Exploring Group Dynamics of Primary 6 Students Engaged in Mathematical Modelling Activities

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This paper explores the group dynamics among three groups of students involved in collaborative learning in mathematical modelling activities. It reports how group dynamics were established and their influence on the students' mathematical problem-solving endeavours. Through video analyses, discourse structures were identified to suggest the dominant roles students play within the group. Frequency counts of the discourse structures accounted for the group dynamics that shape the effectiveness of the learning that takes place in the groups. Implications from the findings are discussed.

Introduction

The field of mathematical modelling is a relatively new domain in the Singapore mathematics curriculum. In order to have students to work in groups on open-ended model-eliciting activities, the instructional platform shifts from a teacher-expository to a student-centric one. Learning is seen as taking place in a socio-cultural environment where students are to develop and construct knowledge as they interact with the environment. Collaborative learning is encouraged as it involves students working together to achieve common goals through the identity developed as they help and support one another communally (Slavin, 1996). As well, the collaborative learning affords the students to engage in more diverse and flexible thinking as they participate in the problem-solving situation (Lowrie, 2011). Studies have found that students working collaboratively were presented opportunities to share ideas, consider the appropriateness of one another's solutions, discuss different representations of the same problem and consider the extent to which solutions can be applied to different contexts (Lowrie, 2004; Schorr & Amit, 2005). Some studies on collaborative learning have focused on the cognitive and metacognitive aspects of collaborative learning and interactions (e.g. Goos, Galbraith, & Renshaw, 2002) as well as types of collaboration that impact problem solving (e.g. Leikin & Zaslavsky, 1997; Watson & Chick, 2001). However, there is limited research in the area of group dynamics involving students and how this impacts problem solving in the field of mathematical modelling. This paper reports how the group dynamics of three groups of Primary 6 (Grade 6) students were established and discusses the impact of the group dynamics on group problem solving. The research is part of a larger study on investigating Primary 6 students' mathematical modelling processes in a problem-based learning setting. The aim of this paper is to discuss how group dynamics were ascertained towards making inferences concerning their problem-solving endeavours.

Theoretical Perspective and Literature Review

Group dynamics is a field of inquiry on the nature of groups, their development and their interrelations with individuals, other groups and larger institutions (Cartwright & Zander, 2000). Kurt Lewin (1890-1947) was believed to have popularised the term "group

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dynamics" and research in this field has indicated potential benefits of work groups through investigating constructs involving size, goals and performance (e.g. Cohen, 1994; Forsyth, 2010). As well, there have been studies indicating that the potential benefits of groups has not been always been positive (Hackman, 1990; Robbins & Finley, 2000).

One of the ways to identify the dynamics of a group is to study the collaborative discourse of the group. Discourse refers to the processes by which individuals communicate with one another (Cazden, 1988; Cobb, 2002). In the mathematical classroom, the collaborative discourse exemplifies the pupils' knowledge that is constructed and shared with other members of the group and this view encompasses actions and processes of speaking and writing, as well as what members are doing and what is being perceived during the interaction (Ball, 1991; Sfard, 2000). Research has shown that discourse in the classroom takes a variety of strands mainly based on the process of discourse (how the discourse is formed—the communication aspects related to who talks, to whom, who listens, how ideas are gathered, who exercises dominance, etc., the content of discourse that is related to the mathematical ideas being talked and also the strategies to support discourse in the classroom).

Discourse structures are commonly the specific components of mathematical talk. According to Van Meter and Stevens (2007), the structure of collaborative discourse (not grouping per se) has implications for knowing "the nature of effective discourse patterns, context factors that support the discourse and the individual learning gains that can be expected to result" (p. 123). Some structures of mathematical talk include *defending*, *arguing*, and *conjecturing* about mathematical ideas (Ball, 1993; Lampert, 1990), or *giving the answer* and *partial explanation*, *explicit explanation*, *making extension*, comparison, *conjecture* and *justification* (Kalathil, 2006). Discourse structures are seen as the building blocks for talking about students' solutions and mathematical problems. In knowledge building discourse, Hmelo-Silver and Barrows (2008) identified three types (which they termed as *moves*), namely, *questions* for specific purpose to open up dialogue, *statements* which may be simple assertions or development of a new view, reformulation or elaboration of an idea, and *regulatory statements* that are directed at collaboration and learning processes. They found that groups worked collaboratively to improve their ideas through the use of those three structures in a problem-based learning setting.

The analysis of discourse structures paves the way for understanding discourse patterns in the mathematics classroom. According to Cobb and Yackel (1996), it is through the pattern of interaction and discourse created in the classroom that students are able to ascribe meaningfulness to one another's attempts to make sense of the world. Learning about how to think about ideas, engaging in discussion, arguing, clarifying and revising thinking is fundamental to making progress.

Background to the Study

My study was designed to cover two phases. Phase One involved a pilot study investigating two classes of Primary 6 students' modelling behaviours for the purpose of establishing a set of problem-solving behaviours (the perspective the investigation took was to see mathematical modelling as problem-solving) and a coding scheme through video analysis. A think-aloud protocol analysis method (Ericsson & Simon, 1993) and parsing of episodes for meaningful chunks (Schoenfeld, 1992) were carried out. The categories of codes established in Phase One were applied towards establishing the group dynamics of the students in Phase Two over three modelling activities. For this study, the students' discourse structures have been identified as *Trigger*, *Plan*, *Dispute*, and *Impasse*. *Trigger* refers to the behaviours that initiate responses leading to a *plan*. *Plan* refers to the behaviours of generating mathematical ideas (for consideration). *Impasse* refers to the behaviours that seek understanding because of uncertainties that developed. *Dispute* refers to the behaviours displayed towards challenging the current state of thinking. This paper reports the findings of the group dynamics aspect in Phase Two of the study. Through a fine-grained analysis of the protocols, occurrences of the discourse structures were noted to indicate the extent of the discourse structures used. The extent of the occurrences account for knowing the dominant role the student plays in the group, thus providing a means to establish the group dynamics of the group.

Findings

From the frequency counts of the use of the discourse structures, the following group dynamics of Groups 1, 2 and 3 were established. The findings of Group 4 are not reported as it was discovered later that there was a video equipment failure. S1, S2, S3 and S4 refer to the students in the respective groups.

Occurrences of New Learning

Part of the main study investigated students' acquisition of *new learning* in the collaborative discourse. *New learning* refers to expressions about not knowing or having a misconception or disagreement at first and how those uncertainties are eventually clarified or the students have gained new information. In other words, it is inferred that students move from a state of not knowing to knowing. For example, one of the movements of the group collaboration shows the acquiring of new learning as "S3 (Trigger) → S4 (Dispute) → S1 (Plan) → S1 (New Learning)". It suggests S3 initiated a thought on how to proceed to solve the problem, S4 questioned the validity of that idea, S1 expanded on that idea, and finally S1 saw the worth of that idea (new learning). No new learning implies an abandonment of the idea or the mathematical talk has digressed to an unrelated event. Figure 1 shows the outcomes related to the acquisition of new learning by Groups 1, 2 and 3 where group dynamics are deemed to be a construct.

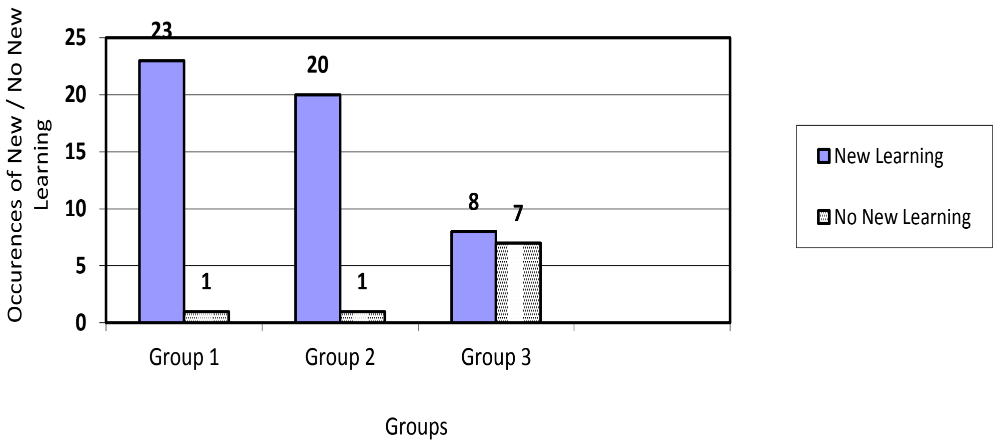

Figure 1. Overall occurrences of new and no new learning by groups across tasks.

Group Dynamics of Group 1

Figure 2 shows the extent of the discourse structures manifested by each student in Group 1 across the three modelling tasks.

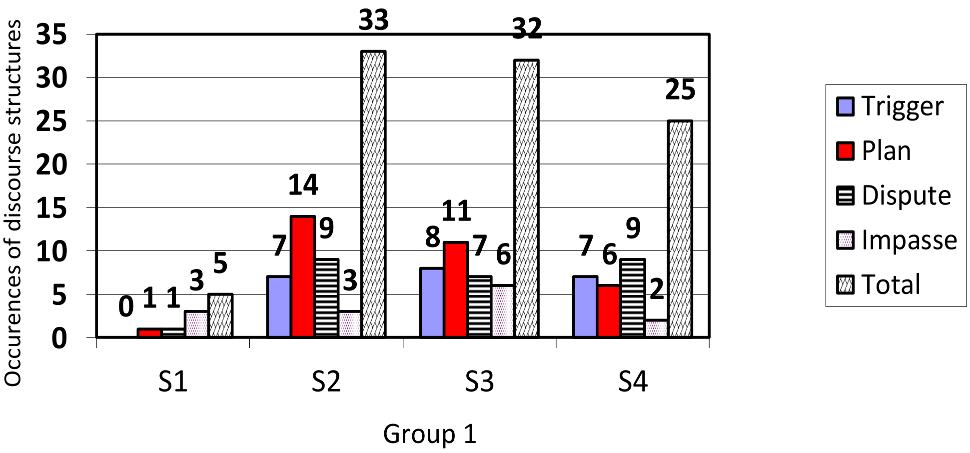

Figure 2. Extent of occurrences of discourse structures by students in Group 1.

Group 1 can be considered a *high-performing group*. A high-performing group is characterised by active engagement and collaboration among members through the interplay of the collaborative discourse structures that lead to instances of new learning. The group has been also more successful in solving the modelling tasks. What constituted Group 1 as a high performing group was that there were three active members, S2, S3 and S4, who registered high incidences of the discourse structures (seen by their totals, 33, 32 and 25 respectively). The three active members played complementary roles. S2, S3 and S4 were *initiators* to engage the group as seen from the comparable number of *triggers* they displayed (7, 8 and 7 respectively). S2 and S3 also functioned more like *strategists* through providing mathematical strategies and methods as they accounted for the relatively high incidences of *plan* (14 and 11 respectively). S2 and S4 were *analysers* as well based on the relatively high incidences of *dispute* (both 9). S3 was more a *seeker* too who tried to get answers in events of uncertainty based on her relatively high number of *impasse* (6 occurrences) compared to the rest. S1 could be considered an inactive *follower* who did not disrupt the group's endeavour. The group dynamics could be represented as a matrix, shown in Table 1.

Table 1
Group Dynamics of Group 1

Student	Initiator	Strategist	Analyser	Seeker	Follower
S1					X
S2	X	X	X		
S3	X	X		X	
S4	X		X		

When S2, S3 and S4 played complementary roles, this suggested a high level of negotiation where *triggers* and *impasses* were followed-up with *plans* or *disputes* that indicated that the problem situations had been thought through deeply towards the acquisition of new learning. In this instance, these group dynamics thus supported the interplay of the use of *plan*, *dispute* and *impasse* discourse structures for successful mathematical modelling and the acquisition of new learning in the modelling environment.

Group Dynamics of Group 2

Figure 3 shows the extent of discourse structures manifested by each pupil in Group 2 across the three modelling tasks.

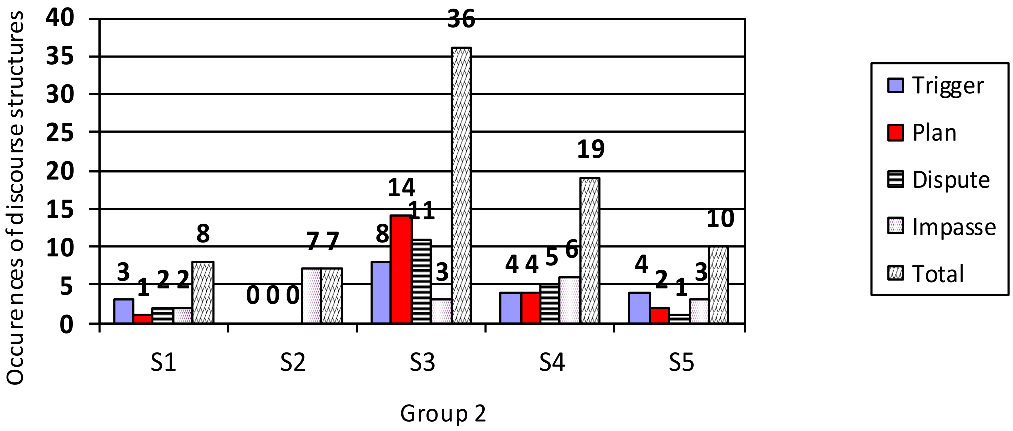

Figure 3. Extent of occurrences of discourse structures by students in Group 2.

Group 2 is identified as a *one-director group*. This is an interesting case where S3 was one who consistently registered substantially high number of instances for *trigger*, *dispute*, and *plan* compared to the other members in the group. S3 functioned more like a *director* who made things happen and who *strategised* and *analysed* matters as well. While S3 was task-oriented, S2 and S4 were more like *seekers* who asked for clarification (relatively high *impasses*), in particular getting S3 to explain his *plans* and *initiations*. S1 and S5 were *followers*. The group dynamics is shown in Table 2.

Table 2
Group Dynamics of Group 2

Student	Initiator	Strategist	Analyser	Seeker	Follower
S1					X
S2				X	
S3	X	X	X		
S4				X	
S5					X

In this group structure, members normally took the cue from S3 to work on the modelling content. Group 2 had seen a fair share of modelling success and acquired relatively high instances of new learning. In this regard, the *director* was deemed to be quite proficient, functioning as an *initiator*, *strategist* and *analyser* to be able to direct the proceedings towards attaining new learning.

Group Dynamics of Group 3

The extent of discourse structures manifested by each student in Group 3 is shown in Figure 4.

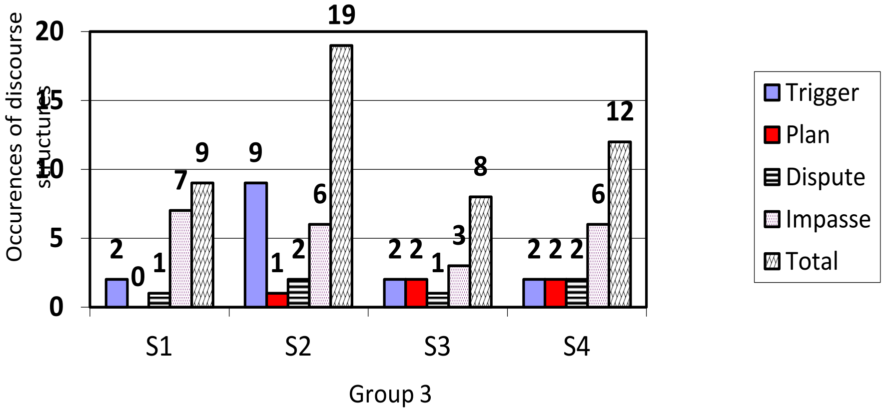

Figure 4. Extent of occurrences of discourse structures by students in Group 3.

Group 3 could be considered to be a *seeking group*. A seeking group comprises mainly members who exhibit high instances of *impasse* compared to the other discourse structure occurrences. In Figure 4, S2 appeared to be the most active with 19 occurrences of discourse structures (total). However, the 19 counts were dominantly based on the high extents of *triggers* S2 had made, otherwise her other discourse structure types were rather comparable with the other members. The group comprised mainly *seekers* and lacked *strategists* and *analyzers* which explained why they encountered more difficulties compared to the Groups 1 and 2. They had many uncertainties and they lacked the know-how to manage them. The group could also be said to be a group of *followers* without a leader who could competently *strategise* and *analyse* details. The group dynamics is shown in Table 3.

Table 3
Group Dynamics of Group 3

Student	Initiator	Strategist	Analyser	Seeker	Follower
S1				X	X
S2	X			X	X
S3					X
S4				X	X

Discussion and Implications

This study, through setting up collaborative discourse structures like *trigger*, *impasse*, *dispute* or *plan* via a fine-grained analysis has enabled the group dynamics of students to be ascertained. By relating dominant roles based on the occurrences of the discourse structures, the group dynamics has allowed for inferences to be made concerning the effectiveness of the groups in their mathematical modelling endeavours. According to Barnes (2003), identifying such patterns of participation is a step in developing a better understanding of factors that promote or inhibit effective collaboration.

The findings of this study revealed that groups that comprise members who dominantly function as strategists and analysers (e.g., Group 1) lead to more effective learning. The mathematical discourse is richer with more instances of students raising queries, probing and arguing, leading to expressing, testing and revising their mathematical representations. In every group, there are seekers and followers. The seekers provide questions in the hope that their queries might be answered. For groups with strategists and analysers, questions raised by seekers are usually taken up by the strategists and analysers. The followers are the doers; they listen and carry out instructions such as using the calculators to make the computations or using rulers to make measurements. A group that comprises members who are dominantly seekers and followers (e.g., Group 3) is unable to go deeper in the discussion and the learning is not as fruitful. Often, there are more questions than solutions. For Group 2, where one particular member dominates, that member does appear to wield considerable powers especially since he is highly proactive in strategising and analysing. According to Douglas (1978) it may mean the members are behaving according to the way they perceive the prevailing atmosphere. In this case, where the dominant member appears to be able to affect the performance of the group and so the members would just “help” him along by doing what he says even though he may not be entirely on the right track at times.

This study shows that every member contributes in a certain capacity towards accomplishing the task. However, where performance is concerned, certain group dynamics do not always have positive effects when there is a lack of key players like strategists and analysers (e.g., Group 3). The findings suggests that group dynamics is an most important aspect to consider as it drives and shape the modelling process as well as is responsible for the evolution of and activation of different sub-activities during the problem-solving process (Ärlebäck, 2009). Thus in considering group composition, having a mix of initiators, strategists, analysers, seekers and followers may result in better productivity than a group that comprises dominantly seekers and followers.

The implications of this study needs to be treated with caution as there are limitations since these are case studies of three groups of students. Although steps are taken to have inter-rater reliability in the coding and convergence of meanings, such coding of human behaviours is still very much a human endeavour and can never be completely accurate. It needs to be noted that the class teachers had participated as facilitators as well and this could also have influenced learning outcomes in the overall scheme of things. Despite the limitations, this study underlines the importance of group dynamics consideration for effective learning to take place. Further research is needed to investigate more cases with students in different schools as well as the impact of training on exercising helping behaviours within groups.

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