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Open-ended Problems for Higher-Order Thinking in Mathematics

Foong Pui Yee

Introduction

Singapore students' participation in Third International Mathematics and Science Study (TIMSS) put the country on the world map when their mathematics performance came in first among competitors from more than 40 countries. Such good results could have been a morale booster for our students, teachers and educators. Nevertheless in October 1996, the Ministry of Education commissioned an External Review Team to review the school curriculum in the light of future economic, technological and social needs, so as to propose changes needed to prepare our students for the 21st century. In its report the Team revealed that:

> *Currently, Singapore's education system is doing well. Our students excel in examinations and integrate successfully into our disciplined workforce. However, for Singapore to remain competitive in the world economy of the 21st century, education must broaden its focus beyond examination results and discipline. The education system must produce creative and critical thinkers who confidently display skills fit for tomorrow's workplace. It must also develop in our people the enthusiasm to learn new skills and ideas and the ability to communicate effectively ...(Curriculum Review Report, 1998)*

No doubt the Singapore students' excellent achievement in TIMSS should be applauded: they did well in acquiring basic mathematical skills and factual knowledge featured in such comparative international tests. The TIMSS items generally assessed lower-order type of mathematical thinking in multiple-choice format and closed-short-answer type of problems. The fact that the TIMSS items had high test-curriculum match with what they were teaching in Singapore schools, was an advantage. This also points to the fact that the existing curriculum may be lacking in developing desirable outcomes such as creative and critical thinking in our learners.

Reforms For Higher-Order Thinking in Mathematics

The trend for reforms in mathematics education is happening worldwide. The concern that schools are producing young adults to be experts at memorizing inert knowledge, performing simplistic tasks and listening passively has prompted initiatives to adopt a so-called 'open approach' in mathematics learning. Teachers are encouraged to develop learning environments where their pupils have more time and room to reflect, discuss and investigate on their own. Teachers provide the class an open learning environment in the form of open-ended problems that are integrated and set in real-life context. Creative and pupil-produced problems will be encouraged to de-emphasize the use of model-type routine problems. The aim is to develop mathematical problem solving and communication skills in the pupils and to give them an opportunity to learn in their own way and at their own pace. Instead of quiet individual computation work there are opportunities for pupils to work and talk mathematics in cooperative group. Fig. 1 lists the ongoing development in mathematics teaching for reforming education for all pupils.

Fig. 1: Ongoing development in mathematics teaching

From	*To*
➢ *Teacher/text steered*	➢ *Pupil participation*
➢ *Isolated problems*	➢ *Themes & integration*
➢ *Rules*	➢ *Insight*
➢ *Formal maths*	➢ *Maths in real-life context*
➢ *Model problems*	➢ *Creative problems*
➢ *Preproduced problems*	➢ *Pupil produced problems*
➢ *'Closed' problems*	➢ *'Open' problems*
➢ *Individual work*	➢ *Group project work*

To gauge whether higher-order thinking is happening in a mathematics classroom, there are three characteristics that can be observed:

- **You can hear students explaining, conjecturing, describing patterns or communicating their ideas** (There is direct teaching of specific problem solving and reasoning strategies).

- **You can hear teachers asking students why, what and how – questions that demand more than one-word answers** (There is an emphasis by the teacher on meaning and understanding).

- **You can observe students making choices about what procedure to use, or how to integrate knowledge to new and non-routine tasks, monitoring progress and evaluating solution** (There is classroom atmosphere that encourages student autonomy, persistence and independent thinking).

These characteristics are aspects of critical thinking skills that we want to inculcate in students for problem solving in the real world. The existing mathematics curriculum in Singapore schools can be said to be a traditional one, emphasizing contents with finished and well-defined inner structures. Mathematics is normally taught using whole class teaching, textbooks and regular testing. For individual work, students do a lot of practice sums to consolidate and reinforce the mathematics concepts or procedures taught by the teacher usually through expository methods. To challenge the pupils' analytical and reasoning skills numerous similar multiple-step problems are normally given where rote learning is involved. Pupils are trained to recognize certain problem types and apply some specific methods such as the "model" method to solve them. Such practices only reinforce instrumental learning that may hinder the development of critical thinking.

Mathematics achievement or competency is not just the number of sums or routine problems a child can do in a test, but the power to think. Now that the mathematics content reduction is in place for all Singapore schools, it will leave more time for teaching to emphasis creativity over rote learning. Fundamentally, the goals of mathematics education set in the 1990's for Singapore schools are sound but it is the traditional practice to attempt to reach these goals that will need to be reviewed and expanded.

What are Open-ended Problems?

The idea of using open-ended problems to improve mathematics teaching to develop and foster methods for teaching problem solving and thinking skills has appeared in the curriculum of many countries, in the form that there is specific curriculum time left for teachers' freedom to adopt an 'open approach'. Very recently in Japan (Landers 1998), the Ministry of Education has approved a new national policy to take effect in 2002 that will cut the prescribed curriculum by 30%. Their aim is to trim the time devoted to traditional subjects and use it for open-ended problems, out-of-school activities and integrated

projects that cut across disciplines. In Germany about one-fifth of the teaching time is left content-free, in order to encourage teachers to use mathematical activities (Pehkonen 1995). In Sweden (Prim-gruppen 1995), open-ended problems have been used in the final assessment of students.

The method of using open-ended problems in the classroom to promote mathematical discussion has been ongoing since the early eighties when the theme of "Problem Solving in School Mathematics" had been adopted as the NCTM's 1980 "Agenda For Action" in America and endorsed by the Cockroft Report in England. From then on, the use of some form of open-ended problems in the classroom has spread all over the world including Singapore. However the concept of 'open-ended' problems was not well defined as there were many types of problems that were put forward as open-ended problems.

In Britain, the use of investigations (Shell Centre 1984) became popular in mathematics teaching; in the Netherlands (Lange 1996) they used real-life situations and called their method 'realistic mathematics' and in Norway and Denmark (Niss 1992), open-ended task through project work were adopted. Problem posing (or problem finding or problem formulating), problems without a question and problem variation ('what-if' method) were some of the many examples of different types of open-ended problems. Broadly underlying these varieties are three basic criteria:

- It should give all students a chance to demonstrate some mathematical knowledge, skill and understanding.
- It should be rich enough to challenge students to reason and think, to go beyond what they expect they can do.
- It should allow the application of a wide range of solution approaches and strategies.

Extending Textbook Exercises to Open-ended Problems

The use of open-ended problems within the existing Singapore curriculum should be encouraged. Problems have always been considered an important part of a mathematics lesson to introduce a new topic or to assess understanding. It is essential, however, to distinguish between a problem and an exercise commonly found in end-

of-chapter of a school mathematics textbook. The typical primary and secondary school textbook exercises, worksheets and assessment test items are usually presented in short answer or multiple-choice formats. There are also complex word problems to exercise application and analytical skills. They are designed with a narrow focus on a few specific skills, concepts and procedures that have been previously covered. They constitute the 'mechanical' aspect of learning and practicing mathematical procedures and formulas. Consider, for example, these two typical textbook sums on area and perimeter:

Textbook Exercises:

1. ***Find the perimeter of a rectangle with a length of 8 m and breadth of 17 m.***

2. ***Find the length of a rectangle of area, 48 sq. m and breadth of 6 m.***

These are closed tasks that have a wrong or right answer and is solved by in one step and by one way. Usually, a pupil will work individually to compute the answers after applying the correct formula. Such questions enable the teacher to know whether the pupil has learned the procedure for calculating the perimeter. However, these items do not enhance pupil's understanding of a perimeter or its the relationship with its shape.

However, a teacher can extend and transform such exercises into open-ended problems for group work. Consider the alternative open-ended problem as follows:

Farmer Lee wants to make a fence for his chicken coop in the shape of a rectangle. He has 20 metres of fence. What are some of the sizes of rectangles that he could make? Which shape would be the best?

Think first for a few minutes before beginning to discuss with each other. Listen to each other and try to understand each other's way of reasoning. Solve the problems through discussing, calculating, writing and drawing. When you have reached one or more solutions, each member of your group should be able to explain how you have reasoned.

In solving this problem, the pupils working in a group of three or four develop critical thinking skills: the problem now poses more than one challenge, suggested as follows:

- There are many possible answers

- A variety of solution approaches can be applied
- Reasoning and communication skills have to be used
- Mathematical concepts to real-life application can be applied
- Thinking is extended
- Sophisticated strategies may be used

Similarly, another textbook sum on area can be extended to an even more open-ended problem that requires more divergent thinking with no convergent answer, such as in the following example:

> ***Find the area of a rectangle with length of 12m and a breadth of 4m.***

can be extended to an open-ended problem:

> ***For this school holiday, your teacher, Miss Tan wants to paint the wall at the back of your classroom. The wall is 12 m wide and 4m high. It takes one can of paint to cover 10 square metres, and the paint is sold at $32.50 a can. What else does Miss Tan need to think of? Make a plan for Miss Tan's trip to the store for supplies for this painting job.***

This problem for pupils to do as group work gives them opportunities to exercise the following skills:

- Creativity and imagination
- Estimation, rounding and mental computation
- Strategies: guess & check, make a chart or list
- Consideration of not only the numerical computations but also 'realistic' aspects in planning a shopping list

Getting Started for an "Open Approach"?

According to Kulm (1994), when teachers are asked, "What do you value the most, and what do you think is the most important about mathematics?", most of them would say something like "Being able to solve problems, being able to think, being able to use your mind, being able to actually apply and use ideas in mathematics". Most teachers of

mathematics in Singapore schools would share the same view. However, if the study by Kaur and Yap (1996) is of any indication, then teachers in Singapore schools, whether primary or secondary, would need a lot of support and encouragement to get started in implementing an 'open approach' to promote higher order thinking in our mathematics curriculum.

Kaur and Yap studied 26 mathematics teachers and sixty-four secondary two classes. They found that most of the lessons were highly structured, with specific objectives. Lessons were usually expository in nature, punctuated with class practice. There was more emphasis on procedures, answers and accuracy than on concepts and processes. There was an over emphasis on drilling students for class tests and examinations. Teachers drilled students with questions from the 10 year series of past GCE 'O' level and 'A' level examinations. Completing the syllabus became the priority and teachers complain of little time left for developing process skills or higher order thinking and communication.

In view of such report, teachers, heads of departments, principals, teacher educators and MOE must work hand in hand in designing the best ways to broaden our mathematics education that will be relevant to the vision of "Thinking Schools, Learning Nation".

SUGGESTIONS

Instead of waiting for top-down regulations from the central authority for alternative assessment policies or new textbooks, teachers can on their own work as a team within the school to develop some comprehensive plans for new approaches to foster critical thinking skills in pupils. They should start at the beginning of the school year to incorporate open-ended problem solving and group work within curriculum time in lieu of the content reduction. Pupils need to be trained as early as possible to work cooperatively and given lots of opportunities to talk mathematics in the classroom.

Instead of continuing to give the same amount of workbook exercises or worksheets, reduce them by 30% to make way for open-ended problem solving in class. In the early 90's, when the focus on problem solving of non-routine problems in the mathematics curriculum was first initiated, teachers complained about lack of resources or time to extend textbook exercises to problem solving and

mathematical thinking (Foong, Yap & Koay 1996). By now, the author believes that there is no such shortage of resource materials on problem solving or open-end problems in most schools' libraries and from the internet. There are innovative teachers in some schools who are already using non-routine mathematical problems for enrichment activities with their pupils. For teachers who have not started on such an open approach, it is timely to try it, by observing pupil participation and recording the specific mathematical and reasoning skills involved.

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