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EDUCATION RESEARCH FUNDING PROGRAMME

PROJECT CLOSURE REPORT

Mathematical Problem Solving for Everyone: INfusion and Diffusion

By

TOH Tin Lam (PI), TAY Eng Guan, LEONG Yew Hoong, QUEK Khiok Seng, TOH Pee Choon, DINDYAL Jaguthsing, HO Foo Him (Co-PIs)

National Institute of Education
Singapore

EXECUTIVE SUMMARY

INTRODUCTION/BACKGROUND

This project MInD builds on the success of the previous project Mathematical Problem Solving for Everyone (M-ProSE, OER 32/08TTL). This section provides a brief description of M-ProSE, which we have completed in 2012.

Anecdotal evidence from day-to-day mathematics classrooms shows that students are generally resistant to following any model of problem solving. This is true even for the higher achieving students. The metacognitive part of the problem solving process still left much to be desired. In an attempt to 'make' the students follow through a problem solving model (in MProSE we chose Polya's model because it was used in the Singapore mathematics curriculum, although any other sound problem solving model would be equally viable), especially when the students were stuck with a particular problem. The research M-ProSE achieved a paradigm shift in the way students looked at these 'difficult, unrelated' problems which had to be done in the 'special' problem solving classes (we call it the mathematics practical lessons), in a way that science practical lesson is to science theory lesson.

In these practical lessons, the essential stages and processes of mathematical problem solving was brought to the forefront of consciousness (while in the usual mathematics classroom, the emphasis is on the mathematical content instead of the mathematical processes). Data from the single M-ProSE research school clearly shows that the students were able to successfully solve the problem and even expanded on based on the practical paradigm, which plays a critical role in cutting a groove at the initial stages of students' learning of problem solving. Evidence shows that students at the lower secondary level were able to acquire a problem solving model and develop their own habits of problem solving.

Based on a survey conducted during a problem solving symposium for secondary school mathematics teachers, our team felt that this concept of a "mathematics practical" is equally applicable to other Singapore schools, although the instructional package needed to be tweaked locally to meet the demand of the individual schools.

STATEMENT OF PROBLEMS

Since the 1990s, mathematical problem solving has been the heart of the Singapore Mathematics Curriculum. Subsequent revision and refinement of the mathematics curriculum have maintained this central role of problem solving within the curriculum. School teachers do teach problem solving in their mathematics classes, *but* it is done as how the schools interpret it. Quite commonly, from the researchers' experience interacting with the mathematics departments of the various schools, for instance, teaching of mathematical problem solving in the mainstream schools have been reduced to the teaching of "heuristics", possibly establishing a one-to-one correspondence between a particular problem type and a particular heuristics to be used – the whole problem solving process is 'lost' in the midst of this way of teaching. Also, more schools are equating problem solving to training for mathematics competitions thereby isolating problem solving to the elite few and putting it outside the curriculum, although it is quite clear that the intent of problem solving is its integration within the main school curriculum. Local researchers have also suggested that there is much room for improvement in creating a more effective and efficient learning environment for developing students' problem solving skills (see for example, Fan & Zhu, 2007; Foong, 2009).

PURPOSE OF STUDY

The research project MInD took up this challenge to explore the *infusion* of problem solving in the mathematics curriculum in the M-ProSE research school and identify this as one of the research objectives. MInD built on the problem solving design in M-ProSE (OER 32/08 TTL) research school by adapting it to accommodate the demands of the mainstream schools. This stage of propagation of innovation is to us *diffusion*, in which (1) we adjusted the M-ProSE problem solving module to suit the specific needs of the mainstream schools; (2) we provided professional development and support for teachers in the mainstream schools by both familiarising them with genuine problem solving, the use of the Practical Worksheet as a pedagogical and assessment tool; and also mathematics content upgrading.

The MInD project also contributed towards developing alternative assessment for mathematical problem solving. The Practical Worksheet introduced in M-ProSE was used as a form of assessment both *of* learning problem solving and *for* problem solving. This assessment strategy is based on the theoretical bedrock of problem solving framework developed by Polya and further refined by Schoenfeld (Toh, Quek, Leong, Dindyal, Tay, 2011). Moreover, the assessment adaptations that we made took into account the local factors within the Singapore education landscape, such as the importance of high stakes examinations.

PARTICIPANTS

The MInD project was trialled on five schools (one M-ProSE research school, and four other mainstream schools, stretching from IP school to a typical mainstream Singapore school), attempting to capture a wide spectrum of Singapore schools. All the teachers involved were required to go through a series of professional development lessons as described in the preceding sections. The entire cohort of students from one particular level and stream of the participating schools were introduced to the problem solving module.

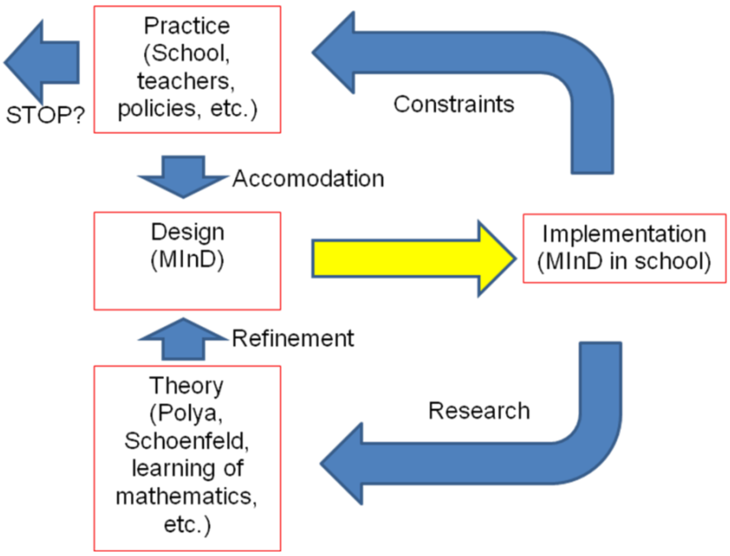
The researchers rode on the affordance of the research to use the M-ProSE approach to teach undergraduate mathematics to their student teachers. It was reported that there was observable positive changes in the student teachers' performance and attitude towards mathematics.

METHODOLOGY / DESIGN

MInD used "design experiment" (Brown, 1992; Collins, 1999; Doerr & Lesh, 2003; Middleton, Gorard, Taylor & Bannan-Ritland, 2006; Quek et al, 2011; Wood & Berry, 2003) as the methodological backbone. Design experiment arose from the attempts of the education research community to address the demands of research in real-life school settings in all its complexity. The methodology of design experiment argues for the application of multiple techniques to study a complex phenomenon in education. As such it permits the use of a whole spectrum of methods which include participant observation, interview, video-taping, and paper-and-pencil testing to provide corroborative evidence for findings.

In our case, we began with a theory regarding mathematics education. Based on this theory, a design is conceptualised to improve learning. The initial design is then implemented in a suitable 'real-world' context (the school). Research methods are then used to examine both the "accuracy of the underlying … theory and the power of the intervention." Based on the findings of the research, refinement is made to the design which is further implemented and the implementation-research-refinement cycle is iterated until the design-experimenter is satisfied.

In our work in the earlier research M-ProSE (OER 32/08 TTL), we were convinced that the teachers and school that have to implement the design have a key role in the final design itself. We adopted the model of design experiment which includes the designer, the researcher and experienced teachers (Black, 2009). We observed that there were two types of changes we had to make along the design process: (1) refining; and (2) accommodation. *Refining* is quite widely discussed in literature; while *accomodation* involves making changes to meet the realistic constraints faced in practice. We summarise the underlying structure of our proposed design-experiment using the following figure.



FINDINGS / RESULTS

We classify the findings of our research project under three main categories:

1. Students: As evident from the students' work, they were able to respond to the problem solving processes as required by the MInD problem solving module. Not only were they able to apply various heuristics to attempt solving a given problem (Stage 3 of Polya), they were able to communicate their understanding of the problem and how they decided on a particular plan in solving a problem (Stages 1 and 2 of Polya). More importantly, most students proceeded to check and expand the problem (Stage 4 of Polya) after they had solved the problems. In short, they were able to exhibit the entire processes of problem solving.

We found significant correlation between students' performance in MInD assessment and their school mathematics achievement test. Not only that, students' attitude towards the MInD problem solving module was an important contributor to their mathematics achievement. One possible explanation was that students' positive attitudes towards Polya's problem solving model and the learning module may allow them to appreciate the usefulness and value of our intervention. As put forth by Ma and Kishor (1997), students' engagement in the mathematics learning (in our case, problem solving intervention) would probably become more active and meaningful. Consequently, they could be more willing to apply what they had learnt from our intervention to their daily mathematics learning and eventually their school examinations. For example, the heuristics and checking approaches students acquired from our intervention might enable them to solve the test problems in a more versatile and cautious manner, which would perhaps result in better mathematics achievement.

2. Student teachers: The student teachers who read the undergraduate mathematics courses through the use of MInD approach (mathematics practical) performed better in their continual assessment and the final examination, compared with their seniors in the previous batch. What is heartening to the course instructor, for example, is the ability of the student teachers to check and expand a given problem, thereby discovering "new mathematics" in the process of solving a problem. The student teachers' feedback was generally positive for this new approach, and they were able to solve a rather complicated multi-step problem in the final examination without much difficulty. This could shed light on how mathematics content teacher education course could be re-structured to maximise the benefit to student teachers in undergraduate mathematics education.

3. Schools: We report (1) the sustainability of the problem solving (PS) module; (2) infusion of PS into the regular mathematics curriculum; and (2) teacher readiness in teaching problem solving.

(1) All 5 schools in the discussion kept the PS module. Schools A and E maintained the PS module as an examinable 9 lesson module in its modular curriculum. The module was also endorsed by the parent university of School A. Schools B and D reduced the number of lessons to 7 and 6 respectively, while School D modified the module to five double period lessons. The modifications were made to align with their tight schemes of work. Modifications were also made to the numbers, types and difficulty of the problems. The schools used props and videos to support the PS module. For example, the participant from School A said in the Focus Group Discussion: "… video showing how the 4 stages are done to the students, and introducing more concepts for problems during our 'E-learning' week. Whereby students can watch on-line video. So teaching is not only in the classroom, students can themselves do self-directed learning in terms of PS techniques." Data of School E was collected after the Focus Group Discussion conducted for Schools A, B, C and D.

(2) The language of PS (understand the problem, heuristics, stuck, etc.) permeated to many other mathematics lessons in at least three of the schools. School C: "And we also come out with an explicit way of maths language for our teachers so that for every problem that you do in class, you will do the same, 'how do you understand the problem', 'what is given', so we have a language for the teacher to help them to facilitate as a way for PS, as a whole curriculum be it in the syllabus or MProSE." School D worked with the researchers to redesign 3 units in the mathematics curriculum to include PS as a pedagogical approach. "And because we have done it in Secondary 1 as a whole module and in Secondary 2 we do the RU, the Replacement Unit. So when we teach the topic with the MPROSE problem,

either in the beginning, in the middle or towards the end also. So it's an infusion of MPROSE. So they still keep in touch with it."

(3) School A reported that teaching materials and support have been systematically developed for new teachers. School B suggested that teachers in pre-service training should learn how to teach PS (We note here that the schools use videos provided by the research team to train new teachers as well.) Schools B, C and D reported that their teachers work on solving new problems themselves. School E reported (after the Focussed Group Discussion) that the Secondary Two level coordinator is in-charge of conducting professional development for their teachers new to problem solving based on the training video.

IMPLICATION TO POLICY MAKERS

This project provides ideas on how the "hard and unglamorous" work of pushing mathematical problem solving, which is the spirit of mathematics education, through in practical terms in the school mathematics classrooms. It shows the feasibility of infusion of mathematical problem solving within the school curriculum without compromise to the other aspects of teaching mathematics. More importantly, the process of scaling up of an innovation is fully demonstrated from our first project OER 32/08 TTL to the current project OER 22/12 TTL. It provides the Ministry of Education an alternative model of scaling up of an innovation for consideration, which shows the dynamics between the designer and the teachers.

IMPACT TO SCHOOLS

First and foremost, the problem solving module introduced in this research has become a mainstay in the mathematics curriculum in the five research schools. This demonstrates the level of buy-in and sustainability of our approach in the research schools. As far as the researchers are aware, several other schools are adapting some ideas of our problem solving research to design their own models of mathematical problem solving within their school curriculum.

The findings of this research have resulted in a series of professional development workshop on mathematical problem solving designed for practicing teachers. This approach has been introduced in the pre-service teacher education programme for PGDE (Sec) and PGDE(JC); and has resulted in a new problem solving module for all undergraduate mathematics students at Year One, after the most recent AS curriculum review.

The Ministry of Education has also adopted some ideas from our research in their design of the new H3 mathematics curriculum. In particular, Ministry of Education has approached Head MME to conduct the problem solving part of the H3 mathematics curriculum for A-level students in 2017.

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KEYWORDS

Mathematical Problem Solving; Practical Paradigm; Infusion; Diffusion

Mathematical Problem Solving for Everyone: INfusion and Diffusion

TOH Tin Lam (PI), TAY Eng Guan, LEONG Yew Hoong, QUEK Khiok Seng, TOH Pee Choon, DINDYAL Jaguthsing, HO Foo Him (Co-PIs)
National Institute of Education

INTRODUCTION/BACKGROUND

This project Mathematical Problem Solving for Everyone: Infusion and Diffusion (abbreviated as MInD, OER 22/12 TTL) builds on the success of the previous project Mathematical Problem Solving for Everyone (M-ProSE, OER 32/08TTL). This section provides a description of M-ProSE (OER 32/08 TTL), which we have completed in 2012.

Anecdotal evidence from day-to-day mathematics classrooms shows that students are generally resistant to following any model of problem solving. This is true even for the higher achieving students. The metacognitive part of the problem solving process still left much to be desired.

This project is an attempt continuing the work of M-ProSE (OER 32/08 TTL) to 'make' the students follow through a problem solving model (in M-ProSE, we chose Polya's model because it was used in the Singapore mathematics curriculum, although any other sound problem solving model would be equally viable), especially when the students were stuck with a particular problem. The research M-ProSE achieved a paradigm shift in the way students looked at these 'difficult, unrelated' problems which had to be done in the 'special' problem solving classes (we call it the mathematics practical lessons), in a way that science practical lesson is to science theory lesson.

In these practical lessons, the essential stages and processes of mathematical problem solving was brought to the forefront of consciousness (while in the usual mathematics classroom, the emphasis is on the mathematical content instead of the mathematical processes). Data from the single M-ProSE research school (OER 32/08 TTL) clearly shows that the students were able to successfully solve the problem and even expanded on based on the practical paradigm, which plays a critical role in cutting a groove at the initial stages of students' learning of problem solving. Evidence shows that students at the lower secondary level were able to acquire a problem solving model and develop their own habits of problem solving.

Advancing from here, the M-ProSE team felt that this concept of a "mathematics practical" is equally applicable to other Singapore schools (in the project MInD OER 22/12 TTL, we included, in addition to the single M-ProSE research school, an IP school, an autonomous school and two typical mainstream schools), although we believed that appropriate adjustment must be made to the original design of M-ProSE.

MInD (OER 22/12 TTL) built upon this initial foundation of M-ProSE (OER 32/08 TTL) to scale out (Infuse) and scale up (Diffuse) the innovation to other Singapore schools in Singapore. One of the key enablers of scaling innovations is a productive collaborative researcher-practitioner partnership. This is an enterprise that requires a careful cultivation and an ongoing re-commitment. In fact, according to Lemke and Sabelli (2008), "[t]he development of effective partnerships takes 5 – 10 years" (p 125). Seen from this perspective of partnership building, M-ProSE was the beginning of the process which MInD proposes to continue. In terms of this partnership building exercise, M-ProSE research school recommitted in writing to infusion under MInD. In addition, 13 schools expressed interest in the proposal of MInD.

Scaling up research involves the effort to "reproduce an effective practice in a considerably greater number of classrooms and schools" (Fuchs and Fuchs, 1998) and the investigation of "the effectiveness of education interventions … when applied on a large scale" (Institute of Educational Science, 2004). The notion of scale-up is clarified in two comprehensive volumes—*Scale-up in education: Ideas in principle, Volume 1,* and *Scale-up in education: Issues in practice, Volume* 2—edited by Schneider and McDonald (2007).

Consisting of contributions from a multidisciplinary and interdisciplinary team of renowned researchers, these two volumes discuss the conceptualization of scale-up, clarify methodological concerns, highlight issues, and present principles for successful scale-up. In the editors' words "Scale-up is the enactment of interventions whose efficacy has already been established in new contexts with the goal of producing similarly positive impacts in larger, frequently more diverse populations [mainstream Singapore schools in our case]." (p. 5). Efforts and interests in scaling up are not new. Within mathematics education, a well-known scaling up research was done by Jim Kaput in his effort to scale up his innovation in teaching mathematics of change and variation (Kaput, 1994, 1997). Kaput was keen to find out: What can we learn from research at the next level of scale (beyond a few classrooms at a time) that we cannot learn from other sources? Scaling up research helps researchers to focus on the robustness of an innovation when used by varied students, teachers, classrooms, schools and regions (Roschelle, Tatar, Schechtman & Knudsen, 2008). Even with a single scaling up study, researchers can characterize how an innovation holds up as it spreads beyond the original trial classroom (Baker, 2007).

The diffusion of innovation was first explained by Rogers (1983) as one which is dictated by uncertainty reduction behaviour among potential adopters during the introduction of innovations. Innovative practices offer its potential adopters new ways of handling the problems, but the uncertainty as to whether it will be better than the existing method poses an obstacle to the adoption process. To overcome this, potential adopters must seek additional information, particularly from their peers. Rogers (2003) also proposes five factors that influence the rate of adoption: observability, trialability, compatibility, complexity and relative advantage.

STATEMENT OF PROBLEMS (I.E., JUSTIFICATION FOR THE STUDY)

In Singapore, mathematical problem solving has been established as the central theme of the primary and secondary mathematics curriculum since the 1990s. Subsequent refinements of the mathematics curriculum have maintained this central role of problem solving in the SMC. The Ministry of Education (MOE) syllabus document states explicitly the importance of problem solving: "Mathematical problem solving is central to mathematics learning. It involves the acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real-world problems." (MOE, 2007, p. 3).

There is teaching of problem solving in Singapore schools but it is done as the school understands it. For instance, from our experience interacting with schools, it appears that teaching problem solving in the mainstream schools have been reduced to the teaching of "heuristics", possibly establishing a one-one correspondence between a particular problem and the heuristics to be used. Also, more schools are equating problem solving to training for mathematics competitions thereby isolating problem solving to a select few and putting it outside the curriculum. This runs contrary to the intent of the SMC's vision of problem solving as integrated in the curriculum. Research conducted in the local setting have found that there is much room for improvement in creating a more effective and efficient learning environment for developing students' problem solving skills (see for example, Fan & Zhu, 2007). Furthermore, although the international comparative studies PISA and TIMSS have revealed that Singapore has achieved a high level of competence in mathematics in schools, these studies have also noted a relatively weaker performance of Singapore students on solving unfamiliar problems (Kaur, 2009).

As the overarching aim of the Singapore mathematics curriculum at all levels of schooling is the development of problem solving ability, research in problem solving in school mathematics in order to support classroom practice or inform curricular policy with research-based evidence is extremely important in the Singapore context.

As to the direction of problem solving research, Alan Schoenfeld wrote in the 2007 special issue on problem solving of the journal ZDM that the current focus should lie in translating decades of theory building about problem solving into workable practices in the classroom:

> That body of research—for details and summary, see Lester (1994) and Schoenfeld (1985, 1992)—was robust and has stood the test of time. It represented significant progress on issues of problem solving, but it also left some very important issues unresolved. … The theory had been worked out; all that needed to be done was the (hard and unglamorous) work of following through in practical terms. (Schoenfeld, 2007, p. 539).

Our proposed project is an attempt at doing the “hard and unglamorous” work of realising the ideals of mathematical problems solving—as envisioned to be at the heart of the Singapore mathematics curriculum—into the daily practices of mathematics classrooms. That problem solving is currently mostly theoretical talk and not common classroom enactments is attested by numerous local studies (see for example, Foong, Yap, & Koay, 1996).

To us, the hard and unglamorous work involves three major steps: (1) *initialisation* of problem solving as an essential part of the mathematics curriculum in a school at a foundational year level; (2) *infusion* of problem solving as an embedded regular curricular and pedagogical practice across all year levels in the school; and (3) *diffusion* of this innovation from this school to the full range of schools in Singapore. In each of these steps, we take a complex systems approach: instead of zooming in at only one component of the system, such as curriculum, instructional practices, assessment, and teacher development, we include all these aspects in our overall design research process.

In line with design research, the initialisation stage of the process involves the creation and trialling of a design. This we have done successfully under M-ProSE (OER 32/08 TTL). In that project, we redesigned the curriculum and structures, including assessment practices and teacher development, within an Integrated Programme (IP) school (hereafter known as the initial school) in Singapore. Through a series of adjustments to the design, we have developed a problem solving curriculum that is now an integral part of the school’s mathematics curriculum at the foundational level. The M-ProSE design comprises a problem solving curriculum and professional development to equip teachers to deliver the curriculum. The details of the curriculum—a Scheme of Work, a series of lesson plans, pedagogical suggestions, the Practical Worksheet (which functions as a pedagogical and assessment tool), and assessment strategy—are set out in the book we authored: *Making Mathematics Practical: An approach to problem solving.* The problem solving curriculum and the research outcomes of that project are well reported in the literature. (For a full list of M-ProSE publications, see Appendix A.)

PURPOSE OF STUDY (INCLUDING RESEARCH QUESTIONS AND/OR OBJECTIVES)

Having learnt *about* problem solving in the initialisation stage, students will learn mathematics topics in the syllabus *through* problem solving in the infusion stage. In other words, problem solving no longer stands outside the main curriculum but is infused into it. Insofar as infusion alters radically the way curriculum and teaching will be carried out, it is an innovative method that would potentially enhance pedagogy.

The design for the initial school needs to be adjusted as it is diffused throughout the mainstream schools. On the other hand, the school has to make changes (for example, curriculum reorganisation and teacher capacity) to accommodate the problem solving design. In our conception of diffusion, this notion of scalability is closely tied to sustainability. Without sustainability, there is no foundation to build upon for scaling out and scaling up. Similarly, the goal of scaling of innovations is for sustainable improvements. Such visions of sustainable innovations require systemic adjustments. We take the view that diffusion of innovations should carefully take into consideration broad scale buy-in and commitment (including teachers, school leaders and policy makers).

In summary, this research aims to answer the following research questions:

<u>Infusion Question</u>

1. For a school where the design is initiated, how can we embed the innovation into the structures and processes within the regular mathematics curriculum of the school?

Diffusion Question

2. What are the issues, challenges and critical success factors in scaling up an innovation within a school to mainstream schools?

In addition, we ride on the affordances within the project to answer the following questions:

3. Do the students who participate in the MInD programme improve their performance in the mathematics achievement tests?

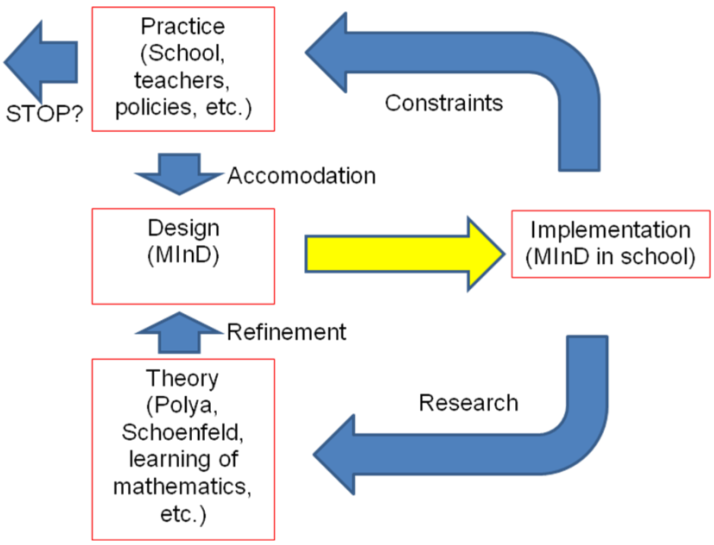
4. What are the protocols to establish for a collaborative researcher-practitioner partnership that would advance the goals of infusion and diffusion of the project?

METHODOLOGY/DESIGN

MInD proposes to use "design experiment" (Brown, 1992; Collins, 1999; Doerr & Lesh, 2003; Middleton, Gorard, Taylor & Bannan-Ritland, 2006; Quek et al, 2011; Wood & Berry, 2003) as the methodological backbone. Design experiment arose from the attempts of the education research community to address the demands of research in real-life school settings in all its complexity. The methodology of design experiment argues for the application of multiple techniques to study a complex phenomenon in education. As such it permits the use of a slew of methods such as participant observation, interview, video-taping, and paper-and-pencil testing to provide corroborative evidence for findings.

To Schoenfeld (2009), design experiment is built on a "design-theory dualism". In our case, it begins with a theory regarding mathematics education. Based on this theory, a design is conceptualised to improve learning. The initial design is then implemented in a suitable 'real-world' context, such as the school. Research methods are then used to examine both the "accuracy of the underlying … theory and the power of the intervention." Based on the findings of the research, refinement is made to the design which is further implemented and the implementation-research-refinement cycle is iterated until the design-experimenter is satisfied. "The coherence of theory to methodology … is of fundamental importance in the evaluation of design experiments, and is critical to explicate for any future scholar or practitioner who attempts to replicate or implement the findings of a design study." (Middleton et al., 2006).

Schoenfeld's (2009) model of design experiment involved only the interaction between the designer and the researcher in a design-theory dualism. In our work in M-ProSE, we are convinced that the teachers and school that have to implement the design have a key role in the final design itself. Black (2009) advocates this model of design experiment which includes the designer, the researcher and experienced teachers. We propose that, in effect, a design-theory-practice troika should always be considered for a designed package to be acceptable to the final users, which are the teachers and the schools. In addition, we notice that there are distinct differences in the type of changes we made along the way in the design process. One type of change relates to what is already widely discussed in the literature on design for the purposes of developing the theory and the product—we call this refining. However, there is another type of change we made which is to meet the realistic constraints faced in practice—we call this accommodation. We think it is important in design research to distinguish the two. The figure below shows the design-theory-practice troika underlying our proposed design-experiment for MInD.



The parameters that are underpinned by theoretical justification (see Quek et al. 2011) for the design experiment in our study are:

1. Place in the curriculum: It must be part of the mainstream mathematics curriculum.

2. Model of mathematical problem solving:

i. Polya's model – all four stages

ii. Shoenfeld's framework – teach Heuristics and emphasise Control

3. Teacher autonomy: Teachers in school will ultimately teach the module themselves. Build teachers' capacity in problem solving and to teach it.

4. Infusion into regular mathematics content: Problem solving skills and habits learnt in the module must be infused into other mathematics modules to prevent atrophy

5. Assessment: A valued component in school assessment

MInD taps on three main timely developments to advance its research: (1) the recent advances in methodological thinking which support the use of multiple methods in seeking insights into the complexities of teaching and learning; (2) a new development in the educational landscape of Singapore, namely, the establishment of IP schools which targets educational "peaks" for academically-inclined students and are given relatively greater freedom to explore curriculum innovations; and (3) recent research focus on diffusion of innovations. As to (1), we will use both qualitative and quantitative methods within the overarching design experiment approach to study the infusion and diffusion thrusts of the project. For (2), we use the “Best-case scenario” method to start our investigation with high-ability mathematics students in a school that clearly emphasises the development of the mathematical ability of its students to the fullest. We argue that the testbed for the initialisation phase of an innovation should be at the school that is most conducive for success. M-ProSE started the process at the initial IP school. MInD adopted this methodological stance and seeks to move on to next stages of infusion of problem solving within the same school. For (3), we tap on the growing literature, especially integrative theories across traditional fields such as modelling of complex systems (Lemke & Sabelli, 2008) and organisational change (Larson & Dearing, 2008) to guide our study of diffusing the problem solving design.

In Gorard’s (2004) words, “The emphasis [in design experiments], therefore, is on a general solution that can be ‘transported’ to any working environment where others might determine the final product within their particular context (emphasis added)”. The envisaged outcome of MInD is to produce a workable problem solving design that can be adapted to the setting of the mainstream Singapore schools. MInD’s approach in this design process is via the two main steps of *infusion* and *diffusion*.

Infusion: With the strong support shown by the Principal and the Head of Mathematics Department of the M-ProSE school, the team entered into the next phase of *infusion* of problem solving with students at the higher Secondary levels. The students would have been initialised to learning about problems solving through an 8-hour module at Secondary Two using the M-ProSE problem solving package. The researchers worked with the teachers to modify the existing curriculum material of the higher Secondary levels to engage the students to learn mathematical topics at these levels through problem solving. From the research perspective, we studied the students’ ability to apply the problem solving approach to learn new mathematics content knowledge, and how the instructional methods used by teachers could be altered to facilitate the students’ acquisition of mathematical knowledge. The teachers from the M-ProSE school had felt that the students could benefit from the infusion of problem solving, and that students from the entire spectrum of mathematical ability will be able to benefit from it. For the infusion stage, the problems used in the MInD are directly chosen to fit the school curriculum, hence suitable for the school students (*compatibility*). The research project aligned well to *re-invention* – it being a design experiment – and very clearly *triable*.

Diffusion: This involves the diffusion of innovation to other mainstream schools. We have discussed with various mainstream schools about the feasibility and constraints in implementing a problem solving curriculum. Taking into account the feedback by the principals and school teachers, we have begun the work on adapting from the initial design to fit the curricular and contextual demands for the participating mainstream schools.

The students from the participating mainstream schools are expected to be from the Lower Secondary levels. The choice of starting with Lower Secondary students was deliberate: (a) the schools were concerned that the Upper Secondary students' preparation for the national examinations will not be affected; (b) the schools have greater autonomy over the Lower Secondary curriculum to make changes to its enactment; (c) build the right habits of mind with respect to mathematical problem solving early; and (d) the prospect of following through to the infusion of the design at the Upper Secondary levels, for a successful curriculum change.

We recognised that teachers are the key to any successful curricular or instructional reform. In the area of non-routine problem solving, past research (see, for example, Schoenfeld, 1992; Lester, 1994) has highlighted the need for teachers to experience problem solving themselves. MInD did more than just equipping teachers with the problem solving strategies and heuristics. It introduced to the teachers Polya's (1954) problem solving model and Schoenfeld's (1985) problem solving framework. Most importantly, MInD uniquely takes the teachers through a paradigm shift by engaging them in the use of a "Practical Worksheet" which is crafted to focus attention on fundamental aspects of problem solving in mathematics. In this study, we rode on the affordance on the research project to trial the teaching of undergraduate mathematics to NIE's pre-service teachers through infusion of problem solving. At least one of the undergraduate mathematics content course in NIE was re-structured to fit the concept of theory-cum-practical approach in the learning of mathematics. The result was reported in a research paper.

Data was collected in multiple ways

1. Field notes from participant observation
2. Survey questionnaires for teachers who attended the teacher professional workshops, and thereafter, only those who are assigned to teach the problem solving course
3. Survey questionnaires for participating students
4. Interviews with selected teachers in #2
5. Interviews with selected students in #3
6. Video-recording of problem solving lessons (teachers) and problem solving attempts (students)
7. Continual assessment of students by teachers
8. Assessment of problem solving using the Practical Worksheet and rubrics
9. Assessment worksheet of students' performance in non-routine problems versus the examination-style questions.

As an additional note to #8 and #9, the MInD project (OER 22/12 TTL) contributed towards developing alternative assessments of mathematical problem solving. It involves the use of the Practical Worksheet and an accompanying assessment rubric as a form of assessment both of learning problem solving and for problem solving. This assessment strategy is based on the theoretical bedrock of problem solving framework developed by Polya and further refined by Schoenfeld (Toh, Quek, Leong, Dindyal, Tay, 2011). Moreover, the assessment adaptations that we made take into account the local factors within the Singapore education landscape, such as the importance of high stakes examinations.

FINDINGS / RESULTS

Students: As evident from the students' work, they were able to respond to the problem solving processes as required by the MInD problem solving module. Not only were they able to apply various heuristics to attempt solving a given problem (Stage 3 of Polya), they were able to communicate their understanding of the problem and how they decided on a particular plan in solving a problem (Stages 1 and 2 of Polya). More importantly, most students proceeded to check and expand the problem (Stage 4 of Polya) after they had solved the problems. In short, they were able to exhibit the entire processes of problem solving.

We found significant correlation between students' performance in MInD assessment and their school mathematics achievement test. Not only that, students' attitude towards the MInD problem solving module was an important contributor to their mathematics achievement. One possible explanation was that students' positive attitudes towards Polya's problem solving model and the learning module may allow them to appreciate the usefulness and value of our intervention. As put forth by Ma and Kishor (1997), students' engagement in the mathematics learning (in our case, problem solving intervention) would probably become more active and meaningful. Consequently, they could be more willing to apply what they had learnt from our intervention to their daily mathematics learning and eventually their school examinations. For example, the heuristics and checking approaches students acquired from our intervention might enable them to solve the test problems in a more versatile and cautious manner, which would perhaps result in better mathematics achievement.

Student teachers: The student teachers who read the undergraduate mathematics courses through the use of MInD approach (mathematics practical) performed better in their continual assessment and the final examination, compared with their seniors in the previous batch. What is heartening to the course instructor, for example, is the ability of the student teachers to check and expand a given problem, thereby discovering "new mathematics" in the process of solving a problem. The student teachers' feedback was generally positive for this new approach. In AAM331 Differential Equations, a course which was redesigned using the practical paradigm, the majority of the students were able to solve a rather complicated multi-step problem in the final examination without much difficulty. This could shed light on how mathematics content teacher education course could be re-structured to maximise the benefit to student teachers in undergraduate mathematics education.

In the other course AAM104 Number Theory in which this problem solving approach was used, the course instructor reported that the student teachers were able to solve a challenging problem and their use of Polya’s model at the end of the course. Based on the student teachers’ solutions, almost 70% of the student teachers managed to arrive at the correct answer of what the instructor would consider a difficult problem at the first year of the undergraduate level. More than half of these student teachers also demonstrated clear reasoning in their solutions of the problem. Their responses far exceeded the instructor’s prior conceptions of how the problem would be tackled.

Schools: We report (1) the sustainability of the problem solving (PS) module; (2) infusion of PS into the regular mathematics curriculum; and (2) teacher readiness in teaching problem solving.

1. All 5 schools in the discussion kept the PS module. Schools A and E maintained the PS module as an examinable 9 lesson module in its modular curriculum. The module was also endorsed by the parent university of School A. Schools B and D reduced the number of lessons to 7 and 6 respectively, while School D modified the module to five double period lessons. The modifications were made to align with their tight schemes of work. Modifications were also made to the numbers, types and difficulty of the problems. The schools used props and videos to support the PS module. For example, the participant from School A said in the Focus Group Discussion: “… video showing how the 4 stages are done to the students, and introducing more concepts for problems during our ‘E-learning’ week. Whereby students can watch on-line video. So teaching is not only in the classroom, students can themselves do self-directed learning in terms of PS techniques.” Data of School E was collected after the Focus Group Discussion conducted for Schools A, B, C and D.

2. The language of PS (understand the problem, heuristics, stuck, etc.) permeated to many other mathematics lessons in at least three of the schools. School C: "And we also come out with an explicit way of maths language for our teachers so that for every problem that you do in class, you will do the same, 'how do you understand the problem', 'what is given', so we have a language for the teacher to help them to facilitate as a way for PS, as a whole curriculum be it in the syllabus or M-ProSE." School D worked with the researchers to redesign 3 units in the mathematics curriculum to include PS as a pedagogical approach. "And because we have done it in Secondary 1 as a whole module and in Secondary 2 we do the RU, the Replacement Unit. So when we teach the topic with the MPROSE problem, either in the beginning, in the middle or towards the end also. So it's an infusion of M-ProSE. So they still keep in touch with it."

3. School A reported that teaching materials and support have been systematically developed for new teachers. School B suggested that teachers in pre-service training should learn how to teach PS (We note here that the schools use videos provided by the research team to train new teachers as well.) Schools B, C and D reported that their teachers work on solving new problems themselves. School E reported (after the Focussed Group Discussion) that the Secondary Two level coordinator is in-charge of conducting professional development for their teachers new to problem solving based on the training video.

IMPLICATION TO POLICY MAKERS

This project provides ideas on how the "hard and unglamorous" work of pushing mathematical problem solving, which is the spirit of mathematics education, through in practical terms in the school mathematics classrooms. It shows the feasibility of infusion of mathematical problem solving within the school curriculum without compromise to the other aspects of teaching mathematics. More importantly, the process of scaling up of an innovation is fully demonstrated from our first project OER 32/08 TTL to the current project OER 22/12 TTL. It provides the Ministry of Education an alternative model of scaling up of an innovation for consideration, which shows the dynamics between the designer and the teachers.

IMPACT TO SCHOOLS

First and foremost, the problem solving module introduced in this research has become a mainstay in the mathematics curriculum in the five research schools. This demonstrates the level of buy-in and sustainability of our approach in the research schools. As far as the researchers are aware, several other schools are adapting some ideas of our problem solving research to design their own models of mathematical problem solving within their school curriculum.

The findings of this research have resulted in a series of professional development workshop on mathematical problem solving designed for practicing teachers. This approach has been introduced in the pre-service teacher education programme for PGDE (Sec) and PGDE(JC); and has resulted in a new problem solving module for all undergraduate mathematics students at Year One, after the most recent AS curriculum review.

The Ministry of Education has also adopted some ideas from our research in their design of the new H3 mathematics curriculum. In particular, Ministry of Education has approached Head MME to conduct the problem solving part of the H3 mathematics curriculum for A-level students in 2017.

CONCLUSION

At the outset, this research project studies the implementation of problem solving in the entire spectrum of Singapore secondary schools at the lower secondary level, and an attempt to carry through the “hard and unglamorous” work of teaching problem solving to the truest spirit of problem solving possible. It shows that it is feasible to teach problem solving within the wider constraints of reality, such as high-stake national examinations among many others.

This project also involves the process of scaling up an innovative practice, beginning from initialization within a school to infusion within the particular school, and also diffusion from one school to more schools, with parameters intact but tweaks to other features. It provides a model of scaling up of an innovation and could, perhaps, offer the Ministry of Education an alternative model of how a new practice could be taken up to all Singapore schools.

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