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METACOGNITIVE AWARENESS OF PROBLEM SOLVING AMONG PRIMARY AND SECONDARY SCHOOL STUDENTS

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ABSTRACT
The ability to solve mathematics problems is the central focus of the Singapore mathematics curriculum. This curriculum postulates that metacognition is one of the five key factors that can facilitate success in problem solving. Metacognition means that the problem solvers become aware of their own problem solving process, take control of this process, and seek help whenever necessary. One component of the CRPP-funded project entitled *Developing the Repertoire of Heuristics for Mathematical Problem Solving* (MPS) examines metacognitive awareness among a sample of P5 and S1 students.

The questionnaire on metacognition asks about what the students do during problem solving and their levels of enjoyment and confidence while solving problems. This was administered as a "pretest" around April 2004, and a "posttest" several months later. There were few changes in students' metacognitive awareness between the two tests. Their responses were fairly general, lacking in deep awareness of personal metacognition. Many students wrote about trying to understand the problem, but very few mentioned about checking their work. Most students expected the teachers to teach and explain more and better, but few could give specific suggestions. If metacognition were to become a standard learning and problem solving process as intended by the Singapore mathematics curriculum framework, structured programs that aim to inculcate various aspects of metacognition need to be developed and researched.

THE STUDY

The ability to solve mathematics problems has been the central focus of the Singapore mathematics curriculum since late 1980. To successfully solve various types of problems, in particular the non-routine ones, a motivated student has to apply four types of mathematical competencies, namely, specific mathematics *concepts, skills, processes*, and *metacognition*. Some studies have been conducted on metacognition and problem solving behaviours among Singapore students (Foong, 1993; Tay et al., 2006; Teong, 2003). In addition, Teong (2003) designed an explicit metacognitive instruction called CRIME (Careful reading; Recall possible strategies; Implement possible strategies; Monitor; and Evaluation) to be used in a computer-based environment. Low-achieving P5 pupils trained under her scheme outperformed the control group in solving word problems. Tay and his team (2006) reported that after explicit lessons on mathematics heuristics, a cohort of high ability Secondary 3 students under the Integrated Programme (IP) had changed their attitudes towards problem solving and mathematics. This paper provides further findings about metacognitive awareness as reported by some Singapore Primary 5 (P5) and Secondary 1 (S1) students after they had solved some mathematics problems (see Wong & Tiong, 2006a, about the use of heuristics). This is part of the CRPP project entitled "Developing the Repertoire of Heuristics for Mathematical Problem Solving" (MPS). The findings about metacognition can contribute towards understanding its role in problem solving, an important issue that mathematics educators from all over the world have studied for more than twenty years (Lester, 1994; Roberts & Tayeh, 2007; Schoenfeld, 1987, 1992).

The metacognitive instrument used in this study was designed by Teong Su Kwang, John Hedberg, and Ho Kai Fai. It consists of 9 questions. The first eight questions are open ended (adapted from Callanhan & Garofalo, 1987), and Question 9 consists of six multiple choice items. Three primary schools and two secondary schools participated in this MPS project, with two classes from each school, either at Primary 5 or Secondary 1 level. Two rounds of data were collected from the participating classes, around March 2004 as "pretest" and August 2004 as "posttest". Unfortunately, posttest data could not be collected from one of the two secondary schools, resulting in a sample mortality of two classes. In between these two administrations, the participating teachers attended one 3-hour mathematical problem solving workshop, separately for primary and secondary schools. The questionnaire took about 20 minutes to complete. Some students gave longer responses than others. These student responses do not necessarily cover fully what the students really thought about these problem solving and metacognitive matters, in particular, those who did not write or who wrote only sketchy comments. It is also possible that some students may not be aware of their inner cognitive workings and others may not be able to express themselves well in writing. Despite these shortcomings, the findings can serve to highlight metacognitive areas that the students are conscious of and gaps that require further attention.

Altogether 639 questionnaires were collected. The written responses were typed into Microsoft Excel worksheets and coded by several research assistants to ensure some degree of inter-rater reliability (see Wong & Tiong, 2006b, for details). However, the relatively high percentages (20% to 40%) of responses in the Others category for some items indicate the difficulty of obtaining agreement on many of the students' responses. This shortcoming should be further investigated in new studies that can incorporate interviewing the students or getting them to talk aloud during problem solving.

FINDINGS AND DISCUSSION

The following tables show the percentages of responses for the various codes used in each question. The percentages were computed based on the questionnaires received for each test within each group (primary or secondary): P5, pretest: n = 217; posttest: n = 213; S1, pretest: n = 143; posttest: n = 66. Some responses may be coded into several categories so that some percentages add up to more than 100%.

Question 1. Problem Solving Process and Reasons

This question asks about things that students usually do when they solve mathematics problems and why they do those things. The responses were classified under Polya's four main steps: Understand, Plan, Execute, and Check. The results are given in Table 1.

Most of the P5 students (91%) mentioned trying to understand the problem at pretest, but this dropped to 65% at the posttest. Instead, there were more responses about planning or checking their work. About two thirds of the S1 students mentioned trying to understand or analyze the problem at both occasions, with slightly more making statements about planning. For both groups, the number of statements about checking their work was very low, around 15%. This lends credence to the perception that many students have not developed the habit of checking their work. This is a serious gap in students' metacognition. On the other hand, Polya and other mathematics educators have argued that this final step of problem solving should include not only checking whether the answers make sense or not but also extending the problem or thinking of other problems that can be solved using a similar method. This latter type of looking back is still rare in mathematics teaching here.

The main reason the students gave relates to the need to be clear about what they were supposed to solve. Only about a quarter thought of the strategy value of the problem solving process. There were increases of 5% (P5) and 8% (S1) in the number of reasons related to checking their work or avoiding careless mistakes. This was encouraging, but obviously the change was not strong.

Table 1: Students' Responses for Question 1: Problem Solving Process and Reasons

Q1 (a) You have taken the word problem solving exercise. Write down all the things you usually do when you solve these problems.
(b) Why do you do these things?

	Primary		Secondary	
(a) Processes	Pre	Post	Pre	Post
Understand the problem by reading or analysing it.	91.2	64.8	63.0	69.7
Plan or think about the method.	27.1	39.9	44.8	59.1
Execute the plan.	48.9	37.6	45.5	36.3
Check the answer or working.	10.2	16.4	16.8	16.6
Others.	30.0	28.6	28.7	19.7
(b) Reasons				
Clarity, to reduce confusion.	56.7	49.8	41.3	51.5
Necessity, otherwise would not know what to do.	32.3	24.9	41.3	25.8
Strategy, to simplify.	24.9	22.5	24.5	31.8
Correctness, to avoid careless mistakes.	22.1	27.2	28.0	36.4
Others.	22.1	26.3	28.7	30.3

Question 2. Handle Mistakes

Being aware of the mistakes one commonly makes when solving problems can be helpful. Instead of writing about mistakes that are specific to particular problems, for example, adding fractions by adding the numerators and denominators separately, the students wrote statements that referred to the problem solving process. Most of these statements were about mistakes at the execution stage (78% to 92%), especially for the S1 students. See Table 2. Nearly two thirds of them attributed their mistakes to carelessness. This attribution seems to support what some Singapore teachers complain at inservice courses that their students are often very careless. However, carelessness is an ill-defined phenomenon. The same mistake, say, writing $2(x - 5)$ as $2x - 5$, could be "careless" for one student but is a misconception of another student. Indeed, research in mathematics understanding (e.g., Hart, 1981; Olivier, 1989) has consistently found that student mistakes are *not* careless; underlying these mistakes are serious misconceptions that are difficult to rectify. Hence, it is important for students and teachers alike not to be too quick to attribute mistakes to carelessness. Indeed, teachers should try to identify the misconceptions and design appropriate learning activities that address these misconceptions, preferably at the beginning phase of teaching the respective topics (e.g., Ashlock, 2002; Borasi, 1994).

In contrast to "carelessness", only about 12% of the statements referred to understanding the questions. Yet, mathematics teachers often complain that their students do not really understand what the problems require them to do, especially for word problems. This finding highlights a mismatch in perceptions between the students and teachers, and knowing about this difference should alert the teachers to find ways to help their students understand the problems. These could include asking the students to read the problems aloud, getting them to paraphrase the problems in their own words, underlining and explaining key words and symbols found in the problems, modelling the problems using concrete materials and actions, and so on. In short, the teachers should make the understanding phase an active component of the problem solving process.

Table 2: Students' Responses for Question 2: Handle Mistakes

Q2 (a) What kind of mistakes do you usually make when you solve word problems?
(b) Why do you make these mistakes?
(c) List the ways which you think will prevent yourself from making these mistakes.

	Primary		Secondary	
(a) Mistakes	Pre	Post	Pre	Post
Executing, in particular careless mistakes.	77.9	83.6	90.2	92.4
Understanding the questions.	12.4	10.8	14.0	9.1
Others.	10.6	5.6	4.9	3.0
(b) Reasons				
Careless.	61.8	67.1	69.9	66.7
Do not understand the questions.	17.5	8.5	8.4	6.1
Not enough time, rush to complete the questions.	5.5	0.9	9.1	10.6
Not focused, dreaming.	4.6	3.3	6.3	4.5
Others.	28.6	29.1	37.1	42.4
(c) Prevention				
Check the work.	43.8	44.1	39.9	43.9
Be careful.	22.1	26.8	23.8	31.8
Read the questions carefully.	11.1	11.7	11.2	16.7
Practice on more questions.	7.4	6.6	7.0	7.6
Draw model.	3.2	1.9	0.7	
Others.	35.5	23.0	36.4	37.9

Having identified the causes of mistakes, the students need to think about possible ways to prevent mistakes from happening. Table 2 shows that about 40% of the students mentioned checking their work, with about 22% to 32% saying that they should be more careful in their work, for example, to write more slowly or to avoid mental calculation. These suggestions appear to be reasonable to address careless mistakes, but, if the real cause were misconceptions, other techniques are called for. The students could also keep a notebook of their mistakes with corrections and to review these notes regularly. This is quite an effective learning strategy, but it is not known whether Singapore students use it consistently in their learning of mathematics.

About 7% of the students mentioned having more practice. It turns out that lack of practice was cited by some students why they may do badly in solving some problems; see Question 5 below. However, practice without conceptual and logical understanding is unlikely to be effective.

Question 3. Keep Track

This question explores monitoring during problem solving. This monitoring of one's problem solving process requires high level of awareness, but this may be lacking in students at the upper primary and lower secondary levels. The data seem to support this perception as only slightly less than half of the P5 students and slightly more than half of the S1 answered "Yes". The percentages answering "Yes" were slightly higher at the posttest, which is expected. See Table 3. It is very difficult to classify the students' ways of keeping track into distinct categories. Some examples are: "look back on the previous steps", "make sure I got the correct answer for each equation", "I check my answers with the calculator", "read my answers again if I am stuck", "try to draw a model" (note: Singapore students are very familiar with drawing model to solve word problems), and "I go through the steps in my head before I attempt the questions". Further investigation in this area is needed.

Table 3: Students' Responses for Question 3: Keep Track

Q3 (a) Do you keep track of what you are doing when you solve a word problem? (b) If you do, name the way(s) you keep track of what you are doing.				
	Primary		Secondary	
(a) Keep track?	Pre	Post	Pre	Post
Yes	41.9	47.4	51.7	60.6
No	26.7	23.0	39.2	27.3
Both	6.0	3.3	1.4	3.0
(b) Ways to keep track. Difficult to classify.				

Question 4. Look Back

As mentioned under Question 1, looking back or checking at the end of the problem solving process was mentioned by fewer than 20% of the students. However, when they were asked specifically about this phase, much higher percentages of students answered "Yes". Perhaps, the students wanted to give the "expected" answers. See Table 4. It is not clear why quite a few students answered both "Yes" and "No"; this suggests ambiguity about their answers. Those who answered "Yes" gave the reason of trying to spot careless mistakes, while those who answered "No" mentioned lack of time and laziness as the main reasons for not looking at their finished solutions.

Table 4: Students' Responses for Question 4: Look Back

Q4 (a) Do you look back and check your working/answer when you finish a problem? (b) Why or why not?				
	Primary		Secondary	
(a) Look back / check?	Pre	Post	Pre	Post
Yes	60.8	72.8	75.5	51.5
No	18.4	16.0	13.3	27.3
Both	12.9	11.7	7.7	16.7
Yes				
To spot careless mistakes.	57.6	62.0	72.7	54.5
To check that answer makes sense, the best method is used.	0.9	0.9	0.7	3.0
Others.	14.3	13.1	9.8	7.6
No				
Do not have time.	10.1	10.3	9.1	30.3
Laziness.	3.7	6.6	1.4	4.5
Do not need to because of confidence.	1.8		1.4	1.5
Others.	16.1	9.4	7.7	18.2

Question 5. Badly Done Questions: Why?

The students were asked to write down the questions that they felt were badly done and why. Since the questions are not reported in this paper, only the reasons will be discussed; see Table 5.

Among the P5 students, the most important reason given by about 60% of them for doing some questions badly was that they had not practised those questions before. This finding is consistent with the common practice of drilling students to master the techniques to solve typical problems. Thus, when confronted with unfamiliar problems as was the case in this project, the students tended to attribute their perceived poor performance to lack of knowledge or exposure to the problems. About one third of them cited difficulty of understanding the problem. The same reasons were also given by the S1 students, although higher percentages of these students mentioned also their incompetence with the types of topics, questions, or calculations required.

Table 5: Students' Responses for Question 5: Badly Done Questions: Why?

Q5 (a) From the exercise you have just done, write down the question number(s) of the problem(s) which you think you did badly.
(b) Give reasons to why you think you are bad at solving these word problems.

	Primary		Secondary	
(a) Badly done questions; not reported here.				
(b) Reasons	Pre	Post	Pre	Post
Had not done before, don't know formula.	63.1	61.5	53.1	43.9
Do not understand, confused by language used.	30.4	32.9	27.3	21.2
Not good at the topic or type of question.	10.2	7.0	21.0	21.2
Not good at calculation.	9.7	8.5	6.3	15.2
Cannot remember the method.	2.8	5.6	9.8	3.0
Not enough time, too slow, lose concentration.	3.7	5.2	9.8	1.5
Others.	13.8	17.9	14.7	15.2

Question 6. Well Done Questions: Why?

The corresponding results for questions that were perceived to be well done are reported in Table 6. More than half the students mentioned that the questions were well done because they were easy, which does not explain much. Familiarity with the questions was cited as a reason by 22% to 33% of the students. Surprisingly, very few students mentioned being careful, and this perception seems to confound the idea of carelessness when the students answered Question 2.

Table 6. Students' Responses for Question 6: Well Done Questions: Why?

Q6 (a) From the exercise you have just done, write down the question number(s) of the problem(s) which you think you did well.
(b) Give reasons to why you think you are good at solving these word problems.

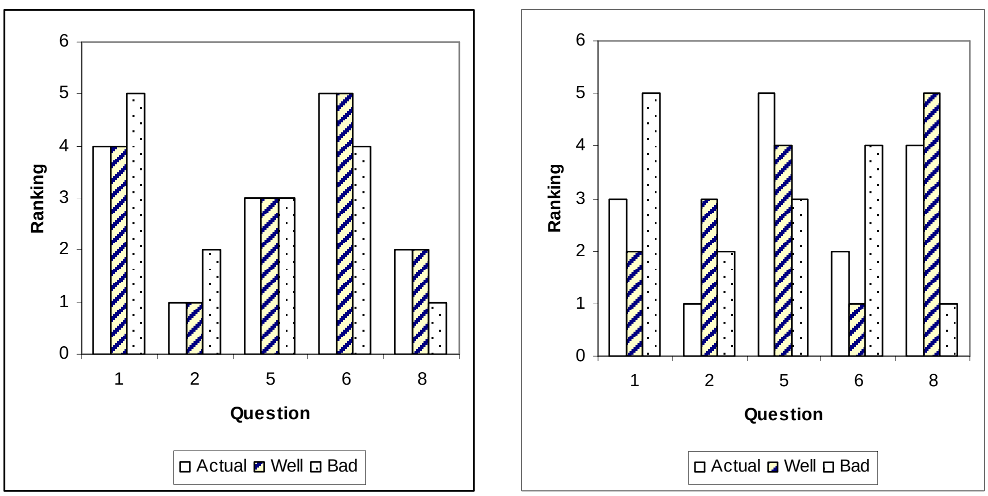
	Primary		Secondary	
(a) Well done questions; not reported here.				
(b) Reasons	Pre	Post	Pre	Post
Easy.	53.5	60.6	55.2	62.1
Familiar with the questions, good at them.	29.9	22.1	23.8	33.4
Good at the calculation, numbers not too big.	2.3	7.0	3.5	3.0
Interested in the questions, challenging.	4.6	6.1	7.7	3.0
Was careful when solving the questions.	2.3		3.5	1.5
Checked the answer.	0.9	2.3		1.5
Others.	9.7	8.0	6.3	1.5

Consistency in Actual and Perceived Performance

It is of interest to determine whether there was any consistency between actual performance based on scores for correct answers and perceived performance based on ratings by students as being well done or badly done. Five of the given problems were ranked using the posttest results from the most positive (rank 1) to the least positive (rank 5): Actual rankings were from highest mean (rank 1) to the lowest mean (rank 5); the Well done rankings were from the most frequently mentioned (rank 1) to the least frequently mentioned (rank 5); and the Badly done rankings were from the *least* frequently mentioned to the most frequently mentioned. The results are shown in Table 7 and Figure 1. For the P5 group, there were very consistent rankings using these three methods. This indicates that the primary school students were quite accurate in judging the relative difficulty levels of the questions against their own perceived ability. On the other hand, the data for the S1 group did not produce consistent rankings. This disparity requires further research.

Table 7. Rankings of 5 Questions Based on Posttest Results

		Questions				
		1	2	5	6	8
Primary	Actual	4	1	3	5	2
	Well done (perceived)	4	1	3	5	2
	Badly done (perceived)	5	2	3	4	1
Secondary	Actual	3	1	5	2	4
	Well done (perceived)	2	3	4	1	5
	Badly done (perceived)	5	2	3	4	1



(a) Primary (b) Secondary

Figure 1. Rankings of 5 Questions by Three Methods Based on Posttest Results.

Question 7. Ways to Become Better at Solving Word Problems

This question asks students to suggest ways to help themselves to become better at solving word problems. The results are summarised in Table 8. Almost all of them had mentioned some strategies to use. The most frequently mentioned strategy was to practice/study more, including learning about different methods. Very few students asked for help; however, higher percentages of the S1 students realised the need to ask for help at the posttest than at the pretest. Question 8 asks about the type of help that the teacher can provide.

Table 8: Students' Responses for Question 7: Ways to Become Better at Solving Word Problems

Q7 What do you think you can do to get better at solving word problems?	Primary		Secondary	
	Pre	Post	Pre	Post
Practice/study more, use different methods.	43.3	57.3	67.8	66.7
Understand the problems.	6.9	7.0	2.8	4.5
Pay attention during lessons.	5.5	3.8	1.4	3.0
Be more careful.	5.5	7.5	4.9	6.1
Check, check answer, check working	4.1	5.2	1.4	7.6
Learn more from books, ask teachers and friends for help.	5.1	7.5	10.5	13.6
Do not know.	1.4	0.5	2.8	
Others.	30.8	20.7	21.7	30.3

Question 8. Teacher's Help

As discussed above, most students did not mention on their own that asking for help can be one way to help them become better at solving problems. However, when they were asked specifically about how teachers can be helpful, about one third of the P5 students thought the teacher could be helpful but only 17% of the S1 students agreed at the pretest and only 10% agreed at the posttest. Most of these students did not elaborate on what they meant. Surprisingly, about 10% to 20% of the students felt that the teacher could not help them.

Among those who gave more elaborate answers, they felt that the teacher could give better or more explanations on the difficult steps, give more practice, and conduct remedial lessons. It is not surprising that the students mentioned these traditional teaching techniques because they had much experience with them in mathematics lessons. Hardly any students mentioned that the teacher could help by organising group work, asking them to keep reflection journal, assigning activities using ICT, or facilitating instead of controlling their learning. It would take many more years before our students become sufficiently comfortable with these "constructivist" methods to associate them with what the teacher could do to help them learn better.

Table 9: Students' Responses for Question 8: Teacher's Help

Q8 Are there things that the teacher can do to help?	Primary		Secondary	
	Pre	Post	Pre	Post
Yes.	33.6	34.7	17.5	10.6
Cannot help.	10.1	15.0	19.6	19.7
Do not know.	1.4	0.9	0.7	4.5
Teach or explain more, teach better, help when we do not know.	31.4	25.3	24.5	18.2
Give more practices/exercises.	7.4	10.3	14.7	28.8
Others.	18.9	25.3	37.1	36.3

Question 9. Perceptions of Problem Solving

This question consists of six items that probe general perceptions about problem solving. The findings are given in Table 10, with similar items re-arranged together.

Table 10: Students' Responses for Question 9: Perceptions of Problem Solving

Q9. For the following questions, please put a tick at the appropriate box:				
	Primary		Secondary	
	Pre	Post	Pre	Post
(a) Did you enjoy solving the problems in the problem solving exercise?				
Yes	22.6	22.5	26.6	36.4
OK	51.6	55.4	55.2	48.5
No	18.9	21.1	14.7	12.1
(f) How confident do you feel about doing mathematics now?				
More	32.7	32.4	27.3	10.6
Same	42.9	54.0	55.9	78.8
Less	16.6	12.2	12.6	7.6
(c) How did you normally get on with Maths in class?				
Easy	21.2	27.7	18.2	31.8
OK	63.6	66.2	72.0	60.6
Difficult	7.4	5.2	6.3	4.5
(d) How did you get on with the problems in the problem solving exercise?				
Easy	3.2	5.6	7.7	16.7
OK	38.2	68.1	63.6	77.3
Difficult	51.2	25.4	25.2	3.0
(b) Were the problems in the exercise like the kind of problems you normally do?				
Yes	4.6	8.5	19.6	34.8
A bit	54.4	63.4	52.4	54.5
No	33.6	27.2	24.5	7.6
(e) Has doing this problem solving exercise helped you see that there are many different types of problems other than the ones in the school textbook and workbooks?				
Yes	58.1	57.7	51.7	33.3
A bit	24.4	29.6	28.7	47.0
No	10.1	11.3	16.1	16.7

On the enjoyment of problem solving, slightly more than half of the students felt that it was just "OK". This perception hardly changed from pretest to posttest. After doing their pretest, about one third of the P5 students felt their confidence had improved, and the same happened after the posttest. On the other hand, for the S1 students, about 78% thought taking the posttest did not change their confidence with only 10% said this had improved their confidence. However, this finding about S1 students should be treated with caution because two S1 classes did not take part in the posttest.

As far as the problems in the problem solving exercise were concerned, 51% of the P5 students found them difficult at the pretest, but only 25% thought so after the posttest. Apparently these students became more familiar with these problems after the pretest and hence found them less difficult now. For the S1 students who did the posttest, only very few thought the problems were difficult at the second attempt. Results for item (b) also show that higher percentages of students at the posttest compared to the percentages at the pretest felt that the given problems were like those in their textbooks. These results are consistent with the rather obvious observation that non-routine problems can become routine with exposure and practice.

IMPLICATIONS AND CONCLUSION

Some implications for practice and research have already been discussed under the specific sections above. A few key findings bear repetition here. Many students wrote about trying to understand the problem, hence, giving most attention to Polya's first stage of problem solving. The other three Polya's stages, in particular, check/look back were mentioned by much fewer students. Familiarity with the problems was given as an important factor about perceived performance, but for the S1 students there was inconsistency between perceived and actual performance. Many students thought of their mistakes as due to carelessness at the execution stage, but the earlier section on Question 2 attempts to argue that this is a common but an untenable perception. Many students wished that their teachers could explain more and better and give more practice, but few of them were aware of alternative pedagogies, in particular learning activities of the inquiry or constructivist type. If metacognition were to be promoted as a serious process as intended by the Singapore mathematics curriculum framework, more concerted effort should be expended into designing and researching structured programs such as CRIME (Teong, 2002) and SOLVED (Hohn & Frey, 2002).

The above findings from the student questionnaire are dependent on the codes used. The coding can be further refined because of the significant percentages of the Others categories and overlaps in some of the identified categories. Further refinement may not provide much deeper insight into student metacognition because, as reported above, most of the students' responses were fairly general, lacking in deep awareness of personal metacognition during problem solving. This could be due to the lack of training for students to become more aware of their inner thinking and inner speech. Some students may not be able to express their inner processes clearly in English due to weak language competence or unfamiliarity with this genre of expression. Other students may not have adequate time to reflect carefully before they put their thoughts in writing during the administration of the questionnaire. These are limitations about research design, data collection, and data analysis that future studies could try to address.

These findings from a small and convenient sample (with the unfortunate loss of two S1 classes) cannot be readily generalised. However, individual teachers could make a judgement whether their students would behave metacognitively in similar ways when they solve mathematics problems by linking the findings to what they already know about their students through non-systematic observations. Indeed, some Singapore teachers need training about metacognition as this seems to be the most difficult of the five factors of the Singapore mathematics curriculum for many teachers to incorporate into their lessons (Wong, 2002).

The above findings were based on written questionnaire using self-reporting, which can give a broad picture but have well-known limitations. Future research should include complementary methods, such as clinical interviews, Newman error analysis (Newman, 1977; Wong, 2003), observations and/or videotaping of students solving given problems with think aloud and probing. These techniques will enable researchers to gather rich data to uncover the complexity and dynamic nature of metacognition, resulting in more appropriate training for both the students and their teachers.

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