagi, S., & Connell, M. (1994, April). *Concept mapping approach and its applications in instruction and assessment.* Paper presented at the meeting of the American Educational Research Association, New Orleans, LA.

Jonassen, D.H., Beissner, K., & Yacci, M. (1993). *Structural knowledge. Techniques for representing, conveying, and acquiring structural knowledge*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Kilpatrick, J., Swafford, J., & Findell, B. (Ed.) (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academic Press.

Kinchin, I. M., & Hay, D.B. (2000). How a qualitative approach to concept map analysis can be used to aid learning by illustrating patterns of conceptual development. *Educational Research, 42*(1), 43-57.

Liu, X., & Hinchey, M. (1996). The internal consistency of a concept mapping scoring scheme and its effect on prediction validity. *International Journal of Science Education, 18,* 921-937.

Ministry of Education. (2007). *A guide to teaching and learning of O-level mathematics.* Singapore: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Nesbit, J.C., & Adesope, O. (2005). Dynamic concept maps. In P. Kommers & G. Richards (Eds.), *Proceedings of World Conference on Educational Multimedia, Hypermedia and Telecommunications 2005* (pp. 4323-4329). Chesapeake, VA: Association for the Advancement of Computing in Education.

Niemi, D. (2001). Assessing conceptual understanding in mathematics: Representations, problem solutions, justifications, and explanations. *The Journal of Educational Research, 89*(6), 351-363.

Novak, J.D. (1998). *Learning, creating, and using knowledge: Concept maps as facilitative tools in schools and corporations.* Mahwah, NJ: Lawrence Erlbaum.

Novak, J.D., & Gowin, D. B. (1984). *Learning how to learn.* London: Cambridge University Press.

Novak, J.D., & Musonda, D. (1991). A twelve-year longitudinal study of science concept learning. *American Educational Research Journal*, *2,* 117-153.

Ruiz-Primo, M. A. (2000). On the use of concept maps as an assessment tool in science: What we have learned so Far. *Revista Electrónica de Investigación Educativa*, *2* (1). Retrieved Oct 20, 2008 from: http://redie.uabc.mx/vol2no1/contents-ruizpri.html

Ruiz-Primo, M.A. (2004). Examining concept maps as assessment tool. In A.J. Canas, J.D. Novak & F.M. Gonzalez (Eds.), *Concept maps: Theory, methodology, technology.* Proceedings of the First Conference on Concept Mapping. Retrieved March 25, 2008, from http://cmc.ihmc.us/CMC2004Programa.html

Ruiz-Primo, M.A., Schultz, S.E., Li, M., & Shavelson, R.J. (2001). Comparison of the reliability and validity of scoring from two concept-mapping techniques. *Journal of Research in Science Teaching, 3*(2), 260-278.

Ruiz-Primo, M.A, & Shavelson, R.J. (1996). Problems and issues in the use of concept maps in science assessment. *Journal of Research in Science Teaching, 33*(6), 569-600.

Schau. C., & Mattern, N. (1997). Use of map techniques in teaching statistics courses. *The American Statistician, 51*(2), 171-175.

Shavelson, R.J., Ruiz-Primo, M.A., & Wiley, E.D. (2005). Windows into the mind. *Higher Education, 49,* 413-430.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Arithmetic Teacher, 26*(3), 9-15.

Skemp, R. (1986). *The psychology of learning mathematics* (2nd ed.). Harmondsworth: Penguin.

Webb, N., & Romberg, T. A. (1992). Implications of the NCTM Standards for mathematics assessment. In T. AA. Romberg (Ed.), *Mathematics assessment and evaluation: Imperatives for mathematics educators* (pp. 37-60). Albany, NY: State University of New York Press.

White, R.T., & Gunstone, R. (1992). *Probing understanding.* Pliladelphia, PA: The Falmer Press.

Williams, C.G. (1994). *Using concept maps to determine differences in the concept image of function held by students in reform and traditional calculus classes.* Doctoral thesis, University of California, Santa Barbara. ProQuest document ID: 741991081.

Wilson, M. (1992). Measuring levels of mathematical understanding. In T. A. Romberg (Ed.), *Mathematics assessment and evaluation: Imperatives for mathematics education* (pp. 213-241). Albany, NY: State University of New York Press.