Title	Socioeconomic status, home mathematics environment and math achievement in kindergarten: A mediation analysis
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Socioeconomic status, home mathematics environment and math achievement in kindergarten: a mediation analysis

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Abstract

Growing evidence suggests that parents' practices contribute to their children's cognitive development and that such practices may reflect SES disparities. This study investigated longitudinal interrelations between home mathematics environment (HME), children's math achievement, and two facets of SES (mother's educational attainment and household income—subsidy status) during the first year in kindergarten (n= 500 children; $M_{age\ at\ T1}$ = 57.3 months, SD= 3.8). Results revealed that these facets of SES operated through different mechanisms in kindergarten—the association between mothers' education and math growth at the end of K1 is fully mediated by HME and children's baseline math knowledge. Furthermore, only home math activities that explicitly supported the understanding of addition and subtraction contributed to children's math growth independently of SES background. The pattern of longitudinal associations suggests that the provision of home math activities may reflect children's mathematical abilities rather than SES disparities.

Keywords: home mathematics environment, math, early childhood, SES

Introduction

The home mathematics environment (HME), that is, activities and opportunities related to mathematics that parents provide at home, is related to children's mathematical abilities (Dunst, Hamby, Wilkie, & Dunst, 2017; Mutaf-Yıldız, Sasanguie, De Smedt, & Reynvoet, 2020). The strength of the association between the HME and children's mathematics may be greater for kindergarten children (i.e., aged 5 and 6 years)—a developmental period when children start grasping basic math skills such as identifying numbers, counting marbles, or ordering tokens by size. Whilst the effect of a supportive home environment on child development is widely acknowledged, a robust finding in the literature is that the quality of the home environment is also associated with the socioeconomic status of a family—SES (Galindo & Sonnenschein, 2015; Zadeh, Farnia, & Ungerleider, 2010). Children from higher-SES backgrounds are exposed to richer interactions that are thought to contribute to the acquisition of basic competencies. Indeed, it has been suggested that parental practices mediate SES disparities in academic achievement (Bradley & Corwyn 2002; Sirin, 2005).

Surprisingly, limited research has disentangled these relations. Thus, the goal of the present study was to explore the role of SES in mediating the relations between the HME and early math achievement for children in kindergarten. Specifically, we look at how different aspects of HME may affect math achievement over the first year in kindergarten and whether home math activities constitute a mechanism that explains the association of distinct facets of SES with math. Furthermore, we investigated the underlying factor structure of HME with a sample of Singaporean children.

The home mathematics environment: supporting early math knowledge

The role of the home environment as a predictor of children's learning and development has a long tradition in ecological and developmental studies (e.g., Epstein, 1983; Evans, 2004; Smith, 1968). Whereas a wealth of research has been devoted to exploring the home literacy environment since the early '90s, research regarding the home mathematics environment is relatively recent. Based on the same theoretical assumptions that give support to the role of the home literacy environment on the acquisition and development of language skills, it is thought that what parents do (mathematically) contributes to their children's understanding of early mathematical skills (LeFevre et al., 2009). It has been suggested that the type and frequency of home-based math activities are particularly relevant at the preschool level, as informal math knowledge may facilitate the acquisition and understanding of formal or more advanced mathematics that include operating with arithmetic rules (LeFevre et al., 2009). Informal math knowledge refers to having a basic understanding of relational terms (i.e., more and less) and numerical magnitudes (i.e., the more numerous arrays of objects are larger), being able to recite the counting sequence and naming shapes, and having basic sorting abilities (i.e., sorting collections of objects according to size).

Although a link between HME and early math is widely assumed (for a systematic review, see Mutaf-Yildiz et al., 2020), findings from empirical studies are somewhat inconsistent; it is not clear what types of activities have a significant effect and whether such association depends on other cultural and developmental factors (Elliott & Bachman, 2018). One of the issues that can cause conflicting findings in the literature relates to how the HME has been measured. Among the studies that have used questionnaires, one of the most common approaches is the aggregation of scores across different items (i.e., a composite score based on parents' ratings on a variety of items). Whilst this approach is appropriate when all of the items are measuring a single dimension, the diversity of the home environment makes the aggregation unsuitable (for a critical review, see Elliott & Bachman, 2017). Other authors have based their conclusions on item-based information, which poses constraints to the interpretation and replicability of the findings, as different cultures may differ in the types of activities that are usually implemented at home.

In an attempt to reduce the heterogeneity in the field and better conceptualize the HME, some authors have suggested measurement models with latent variables that “summarize” the effect of different types of activities. LeFevre and colleagues formulated a model that considered two aspects of the HME (LeFevre et al., 2009), based on Sénéchal and LeFevre’s home literacy model (2002). In this model, (based on parents’ reports about the frequency of a series of activities) two main factors or groups of activities were distinguished—direct activities and indirect activities (or formal and informal factors, respectively). Direct activities explicitly teach quantitative skills or engage children in number-related tasks (e.g., counting objects, practicing number names, and printing numbers) whereas indirect activities involve incidental learning of numbers and relations via games or other activities in non-numerical contexts (LeFevre et al., 2009, p.56). Although the factor structure of that model varies across replications (e.g., Ciping, Silinskas, Wei, & Georgiou, 2015; Hart, Ganley, & Purpura, 2016; LeFevre et al., 2010; Skwarchuk, 2009), the distinction between direct and indirect activities is widely accepted.

Despite the methodological improvement, this conceptualization of direct and indirect activities has not resolved the inconsistency in findings on the role of HME. Findings regarding the role of indirect activities do not provide a clear picture of the association of HME with math achievement. Null effects (LeFevre et al., 2010), positive effects (LeFevre et al., 2009; Niklas & Schneider, 2014), and negative effects have been reported (Huntsinger, Jose, & Luo, 2016).

Indeed, in some studies that have aimed at measuring both types of activities authors have used formal items as a measure of HME because parents did not report being engaged in indirect activities with their children (e.g., Manolitsis, Georgiou, & Tziraki, 2013); these mixed findings have raised additional questions regarding the role of contextual variables such as SES, culture, and region/ethnicity on the structure of HME. In contrast, the role of direct or formal activities is more consistent across studies, independently of contextual variables, which suggests that formal math practices may be more strongly related to children's math skills (e.g., Huntsinger, Jose, Larson, Balsink Krieg, & Shaligram, 2000; Huntsinger, et al., 2016; Kleemans, Peeters, Segers, & Verhoeven., 2012; LeFevre et al., 2010; Mutaf Yıldız, Sasanguie, De Smedt, Reynvoet, 2018a; Rosales, Ramos, Janez, & De Sixte, 2020; Skwarchuk, Sowinski, & LeFevre, 2014; but see Blevins-Knabe, & Musun-Miller, 1996; LeFevre et al., 2009).

Among the studies that have specifically tackled the role of direct (or formal) activities, Skwarchuk et al. (2014) identified a two-factor structure—reflecting advanced and basic activities—with a sample of Canadian families. Advanced activities involved arithmetic procedures whereas basic activities served to activate early numeracy skills such as counting in sequence and reciting numbers. In a regression analysis that considered the cross-sectional link of HME and kindergarteners' symbolic and non-symbolic knowledge, the authors reported that advanced activities related to children's knowledge of Arabic numerals. No association was found with basic activities. Notably, the overall factor structure of this model has been successfully replicated across different socio-cultural contexts. Adopting Skwarchuk and colleagues' model, Susperreguy, Douglas, Xu, Molina-Rojas, and LeFevre (2020) also identified a similar factor structure with Chilean families. Besides, they found a positive role of advanced activities (labeled as "operational activities") on kindergarteners' early numeracy skills (measured concurrently with the HME) and other math skills measured one year later (number line estimation and word problem-solving).

Elliott and Bachman (2017) have provided a detailed and critical account of why the field accumulates such disparate results. Issues regarding methodological rigor and how HME and math skills are measured or conceptualized are highlighted as potential causes of mixed findings. For instance, the effect of HME on math seems larger and more consistent in those studies that have considered specific early numeracy skills such as counting, and numerical magnitude comparison. In contrast, findings are rather inconsistent in studies that have used more general (and standardized) measures of math achievement. Another group of factors that may affect the consistency of findings relates to sample characteristics. For instance, cultural and social differences may affect the factor structure of the HME. Whereas in some cultures the approach to preschool education is more holistic and considers the co-development of multiple aspects, in others emphasis is placed on strengthening pre-academic skills. Similarly, findings also vary across studies that have targeted different SES strata. Age differences may also affect the link between HME and math, as the effect is thought to be stronger during the kindergarten period than at earlier or later developmental stages (Mutaf-Yildiz et al., 2020; Thompson, Napoli & Purpura, 2017). Sample size may also hinder the replicability of findings given that small sample sizes do not allow exploring a measurement model of HME and require authors to rely on item-level information or inappropriate composites that average positive and negative effects across a variety of items.

Socioeconomic status and early childhood

Whilst creating a more stimulating home environment and richer social interactions are thought to promote cognitive and socioemotional development (Crosnoe & Cooper, 2010; Sirin, 2005; Thomson, 2018), evidence suggests that high-quality learning environments are related to high-SES backgrounds (Bradley, Corwyn, McAdoo, & Garcia Coll, 2001; Guo & Harris, 2000; Yeung, Linver, & Brooks-Gunn, 2002). Although SES is usually measured by some combination of income, education, and occupational status, mothers' education alone often reflects accurately the effects of family SES in the context of early childhood (Cheadle, 2008; Haveman & Wolfe, 1995; Reardon, 2011). Mother's educational attainment is strongly associated with children's pre-academic skills (Duncan & Magnuson, 2003; Nguyen et al., 2016; see Klein, Starkey, Clements, Sarama, & Iyer, 2008, for a review). One explanation for this finding is that higher-educated mothers are more likely to create quantitatively and qualitatively richer interactions with their children (Vandermaas-Peeler, Nelson, Bumpass, & Sassine, 2009). For instance, Levine, Suriyakham, Rowe, Huttenlocher, and Gunderson (2010) found that mothers' educational attainment and a measure of the frequency of parents' numerical language combined to explain 36% of the variance in children's understanding of cardinality.

Other studies that have considered alternative aspects of SES such as household income and parents' occupation (and those studies using an SES index that combines different facets of SES) have also found that children from high-SES backgrounds outperform their counterparts from less advantaged backgrounds in measures of school readiness (for a theoretical review, see Brooks-Gunn & Duncan, 1997) and that high-SES backgrounds afford richer learning environments. For instance, Bradley et al. (2001) carried out an item-level analysis of the indicators of environmental quality of the Home Observation for Measurement of the Environment Inventories (HOME; Caldwell & Bradley, 1984) with a sample from the National Longitudinal Survey of Youth and found variation in almost all of the items with respect to poverty status. It is thought that children from higher-income households have more access to resources—i.e., they are exposed to richer learning environments or more stimulating materials that can help parents in engaging their children in math activities. There is also evidence that low-SES backgrounds may induce higher levels of stress, and that stress may affect how parents stimulate and provide care to their children (Bradley & Whiteside-Mansell, 1997; Conger, Ge, Elder, Lorenz, & Simons, 1994).

Home mathematics environment: relations with socioeconomic status

Whereas substantial evidence supports the assumption that parents' practices and the home learning environment reflect SES disparities in children's language development, the evidence is scarce when it comes to the acquisition and development of early numeracy skills and early math abilities. Among the few studies that have specifically investigated this topic, Zadeh et al., (2010) found that different aspects that characterize enriching home environments—measured with the HOME inventory (Caldwell & Bradley, 1984), mediated the association between maternal education and math achievement. Galindo and Sonnenschein (2015) also used the HOME inventory in a similar analysis and found that the home learning environment explained between 11% and 23% of the total effect of SES on kindergarteners' math achievement (SES was a composite variable that included parental education, occupation, and family income). Importantly, the direct association between SES and math remained significant in both studies, which suggests that SES also operates through alternative mechanisms and that the home environment and SES are independently associated with math (see also, Manolitsis et al., 2013; Skwarchuk et al., 2014; Susperreguy et al., 2020). Crucially, there is no evidence of domain-specific influences or whether the provision of home math activities reflects SES disparities in early math achievement.

Findings from studies that have specifically investigated the HME suggest that the relation of HME and SES is not straightforward and that math practices at home may not reflect SES disparities. For instance, Del Río, Susperreguy, Strasser, and Salinas (2017) found that Chilean parents from low-SES backgrounds engaged more frequently in math practices than those from high-SES environments (but see DeFlorio & Beliakoff, 2015 and Susperreguy et al., 2020, for the opposite relation). Others have found that the provision of home math activities does not relate to SES. For instance, Manolitsis et al. (2013) and Skwarchuk et al. (2014) found that maternal education did not explain variance in the frequency of home math activities in Greece and Canada, respectively. Furthermore, different association patterns between distinct aspects of HME and SES have been reported. Whereas some studies have found that more educated parents engage their children more frequently in informal or game-based activities (e.g., LeFevre et al., 2010), others have not been able to identify informal activities (e.g. Manolitsis, et al., 2013), and others have found that more educated parents engage their children more frequently in both informal and formal math activities (e.g., Susperreguy et al., 2020). These conflicting findings may be due to different HME conceptualizations and the degree of SES and HME heterogeneity.

It is worth mentioning that associations between SES and HME may be due to confounding between SES and math ability—children from high-SES backgrounds are more proficient (mathematically) and enter kindergarten with more advanced math abilities. It is feasible that parents' practices and the frequency of home math activities reflect children's math abilities rather than SES disparities and that children elicit different activities based on their skills. For instance, Saxe et al. (1987) found that mothers from higher-SES environments were more likely to adjust the goal structure of a given activity to reflect their children's math abilities. There is also evidence that the concurrent relation between HME and kindergarteners' math achievement may be one in which children's math proficiency affects how frequently parents engage their children in home math activities (Blevins-Knabe & Musun-Miller, 1996). Findings from longitudinal studies also suggest an association in which children's math abilities determine how frequently parents engage their children in home math activities. For instance, some studies have reported that the association of HME with math vanishes after accounting for individual differences in previous math knowledge (e.g., Manolitsis et al., 2013). Others have found more consistent longitudinal effects over the kindergarten years when the effect of previous math knowledge is not directly accounted for (e.g., Susperreguy et al., 2020). Overall, these findings suggest a complex set of relations between HME, SES, and math abilities in kindergarten, and that failing to account for baseline math abilities may alter the interpretation of such associations.

The current study

In the current study, we address some of the methodological issues mentioned above and investigate the role of home math activities in Singapore. Our first aim relates to whether HME explains variability in kindergarteners' math achievement at the end of K1 and whether HME and baseline math abilities (at entry to K1) interact. This is a relevant question because most research is cross-sectional—i.e., data are usually collected simultaneously with children's performance data and neglects the possibility that parents' practices are also affected by their children's abilities. Our second aim relates to whether HME is a common mechanism that explains the association of distinct facets of SES with early math achievement. The review of the literature suggests that maternal education is a stronger predictor of children's development and academic outcomes than other facets of SES and that such association is probably due to different parents' practices. Nonetheless, there is also evidence that other facets of SES such as household income affect how parents provide care and support—i.e., children from high-income families have more accessibility to learning resources. Thus, we do not know whether the association between different facets of SES and children's early math achievement is similarly mediated by HME.

A subsidiary question that needs to be tested first (before the aforementioned questions can be addressed) pertains to the conceptualization of HME in Singapore and whether the factor structure of HME is similar to that reported in other countries. Given the socio-cultural context of the current study, the expectations of Singaporean parents for academic readiness may mean a greater focus on more formal numerical activities in the home. Because the extant findings suggest that informal activities have a limited association with kindergarteners' math achievement, we focused on formal activities (or activities that explicitly engage children in number-related tasks) to define HME.

Method

Sample

Data from the current study were drawn from a large-scale longitudinal study examining the impact of pre-school education and home environment on children's development and academic outcomes in Singapore. Recruitment for the main study followed a stratified sampling strategy to target centers from a range of social strata. The sample for the current study comprised 500 children ($M_{\text{age at T1}}$ = 57.3 months—4 years and 8 months, SD= 3.8 months, 52% females) and was randomly selected from a larger dataset of 1179 children for whom the home mathematics environment questionnaire had been completed by the mother (this corresponds to

90% of the total number of respondents in the main study, approximately). Based on Government data from the same year of data collection (General Household Survey; Department of Statistics, 2015), our sample included a larger low- and mid-class representation. The median monthly household income corresponded to S$7,000 in the current sample, which is below the value for the wider population (S$8,666) corresponding to the same year of data collection (Department of Statistics, 2015). The percentage of mothers having completed post-secondary education (non-tertiary) and that of mothers having professional qualifications in the current study (27% and 41%, respectively) were larger than those reported for the wider population (8% and 14%, respectively). Similarly, the percentage of mothers that reported having a university degree in the current study was substantially smaller (14% in the current study vs 26% for the wider population).

Materials

Math ability

The Test of Early Mathematics Ability-3 (TEMA-3; Ginsburg & Baroody, 2003) was used to assess children's math ability. The test is designed for use with children from 3 to 8 years of age, to assess both informal (acquired outside the context of schooling) and formal (skills and concepts learned in school) mathematical thinking. In this study, Form A, comprising 72 items, was administered. Each item was scored dichotomously (i.e., 0 or 1). Test administration began with an entry point suitable for a child's age and was terminated when the ceiling and basal were established. Raw scores were used in the analyses.

Non-verbal intelligence

Raven's Coloured Progressive Matrices (Raven, 1947) was used as a control variable (see below). It comprised three sets of 12 items (Sets A, AB, and B). Within each set, items were arranged in order of increasing difficulty. In each item, a pattern with a missing element was presented in matrix format (either 2x2, 3x3, 4x4, or 6x6). Children were asked to select the element that completed the pattern from a set of alternatives. The administration of each set was terminated when four consecutive incorrect responses were made. The dependent measure was the total number of correct responses across all three sets

Home Mathematics Environment (HME)

We developed a short HME questionnaire that included 14 questions aimed at capturing different aspects of the home mathematics environment. In this questionnaire, we focused on the formal math activities described in Skwarchuk et al. (2014) and also included in a recent replication (e.g., Susperreguy et al., 2020). Twelve questions were drawn from Skwarchuk et al. (2014) and two additional items were developed to take into account other cultural aspects such as children's math-related gaming activities and their engagement in small talk about money (for specific items, see Table 1). Parents indicated how frequently they engaged in each math activity on a scale from 1 (never/rarely) to 7 (often/daily). The HME questionnaire was part of a larger questionnaire that was distributed to parents online or in paper format. The questionnaire aimed at providing information regarding the home environment, as well as other aspects of child behavior. For the current study, we only included data corresponding to questionnaires that were completed by the mother.

Socioeconomic Status (SES)

Parents were asked to provide home background information (e.g., parents' educational qualifications, housing type, household income, and whether the family qualified for educational subsidies or kindergarten fee assistance) in a short questionnaire. This questionnaire was distributed to parents via their children's kindergarten teachers and also included basic demographics (e.g., age, gender, ethnicity), and other information about the child's early development. Based on prior findings, we focused on the mother's educational attainment and whether the family qualified for childcare subsidies (or the child was receiving any other type of kindergarten fee assistance). The latter is a more direct measure than parental reports on income since it accounts for a variety of aspects that may impact parents' economic status and how household income translates into children's accessibility to learning resources (e.g., age and number of siblings, employment status, being medically unfit for work, taking care of family members). In the current study, 21% of children qualified for childcare subsidies. Mothers' education was coded according to the Singapore Standard Educational Classification (SSEC; Department of Statistics, 2015) (1= below secondary, 2= secondary, 3= post-secondary (non-tertiary), 4= Diploma/Professional qualification, 5= university degree). Groups 4 and 5 correspond to tertiary education. A chi-square test of independence showed a moderate association between both facets of SES, χ^2 (4) = 90.19, $p < .001$ ($\tau b = .37$). A smaller proportion of mothers declared having tertiary studies in the group that qualified for childcare subsidies (17% vs. 65% in the group that did not qualify for childcare subsidies).

Procedure

Child measures were administered individually as part of a larger battery of tasks at the beginning of Kindergarten 1 (K1; wave_1) and towards the end of K1 (wave_2). Information regarding HME was collected during the first half of the year (HME data and children's data were collected simultaneously). Information regarding SES was collected at entry to the study.

Analytical Plan

All descriptive and inferential statistical analyses were estimated using Mplus v.8 (Muthén, & Muthén, 2017). First, we investigated the factor structure of HME. Second, we formulated a SEM to investigate the role of both HME and math at kindergarten entry as parallel mediators of the effect of both facets of SES on math at the end of K1. Thus, in this model, the (longitudinal) association of HME with math at the end of K1 was estimated together with math knowledge at kindergarten entry and SES as explanatory variables. We allowed the residual variances of mediators (HME and math achievement at kindergarten entry) to covary. We also included two variables to account for time-varying observations—children's time in kindergarten at the first time point of data collection and the gap between waves of data collection, as well as a control variable to account for differences in non-verbal intelligence (see Figure 1S). Testing the concurrent association of HME and math is problematic because, as argued above, i) the interpretation of that association is ambiguous when both measures are collected simultaneously and ii) the aim of the current study focuses on the role of HME as a mediator of SES disparities. Instead, we examined the extent to which the influence of HME is stable for children with different math abilities at the beginning of K1 by forming an interaction term with the two variables and examining its influence on math at the end of K1. We also modified our model to test the association between HME and math under conditions similar to those reported in other studies that have not considered children's prior math skills. We did this by removing math achievement at entry to kindergarten to investigate the strength of the longitudinal association of HME with math at the end of K1.

Parameters were estimated with full information maximum likelihood. A robust estimator (MLR) was used to investigate the factor structure of HME since some items showed non-normal distributions. Standard errors and non-symmetric confidence intervals for the population values of the indirect effects of both facets of SES through HME were derived using bootstrapping methods (5000 samples). Model fit was assessed by inspecting the χ^2 test, as well as the Comparative Fit Index (CFI), the Tucker-Lewis Index (TLI), and Root Mean Square Error of Approximation (RMSEA). For both the TLI and CFI, we assumed values above 0.95 to indicate adequate fit (Hu & Bentler, 1999). For the RMSEA, we considered values below 0.06 as indicating good model fit (Chen et al., 2008).

Results

Preliminary analyses: factor structure of HME

First, all of the items in the HME questionnaire were submitted to an exploratory structural equation model (ESEM; Asparouhov & Muthen, 2009) to investigate the appropriateness of the items. We specified a 2-factor model since the questionnaire included two main types of items according to Skwarchuk et al. (2014) (advanced and basic math activities). This solution showed a poor fit (χ^2 (64) = 331.67, CFI = .907, TLI = .867, RMSEA = .091). We also specified an alternative 3-factor solution since several activities were contextualized as play-based activities (i.e., board games and computer games that involve numbers). The fit indices of the 3-factor solution were optimal (χ^2 (52) = 169.20, CFI = .959, TLI = .929, RMSEA = .067) (see Table 1). However, a closer inspection of items for the third factor revealed some with similar loadings across the three factors. Furthermore, several cross-loadings were above .30 and all the dimensions that were modeled captured less than 50% of the variance in some indicators.

TABLE 1

Given the aim of this preliminary analysis is to find an optimal measurement model for the HME, we retained the items for which the total amount of explained variance was above 50% (Kline, 2016) in the 3-factor ESEM solution (corresponding to factor loadings above .70). The third factor was dropped because only one indicator had the requisite loading. The resultant 2-factor model was subjected to a CFA and it rendered a good fit to the data (χ^2 (8) = 24.50, CFI = .984, TLI = .970, RMSEA = .064). Parameter estimates are shown in Figure 1. More importantly, the factor structure was theoretically acceptable and showed two qualitatively different[1] aspects of the home mathematics environment that align with those identified in studies that have investigated formal math activities in very different socio-cultural contexts (e.g., Skwarchuk et al., 2014; Susperreguy et al., 2020). One of the factors related to activities that involved more advanced math knowledge such as arithmetic procedures (*Advanced math activities*) whereas the other factor related to basic skills such as counting and recognizing numerals (*Basic math activities*)[2]. These two factors are conceptually similar to those defined in Susperreguy et al. (2020) as *operational activities* and *mapping activities*, respectively.

FIGURE 1

Structural Equation Model

The model that was specified to explore the longitudinal effect of HME, as well as the role of both HME and math at kindergarten entry as parallel mediators of SES, showed a good fit (χ^2 (41) = 80.82, CFI = .986, TLI = .977, RMSEA = .044). Descriptive statistics and standardized

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[1] A 1-factor CFA rendered a worse fit (χ^2 (9) = 229.39, CFI = .783, TLI = .639, RMSEA = .221).
[2] We tried to replicate the factor structure of the formal HME that was suggested by Skwarchuk et al. (2014). Nonetheless, the CFA showed a poor fit (χ^2 (19) = 196.18, CFI = .858, TLI = .791, RMSEA = .137).

parameter estimates are shown in Table S1 and Figure S1, respectively (Supplementary file). For clarity, Figure 2 only shows parameter estimates of the paths that were significant in the analysis.

FIGURE 2

This model revealed a positive association between *Advanced math activities* and children's math achievement at the end of K1. Nevertheless, no effect was found regarding the role of *Basic math activities*, which underscores the role of activities that promote an early understanding of addition and subtraction over those that focus on basic numeracy skills when it comes to explaining differences in children's math gains in kindergarten. Importantly, the association of *Advanced math activities* and math at the end of K1 was significant after accounting for individual differences in math at kindergarten entry and SES disparities, which are aspects that contribute to the frequency of *Advanced math activities* ($r = .255$, $p < .001$ and $r = .165$, $p < .01$, respectively).

The model also revealed that both types of home math activities are related to the mother's educational attainment. More educated mothers engaged their children more frequently in both *Advanced* and *Basic math activities*. Indeed, the factor *Advanced math activities* was a significant mediator of SES disparities related to maternal education ($b = .156$, SE = .078, C.I. [.033, .331]) but not subsidy status, which suggests that maternal education and subsidy status operate through different mechanisms. Notably, the indirect effect of mother education through HME was very small—equivalent to .02 SD—compared to that through math skills at K1 entry—equivalent to .14 SD.

Note that the non-significant association of the mother's educational attainment with math at the end of K1 (as shown in Figure 2) does not mean that *Advanced math activities* is a full mediator of this facet of SES. It is feasible that such association vanishes after accounting for the effect of baseline math knowledge. In fact, removing children's math knowledge at kindergarten entry from the model (χ^2 (36) = 77.09, CFI = .981, TLI = .971, RMSEA = .048) revealed that both *Advanced math activities* and maternal education remained as predictors of math at the end of K1 (b = 1.76, SE = .34, $p < .001$, and b = 1.25, SE = .37, $p < .001$, respectively) and that the magnitude of the indirect effect of SES through *Advanced math activities* remained small (.05 SD). Crucially, failing to account for baseline knowledge inflated the association of *Advanced math activities* with math at the end of K1—i.e., equivalent to .30 of a standard deviation (compared to .11 when previous math knowledge at kindergarten entry was included in the model). Furthermore, this modified model revealed a significant (but negative) association of *Basic math activities* with children's math achievement at the end of K1 (b = -.897, SE = .39, $p < .05$). To summarize, the association between mother's education and math at the end of K1 was mediated fully by both *Advanced math activities* and baseline math knowledge, but with baseline knowledge playing a stronger mediation role than *Advanced math activities*. This is in contrast to how the other facet of SES operates—the effect of subsidy status was partially mediated by baseline math knowledge (b = 2.13, SE = .072, C.I. [.074, 3.588]) and still had a direct impact on children's math achievement at the end of the first year in kindergarten.

Finally, we conducted two separate analyses to investigate baseline interaction of math achievement at kindergarten entry with each HME factor. This analysis is important because it clarifies for whom home math activities were more beneficial. The interaction of each HME factor with math abilities at entry to K1 was investigated in separate SEM where math achievement at the end of K1 was the dependent variable and the remaining manifest and latent variables were predictors. For instance, math at the end of K1 was regressed onto the factor *Advanced math activities*, baseline math skills (math at entry to K1), an interaction term formed by multiplying this HME factor and math at entry to K1, the HME factor *Basic math activities*, both SES facets, and the control variables (time in kindergarten, gap between both waves of data collection, and non-verbal intelligence). We allowed all these predictors to covary. A similar model was specified to investigate the interaction of *Basic math activities* and baseline math skills. Parameter estimates are reported in Table S2 (see supplementary material). This analysis revealed a significant interaction between *Advanced math activities* and baseline math skills (b = .037, SE = .02, $p < .05$). Only children with average or better math skills at kindergarten entry benefitted from *Advanced math activities*. Figure 3 shows the interaction effect. Each point in the straight line represents the value of the slope of the regression of *Advanced math activities* on math at the end of kindergarten (or math gains per unit of frequency of *Advanced math activities*). The left and right ends of the line correspond to low- and high-achievers at kindergarten entry (-1SD and 1SD below and above the sample mean, respectively). As can be observed in the bottom left quadrant of Figure 3, the confidence interval crossed zero at values corresponding to low-achievers at kindergarten entry, which indicates that only children with average and above-average math proficiency at K1 entry benefited from *Advanced math activities*. The interaction of *Basic math activities* with children's math abilities at K1 entry was not significant (b = .030).

FIGURE 3

Discussion

Understanding the factors that predict variability in parents' promotion of the home mathematics environment and how this environment affects children's math achievement during early childhood is critical to parents, educators, and policymakers. Although substantial theoretical support has been given to the association between SES, HME, and early math achievement, empirical findings are mixed, and few studies have explicitly investigated the nature of these associations. In what follows we address each of our research questions separately before we draw some general conclusions and discuss the implications of our findings.

How does HME relate to children's math achievement over the first year in kindergarten?

In line with other studies that have (a) used a latent model of HME, (b) sampled similar-age children, and (c) factored in the effects of previous math knowledge (e.g., Susperreguy et al., 2020; Skwarchuk et al., 2014), our findings indicate that only math activities that promote addition and subtraction skills contribute to math achievement at the end of the first year in kindergarten (see also, Del Rio et al., 2017; Zippert & Ramani, 2017). Because baseline math knowledge was included in the model, this finding suggests that children who received more support in understanding addition or subtraction at home also experienced steeper math growth. Such home activities largely align with the type of math knowledge that is expected at the end of K1. For instance, children are expected to develop an understanding of additive properties and part-whole relations during this period (*Nurturing Early Learners Framework*; Ministry of Education, 2013). The set of mathematical abilities that were evaluated with the TEMA at the end of K1 also included a majority of activities that relied on arithmetic skills (or at least involved other than basic numeracy skills).

Our study did not reveal a significant association between children's math gains at the end of K1 and the frequency with which parents engaged their children in basic math activities. It is possible that if the children were younger, the correlation between math skills and the HME would be higher for basic than for advanced activities that focus on arithmetic. It has been suggested that “basic home numeracy activities are more related to mathematical skills than advanced activities in relatively younger samples, whereas the opposite pattern is to be expected in relatively older samples” (Mutaf-Yildiz et al., 2020, p. 16). However, research is limited regarding the basic/advanced distinction across a range of age groups. Thus, this explanation is tentative and needs more evidence. An alternative explanation is that the null association with *Basic math activities* indicates that the TEMA (as a measure of overall math ability) may not capture differences in specific basic numeracy skills such as numbering (e.g., verbal counting by ones) and number comparison (e.g., choosing the larger number), which align more closely with *Basic math activities*. Although this explanation is feasible, the association of such basic numeracy skills with similar standardized measures is quite robust (e.g., Nguyen et al., 2016) and longitudinal studies that have investigated the association between parents’ provision of basic math activities and specific numeracy skills in kindergarteners have failed to identify that particular link (e.g., Susperreguy et al., 2020).

Basic math activities emerged as a significant negative predictor of math at the end of K1 when baseline math skills were not considered. This negative relation between basic math activities and TEMA scores indicates that children who were exposed less frequently to basic math activities at K1 entry showed better math abilities at the end of the first year in kindergarten. A possible explanation of this finding is that parents of children with better math performance at the beginning of K1 (those who also showed better math skills at the end of K1) only focused on math activities that aligned with their children’s math skills—which probably involved already a basic understanding of addition and subtraction. This explanation is also supported by the interaction effect that was found—advanced math activities were more relevant for children with better math skills at entry to K1. In contrast, the parents of children with poorer math skills at K1 entry (those who also showed poorer math skills at the end of K1) probably focused on basic math activities to support the understanding of counting procedures, or numerical magnitude. Similar findings have been reported in other studies. For instance, in a cross-lagged analysis of the effect of HME on math in grade 1 students, Ciping et al. (2015) found a negative correlation between math skills in grade 1 and the frequency of formal home math activities in grade 2 (see also, Blevins-Knabe & Musun-Miller, 1996). Parents engaged more frequently in home math activities when they noticed (or received feedback from teachers) that their children were not performing at the same level as their peers. Silinskas et al. 2020 also reported negative relations between basic activities and children's math performance in a longitudinal study, with a similar argument—that parents are providing activities in response to their children's skills. Other studies that have considered a longitudinal approach to explore cross-relations between the home literacy environment and academic outcomes have also revealed a similar remediation effect (e.g. Silinskas, Leppänen, Aunola, Parrila, & Nurmi, 2010; Silinskas et al., 2012).

The fact that such association of *Basic math activities* with math is not observed after accounting for previous math knowledge at K1 entry and that the magnitude of the association of *Advanced math activities* on math decreases substantially (and changes as a function of baseline math knowledge—i.e., the interaction effect) suggests that parents' math practices (i.e., the type and frequency of home math activities) are strongly influenced by their children's math abilities. Indeed, it has been argued that parents adjust or modulate their math-related interactions, according to their children's performance (Saxe et al., 1987). Although evidence from cross-sectional studies is usually interpreted unidirectionally (even though HME and math measures are collected simultaneously), it is feasible that the provision of home math activities and children's development of math skills are reciprocal causes that work simultaneously over the first year in kindergarten.

Is HME a mediator of different facets of SES?

We found that subsidy status and mother's education operated through different mechanisms in predicting children's end of K1 math skills, which aligns with studies that have suggested that different facets of SES have distinct impacts on children's development (Duncan & Magnuson, 2003). The frequency of home math activities only related to the mother's education, not subsidy status. More educated mothers engaged their children more frequently in advanced math activities, which in turn affected their children's math achievement at the end of K1. It is feasible that less educated mothers relied on preschool to support their children's basic numerical knowledge. For instance, Holloway, Rambaud, Fuller, and Eggers-Pierola (1995) found that mothers from low-SES backgrounds expected preschool teachers to provide instruction in math skills to prepare their children for school (see also, Starkey & Klein, 2008). Our findings align with those from studies that have exclusively considered the role of the mother's educational attainment (and a representation of various SES strata). For instance, Susperreguy et al. (2020) also found that more educated Chilean mothers engaged their children more frequently in advanced (operational) math activities. Indeed, the magnitude of that association is similar to that found in the current study ($r = .15$). This correspondence is noteworthy given the substantial socio-cultural differences between Singaporean and Chilean parents.

It is worth noting that the role of mother's education on the frequency of both types of home math activities as well as the indirect effect through *Advanced math activities* were small. Crucially, the association of *Advanced math activities* with children's math achievement at the end of K1 remained significant after accounting for differences in math skills at K1 entry whereas SES disparities in math (related to the mother's educational attainment) vanished. This finding suggests that there are factors other than home math activities that probably contribute to SES disparities in math at earlier stages in development and that such disparities remain during the first year in kindergarten. Furthermore, as suggested by the interaction effect, the provision of home math activities may reflect a child's math ability rather than SES disparities. Thus, the study adds to research that suggests that not all the relation between SES and children's development is mediated through what parents do (Brooks-Gunn & Duncan, 1997), at least with regard to the development of math skills.

In contrast, subsidy status predicted math ability at the end of K1 directly and indirectly via earlier math ability but did not operate through HME. In the context of the current study, several reasons explain why HME is not predicted by subsidy status (independently of maternal educational attainment). There are programs in Singapore that aim at addressing the SES achievement gap early in development. For instance, the Mendaki's KMM@CC and Tiga-M (https://www.mendaki.org.sg) programs provide parents from low-SES backgrounds with resources to learn about early numeracy, educate them on types of activities in the home that might involve math, and aim to increase parents' confidence that they have the skills to engage in those activities. Such programmes may have attenuated the impact that low SES would have had on parents' use of math activities at home. In the same vein, the fact that only subsidy status was directly associated with math growth over the first year in kindergarten suggests that other than home practices or what parents do with their children at home contributes to increasing the SES-achievement gap in kindergarteners. In the context of the current study, the role of extracurricular tuition/enrichment activities or shadow education is notable. Although there is no formal evidence that children from higher-income families attend these activities more frequently than their less-advantaged counterparts, it is assumed that more educated parents adopt a more proactive interventionist parenting style and are willing to pay more for academic and non-academic tutoring (Teng, 2015). Thus, the prevalence of external tutoring may move some math activities from the home to the tutor.

Finally, it is worth mentioning that, despite some differences at the indicator level (which may reflect cultural artifacts), our HME model captured two well-differentiated factors that aligned relatively well with Skwarchuk et al.'s (2014) model of formal home math activities. This model has been tested in other countries and cultural contexts and shows good consistency (e.g, Susperreguy et al., 2020). Indeed, the magnitude of the associations of *Advanced math activities* with math and mother's education in the current study were similar to those reported in Susperreguy et al. (2020). Such cultural artifacts relate, for instance, to failing to identify a factor related to indicators that relied on playful contexts. All of the indicators in the factor structure that was observed included contexts and situations where parents provided direct support and/or instruction of numeracy and arithmetic skills, which may reflect the expectations of Singaporean parents for academic readiness. Other replications of Skwarchuk et al.'s (2014) model in Europe and America have included aspects that are contextualized in a playful context (e.g., playing board games, collecting objects, singing songs). This finding aligns with other works that suggest that Asian parents emphasize strengthening pre-academic skills early in development and that learning is perceived as a serious rather than playful activity (Luo, Tamis-LeMonda, & Song 2013; Rao, Ng & Pearson, 2010).

Conclusions and directions for future research

In line with studies that have used similar conceptualizations of HME and math achievement, our findings revealed a significant (although small; $r = .11$) association between kindergarteners' math achievement and the frequency with which parents engaged their children in math activities that involved basic arithmetic operations when other variables that may affect the provision of home math activities were considered (i.e., maternal education and baseline math knowledge). In the current study, we have also replicated the factor structure of HME (as well as the magnitude of the associations with the mother's educational attainment and math achievement) with a larger sample and a very different socio-cultural context. Besides, we have disentangled the association between SES and HME. We found that parents' provision of home math activities related to the mother's educational attainment rather than to household income (subsidy status in the current study), but that such provision may also reflect the child's math ability. In other words, the provision of home math activities and children's development of math skills are reciprocal causes that may work simultaneously over the first year in kindergarten. Indeed, our findings indicate that both maternal education and baseline math knowledge are sources of heterogeneity that cannot be overlooked when it comes to investigating how parents' practices impact kindergarteners' math achievement.

Several questions remain open for future research. For instance, we have used a frequency-based approach to measure the home mathematics environment, but we know nothing about the quality of parent-child interactions. Although findings from observational studies are mixed and there is poor correspondence between what parents report and what they do (mathematically) with their children (Mutaf-Yildiz et al., 2018b), research may benefit from observational measures to capture more precisely the nature of parent-child interactions. These measures may clarify, for instance, whether creating a supportive home math environment relates to the child being provided with resources but with limited input/interaction from the parent or parents create richer interactions. Whilst findings from the current study suggest that parents can actively contribute to their children's development, these questions may help to understand more specifically how parents nurture such growth and development.

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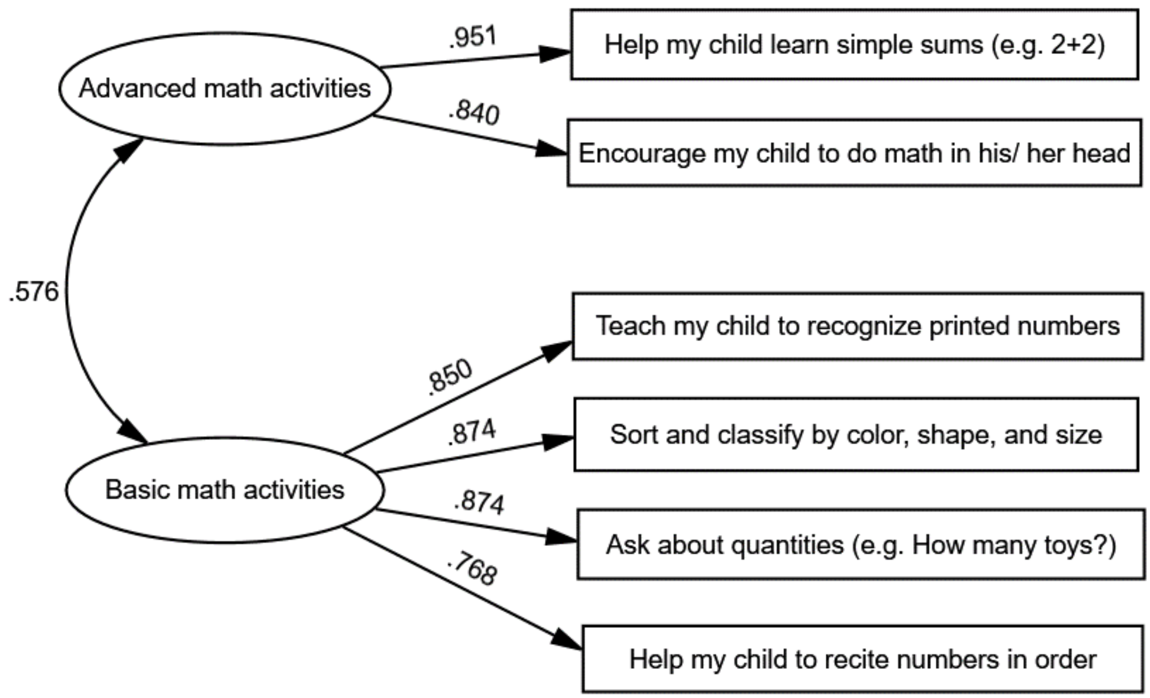
Tables and Figures

Table 1: 3-factor ESEM solution (standardized loadings) and descriptive statistics of HME items

	Items	F1	F2	F3	M	SD
1	Help my child learn simple sums (e.g. 2+2)	**0.85**	0.141	-	4.4	1.71
2	Encourage my child to do math in his/ her head	**0.78**	-	**0.17**	4	1.82
3	Talk about time with clocks and calendars	**0.32**	**0.301**	**0.25**	4.24	1.71
4	Help my child weigh, measure and compare	**0.31**	0	**0.55**	3.3	1.61
5	Play games that involve counting, adding or	**0.31**	**0.277**	**0.36**	3.98	1.61
6	Teach my child to recognize printed numbers	0.10	**0.857**	-	5.03	1.58
7	Sort and classify by color, shape, and size	-	**0.883**	0.02	4.76	1.67
8	Ask about quantities (e.g. How many toys?)	-	**0.837**	0.06	4.96	1.61
9	Play board games or cards	0.03	0.076	**0.59**	3.64	1.74
10	Encourage collecting (e.g. cards, stamps, rocks)	-	-	**0.76**	2.93	1.72
11	Help my child to recite numbers in order	0.07	**0.738**	-	4.89	1.71
12	Sing counting songs (e.g. Five Little Ducks)	-	**0.526**	**0.28**	4.42	1.83
13	Play computer games that involve numbers	0.01	0.095	**0.43**	3.14	1.81
14	Talk about money or compare costs	0.10	0.177	**0.45**	3.71	1.72

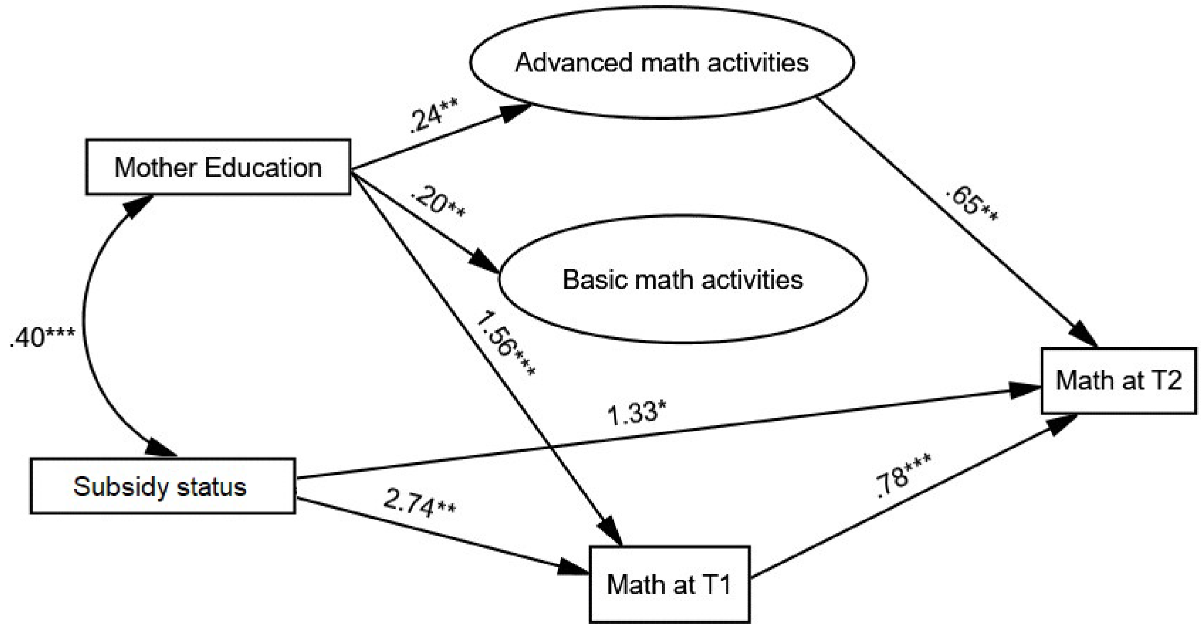
Note: Parameters in bold indicate significant loadings.

Figure 1: CFA structure of the Home Mathematics Environment (formal activities)



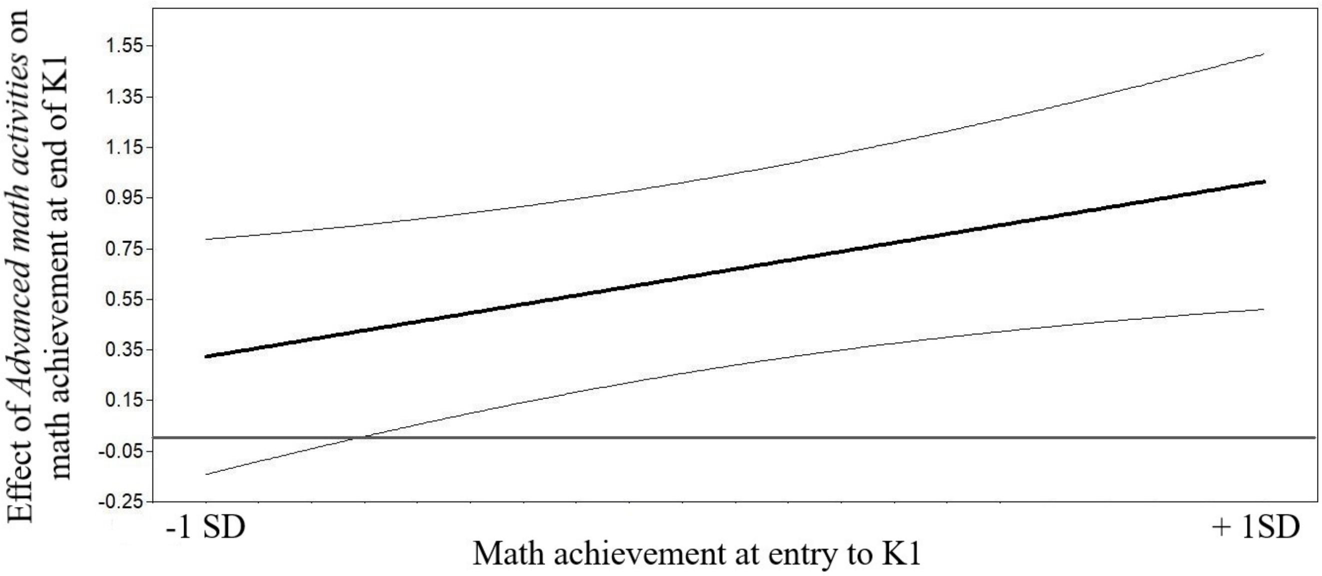
Note: Estimates on single-headed arrows are factor loadings. The estimate on the double-headed arrow relates to the correlation between factors.

Figure 2: Parameter estimates of the mediation model



Note: For clarity, indicators of the HME factors and control variables are omitted and only significant paths of the variables of interest are included in the diagram. Estimates on single-headed arrows are unstandardized regression coefficients. The estimate on the double-headed arrow relates to the covariance between variables (*** $p < .001$; ** $p < .01$; * $p < .05$). T1 and T2 correspond to the first and second time points, respectively.

Figure 3: Conditional effect (and 95% CI) of *Advanced math activities* on math achievement at the end of K1 as a function of math skills at entry to K1



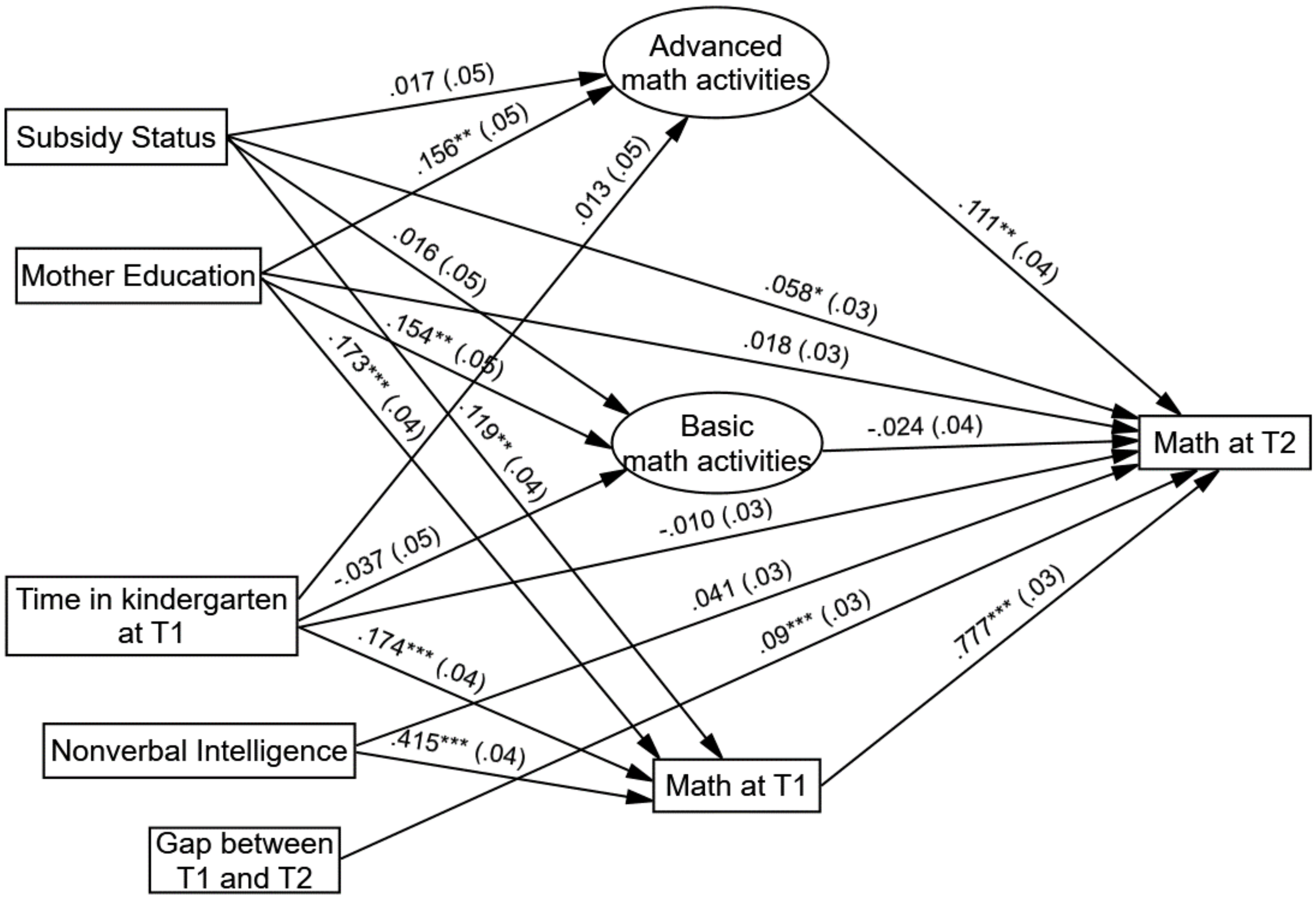
Note: Vertical axis: Estimated effect of *Advanced math activities* on math achievement at the end of the first year in the kindergarten. Horizontal axis: Math achievement at kindergarten entry (the range that is displayed relates to -1SD to +1SD of the centered mean).

Table S1: Descriptive statistics and correlations for the overall sample and by subsidy status

									r				
Overall	**M**	**SD**	**Min**	**Max**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1 Math T1	24.59	9.54	2	68	—								
2 Math T2	29.18	9.49	2	68	0.85***	—							
3 Mother Education	3.46	1.05	1	5	0.3***	0.3***	—						
4 Advanced HME	0.88	1.52	-2.36	3.67	0.26***	0.32***	0.17***	—					
5 Basic HME	0.6	1.29	-2.9	2.55	0.06	0.1*	0.16***	0.63***	—				
6 Nonv. Intelligence	15.32	5.18	2	32	0.48***	0.44***	0.13**	0.13**	0.06	—			
7 Time in K T1 (months)	3.38	1.75	1	7	0.28***	0.22***	0.11*	0.04	-0.02	0.12**	—		
8 Gap T1-T2 (months)	5.85	0.76	4	7	0.06	0.17***	0.09	0.11*	0.05	0.12**	-0.06	—	
9 Income (subsidized program; 0=yes)	.78	.41	0	1	0.29***	0.29***	0.41***	0.09*	0.07	0.1*	0.27***	0.02	—
Low-income (subsidized program)													
1 Math T1	19.32	8.76	2	49	—								
2 Math T2	23.88	9.21	2	53	0.83***	—							
3 Mother Education	2.64	1	1	5	0.3**	0.31**	—						
4 Advanced HME	0.62	1.5	-2.36	3.47	0.31**	0.31**	0.31**	—					
5 Basic HME	0.43	1.28	-2.9	2.53	0.12	0.14	0.19*	0.61***	—				
6 Nonv. Intelligence	14.31	4.67	3	29	0.53***	0.44***	0.17	0.14	0.00	—			
7 Time in K T1 (months)	2.45	1.74	1	7	0.18	0.01	-0.18	-0.17	-0.17	-0.03	—		
8 Gap T1-T2 (months)	5.82	0.71	4	7	-0.03	0.18	0.17	0.04	0.04	0.01	-0.23*	—	—
High-income (not subsidized)													
1 Math T1	26.03	9.24	2	68	—								
2 Math T2	30.61	9.05	2	68	0.83***	—							
3 Mother Education	3.68	0.95	1	5	0.19***	0.17***	—						
4 Advanced HME	0.95	1.52	-2.36	3.67	0.23***	0.31***	0.11*	—					
5 Basic HME	0.65	1.29	-2.89	2.55	0.02	0.07	0.14**	0.63***	—				
6 Nonv. Intelligence	15.6	5.28	2	32	0.46***	0.44***	0.09	0.12*	0.07	—			
7 Time in K T1 (months)	3.63	1.68	1	7	0.23***	0.19***	0.06	0.06	-0.00	0.13*	—		
8 Gap T1-T2 (months)	5.85	0.77	4	7	0.08	0.17**	0.06	0.12*	0.05	0.14**	-0.03	—	—

Note: T1 and T2 indicate first and second time points, respectively. * $p < .05$, ** $p < .01$, *** $p < .001$

Figure S1: SEM and standardized parameter estimates (and SE)



Note: Covariances between independent variables, indicators of latent variables, and covariances between residual variances of the mediators (*Advanced math activities, Basic math activities,* and *Math at T1)* have been omitted for clarity.

Table S2: Standardized parameter estimates (and SE) of the SEM investigating the interaction between HME and math at entry to K1.

	Interaction Math T1 x Advanced HME	Interaction Math T1 x Basic HME
Math T1	.772*** (.02)	.777*** (.02)
Interaction term	.059* (.02)	.041 (.02)
Mother Education	.020 (.03)	.019 (.03)
Subsidy Status	.063* (.03)	.061* (.03)
Advanced HME	.116** (.03)	.115** (.03)
Basic HME	-.028 (.03)	-.030 (.03)
Nonv. Intelligence	.044 (.03)	.041 (.03)
Time in K T1 (months)	-.014 (.02)	-.012 (.02)
Gap T1-T2 (months)	.092*** (.02)	.092*** (.02)

Note: * $p < .05$, ** $p < .01$, *** $p < .001$. For simplicity, factor loadings corresponding to the HME factors are not shown.