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The contribution of working memory to structural and procedural errors in algebraic problem solving

Introduction

A number of studies in the psychological and educational literatures have examined the issues associated with learning and solving word problems (See Kintsch & Greeno, 1985; Mayer & Hegarty, 1996 for reviews). However the number of studies examining the role of working memory and word problem solving is small, with most studies concentrating on arithmetic word problems. Much less is known about the contributions of working memory and reading abilities to the performance of more complex mathematical tasks such as solving algebraic word problems.

How do algebraic and arithmetic word problems differ? The most salient difference is that in algebraic word problems the unknowns are treated as knowns. Singapore primary children are taught to use the model method to solve arithmetic as well as algebraic word problems. In algebraic word problems, rectangles represent unknowns whilst in arithmetic type word problems, the rectangles are known values.

To draw the model, it is necessary for children to read the question presented as text (T), translate this information presented in the textual mode into a cohesive structure known as the model (S). Next, to solve the problem; children translate the information embedded in the model into a set of arithmetic procedures (P). We hypothesise that strong linguistic skills are needed for each phase of the translation process – Text to Structure (TS) and from Structure to the Processes (SP).

Question presented as text (T)

Translate text into arithmetic statements (procedural aspect) (P) ← Translate text into a model (S) (structural aspect)

Figure 1: Modes of representation and possible routes to solve word problems when the model method is used.

Give the dearth of information directly relevant to algebraic word problems; we used a standardised measure, the Working memory Test Battery for Children (WMTB-C) (Pickering & Gathercole, 2001) to examine the working memory span. The contribution of reading ability was measured using a comprehensive test that has been modified and normed for the local population. We controlled for differences in general intellectual functions by using an abbreviated intelligence measure. A mathematics test comprising ten complex word problems was used to assess children's ability to solve algebraic word problems.

In this study we examined the contribution of working memory to the types of errors made in solving algebraic type word problems. Children's responses to each question was analysed for the types of translation errors made, TS, SP. Also the responses were further analysed for the different types of errors. These errors are listed in Table 2.

Method

Please refer to Poster B2-60—The contributions of working memory, language, and intelligence to individual differences in algebraic word problem solving for details on participants, material and procedure.

Children were asked to use the model method to solve the ten items.

Types of Errors for Mathematics Performance Tasks
There were three levels of analysis for the mathematics task.

Level 1 - The word problems were first scored for overall accuracy.

Level 2 - A correct solution was credited with full TS, SP and Computation (C) scores.

TS score was awarded if a response had a correct model but erroneous translation into procedures.

SP score was awarded if a wrong model was constructed but the translation based on the constructed model was correct.

Partial TS and SP scores were awarded if part of the model was correct and the translation was appropriate for those parts.

Examples of children's responses are provided in Table 1.

Table 1: Examples of children's responses

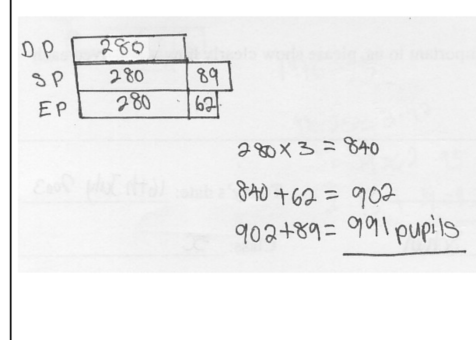
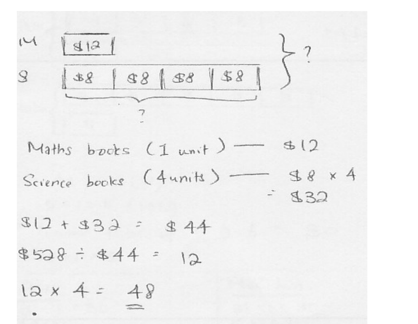
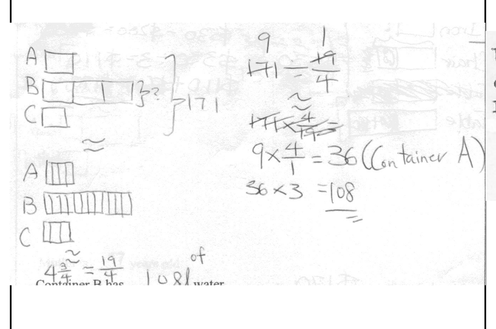
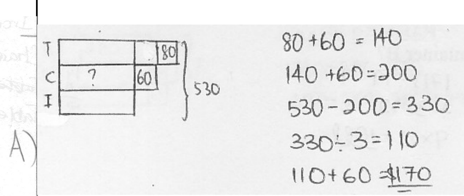
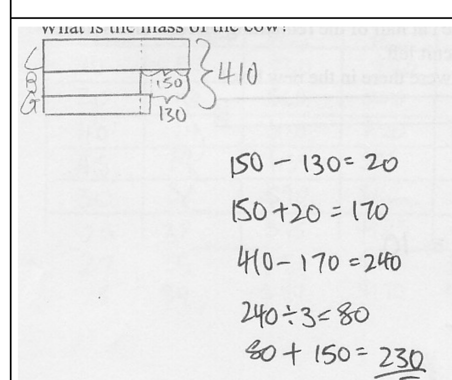
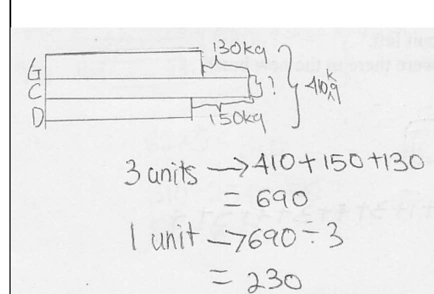
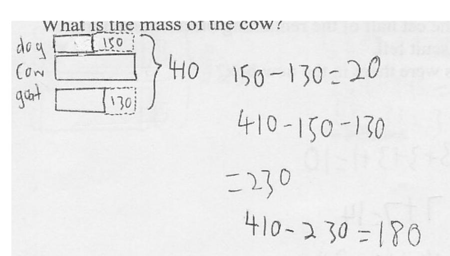
Responses with full TS, SP and C scores :	
	
Item 1	Item 2
	
Item 3	Item 5
	
Item 4: Dog as generator	Item 4: Cow as generator
Responses with full TS and C scores but incomplete SP scores	
	
Item 4	Item 5

Table 2: Categories and Descriptions of Errors

	Catego-ries		Description
1	CA		Correct Answers irrespective of methods used
2	AE		Arithmetic Error – conceptually correct but error in computation
3	G		Generator error – The wrong base or generator was used.
4	CE	LA	Language – Multiplicative and additive errors, more than and less than (subtracts rather than adding), reversal errors
5		F	Fractional part of the whole errors
6		SP	Structural- procedural errors. Correct methods of solution using the model method but erroneous translation into procedures
7.		KP	Knowledge of Procedures: Attempts to solve a given problem but because of gaps in knowledge of the procedures, failed to do so.
8.	UO		No notion of what to do. Numbers were used to arrive at any solution.
9	NR		No response

Results

Inter-correlations

Variables	TS	SP	C	2	3	4	5
Phonological loop	.33**	.32**	.17*				
Visual spatial	.34**	.35**	.09	.21*			
Central executive	.47**	.45**	.24**	.51**	.37**		
Performance IQ	.49**	.51**	.16*	.29**	.40**	.35**	
Literacy	.57**	.54**	.17*	.57**	.40**	.56**	.39**

** p < .01, * p < .05

Multiple Regression analyses on Translation Errors TS, SP and C scores

Variables	TS		SP		C	
	β	*sr*	β	*sr*	β	*sr*
Phonological Loop	-.08	-.07	-.06	-.05	-.03	-.02
Visual spatial	.01	.01	.04	.04	.02	.02
Central executive	.19**	.14	.17**	.13	-.22**	-.17
Performance IQ	.30**	.26	.32**	.28	-.17	-.15
Literacy	.40**	.29	.34**	.25	-.07	-.05

Note,

for TS: . $R^2 = .44$, $R^2_{adjusted} = .42$, $F(5,145) = 22.49$, $p < .01$, ** $p < .05$
for SP: $R^2 = .41$, $R^2_{adjusted} = .39$, $F(5,145) = 20.39$, $p < .01$, ** $p < .05$
for C: *Note.* $R^2 = .14$, $R^2_{adjusted} = .11$, $F(5,145) = 4.79$, $p < .01$, ** $p < .05$

MANOVA analysis
This analysis was performed to examine group differences in working memory, language and intelligence from the types of errors made.

The modal type of error, for the set of 10 questions, made by each individual was obtained. Those who scored perfectly or made multiple types of errors were excluded from this analysis.

- Wilks' Lambda F(15,185.359)=1.799, p<.04
- Tests of Between-Subjects Effects demonstrated that Literacy is the only variable that is significant F(3,71)=3.800, p<.02

Pairwise Comparisons demonstrated that the AE group and UONR group differed significantly in their Literacy measure, D=.934, p<.02

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Discussion

Children with greater central executive function were more accurate in both the translation tasks, TS and SP.

Also children with higher performance IQ and better literacy skills were more successful in these two translation tasks.

Children with greater central executive function demonstrated greater accuracy in their computation skills. Neither performance IQ nor literacy skills were significant contributory factors towards success in this task. One reason for this finding could be because children are so well drilled with arithmetic sums that they are highly competent to complete such tasks. This would suggest that these children would score highly if they were given a test comprising purely of computation sums.

That literacy skills is an important contributory factor towards success in both levels of translation is not surprising. Good command of language is needed to understand what is presented in the question in text mode.

Furthermore language is needed to translate the mathematical information in text mode into pictorial forms as children articulate relational information into appropriate shapes and also how to organize the shapes into a coherent whole in the form of a suitable model.

At the next level of translation, SP, language is again needed to translate information presented in the model into arithmetic operations. For example, 4 equal rectangles placed below another should be translated as four times as many and therefore the operation needed could be multiplication or division.

Performance IQ is a contributory factor towards success suggests that children who can manipulate shapes are likely to translate the text into the correct model and also from the model into the correct set of arithmetic procedures. This suggests that children who are better able to process pictorial information are likely to be able to organize information into pictures, possibly they are able to visualize the model to be in their minds.

Perhaps children with better central executive function, literacy skills and performance IQ may be more successful in the way they organize, visualize and hold the information presented in the questions, hence explaining for their success in these algebraic word problems.

Conclusion

The implications of these findings on pedagogy is significant.

Knowledge of pure computation skills does not mean success in algebraic word problem solving. Rather children who can articulate their thoughts verbally and pictorially are more likely to succeed in such tasks.

The findings suggest that

Children need to understand what they read *and to articulate* their understanding into words, verbally as well as pictorially.

Children need to be exposed to activities where they use language to articulate arithmetic expressions verbally as well as pictorially.

Pickering, S. J. & Gathercole, S. E. (2001). *Working Memory Test Battery for Children*. Kent: The Psychological Corporation.
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Mayer, R. E. & Hegarty, M. (1996). The process of understanding mathematical problems. In R. J. Sternberg & T. Ben-Zeev (Eds.), *The nature of mathematical thinking* (pp. 29 –53). Mahwah, NJ: Lawrence Erlbaum.

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