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Problem Solving in Singaporean Secondary Mathematics Textbooks

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Abstract

This study examined how two widely used Singaporean school mathematics textbooks at lower secondary level represent problem solving. The study established a framework on the classification of problems and problem solving procedures to code and analyze the data from the textbooks. Based on the framework, an overall picture of the representation of problem solving in the textbooks was obtained. The authors concluded that the textbooks present a good foundation for students to development their abilities in problem solving and are strong in aspects such as using fundamental/theoretical knowledge to solve problems, developing students' logical and higher-order thinking skills through solving multistep and challenging problems, exposing students to a variety of heuristics, and leading students to new concepts and algorithms through problem solving. The authors also suggested some areas for further improvement of the textbooks concerning the representation of problem solving.

Introduction

Educational researchers and policy makers have paid considerable attention during the last two decades to cross-national comparisons of students' mathematics achievement (e.g., see Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996; Robitaille & Garden,1988; Song & Ginsburg, 1987; Stevenson, et al., 1990; Stevenson, Stigler, Lucker, Lee, Hsu, & Kitamura, 1986; Stigler, Lee, Lucker, & Stevenson, 1982). In those comparisons, Asian students generally outperformed their western counterparts. Particularly, the mathematics performance of Singaporean students ranked first in all the participating countries in the well-publicized Third International Mathematics and Science Study (TIMSS) (e.g., see Keys, Harris, & Fernandes, 1996 & 1997). This fact has aroused much interest of researchers, teachers, and the general public from western countries especially the United States, as well as Singapore itself, in its educational system and practice.

In searching for the possible reasons to explain the success of Singaporean students, some people have turned their attention to Singaporean textbooks, and believe that textbooks are an important factor. An article on *The Straits Times* claimed that mathematics textbooks used in Singaporean schools provided students with a firm grasp of the subject (Quek, 2000). In the United States, some professors and educators have been encouraging and helping school districts in many states such as Colorado, Illinois, Maryland, and New Jersey to use Singaporean textbooks. They praised the texts for "their clear, simple prose, their novel problem solving approaches, and the complex, multistep problems they present to students", and believe that the books really "empower students" as problem solvers. It has also been reported that, in American schools using Singaporean mathematics textbooks, both teachers and students seem to like the textbooks, especially for the problems and the explanations for the solutions presented in the texts (Viadero, 2000).

There is no doubt that problem solving should be the center of mathematics curriculum and instruction in primary and secondary schools, a consensus widely held in the community of mathematics education since the 1980s (e.g., see Krulik & Rudnick, 1987; National Council of Teachers of Mathematics, 1989). The Singapore mathematics syllabus stated clearly that the primary aim of the mathematical curriculum is to enable pupils to develop their ability in mathematical problem solving (Ministry of Education, 1990). Nevertheless, academic inquiry on how Singaporean textbooks actually represent problem solving in mathematics is lacking.

The study reported herein is part of a larger research effort, which aims to investigate how mathematics textbooks in three different countries, China, Singapore, and the United States, represent problem solving in mathematics. This study, as a case study, is particularly to examine how Singaporean mathematics textbooks represent problem solving. More specifically, the study is intended to address the following two research questions. First, how different kinds of problems are represented in two widely used Singaporean mathematics textbooks? Second, how problem solving procedures, including general strategies and specific heuristics, are represented in the textbooks?

Textbooks are a key component of intended curriculum. Their importance in classroom teaching and learning has been increasingly recognized in research literature during the last 15 or so years. In fact, researchers have paid growing attention to the study of textbooks with focus on their influences on teachers' teaching practice (e.g., see Fan, 1999; Fan & Kaeley, 2000; Graybeal, 1988;

Krammer, 1985; Sosniak & Stodolsky, 1993). According to Robitaille, an analysis of textbooks can make an important contribution to understanding of curricula in a particular country (Howson, 1995, p. 6).

Available researches generally revealed that textbooks affect, to a varying degree, not only what teachers teach, but also how teachers teach (Fan & Kaeley, 2000; Robitaille & Travers, 1992). In particular, according to a TIMSS report (Beaton, Mullis, Martin, Gonzalez, Kelly, & Smith, 1996), in almost all the participating countries, the textbooks were the major written source mathematics teachers used in deciding how to present a topic to their classes. In the case of Singapore, 89% of the eighth grade (Secondary 2) mathematics teachers in the TIMSS study reported they decided how to present a topic mainly based on textbooks. Therefore, the role that textbooks play in mathematics instruction is very important.

Having said that, we wish to point out that how textbooks are used in classrooms and how textbooks are designed are two related but different issues. The former is implemented curriculum, while the later is intended curriculum. Textbooks as intended curriculum are only one of the many factors that affect teachers' teaching practices in classrooms; other factors include teachers' own knowledge and ability, students' background and learning behavior, school environment and teaching facilities, and social and cultural tradition, to name a few. Therefore, what are written in textbooks do not necessarily reflect what exactly happen in classrooms; readers are reminded to be cautious when relating the results of this study to the actual mathematics teaching practices in Singaporean classrooms.

Methodology

To investigate how Singaporean mathematics textbooks represent problem solving, this study chose two widely used textbooks at the lower secondary level. It also established a conceptual framework about problem solving in order to analyze the textbooks and address the two specific research questions.

Grade level

Dacey (1989) identified six peak periods in the growth of human creativity across the lifespan. The first three periods include both preschool and school years and of particular importance is the 10- to 14-year-old period. According to Dacey, at that age level, students are attempting to define their self-concepts and they are open to new ideas as they are intensifying their researches for their identities. Other researchers also have found that, although older children have higher memorial and cognitive level, the difference between teens and much older students is not large as that between them and the younger children (e.g., Moely, 1977; Ornstein & Liberty, 1973). In other words, children at teens are in the best state in intelligence and that period is the optimal stage to develop their abilities in problems solving. Moreover, some researchers have discovered that students' problem-solving strategies become more systematic and logical as they get older and a marked change in their problem-solving strategies occurs between the ages of 11 and 13 (e.g., see Days, Wheatley, & Kulm, 1979; Hembree, 1992; Yudin & Kates, 1963).

Based on the above, this study focused on the lower secondary level, namely the first two years of students' secondary education. At this grade level, students are 13 and 14 years of age, and it is one of the more important stages for students to develop their abilities in problem solving.

Textbooks

In Singapore, there are four streams at the lower secondary level for students to select mainly based on their needs and abilities. They are express course, special course, normal course, and normal technical course. The mathematics curriculum for express and special streams is the same. It is different for the other two streams. According to the available statistics provided by the Ministry of Education (1999), there are usually about 60% of the students taking the express course and special course in the recent years. For those students, *New Syllabus D Mathematics*, a series of four mathematics textbooks first published in 1982, has been most commonly used in their learning of mathematics (Cheung & Chong, 1993). Therefore, we selected this series of textbooks as the subject of our study. At the lower secondary level, this series contains two textbooks, *New Syllabus D Mathematics 1* (Teh & Looi, 1997a) and *New Syllabus D Mathematics 2* (Teh & Looi, 1997b), for students to use in the two years.

Conceptual framework

A general conceptual framework was established for the study. It starts with the definition of problems from the perspective of textbook analysis, and includes the coding schemes about the types of problems and problem solving procedures to be used for data coding in the study.

What is a problem?

Different researchers often have different understandings of what a problem is in problem solving. According to Kilpatrick, "a problem is defined generally as a situation in which a goal is to be attained and a direct route to the goal is blocked" (Kilpatrick, 1985, p. 2). The definition stresses the point that the solution to the problem is not readily available in problem solving. Some researchers further argued the questions that could be easily solved by using algorithms or routine "problems" were not real problems (Fong, 1996). This stricter definition could cause some difficulty in textbook analysis, as a question according to this definition might be a problem to some students, but not to others, thus resulting in some uncertainty.

In this study, a problem is defined as a situation that requires a decision and/or answer, no matter the solution is readily available or not to the solver. This broader definition is more operational in textbook analysis where a textbook is treated as intended curriculum and the students who will use the textbook in their learning of mathematics are not exactly known. The definition is also consistent with the description given in Singapore mathematics syllabus, in which the term "a problem" is used to cover "a wide range of situations from routine mathematical problems to problems in unfamiliar context and open-ended investigations" (Ministry of Education, 1990, p. 3). In addition, researchers have argued that a major objective of mathematics instruction is for students to learn to solve routine problems, as life is "full of routine problems" (Holmes, 1995, p. 35). How students' ability in solving routine problems is related to their ability in solving non-routine problems is an interesting question that needs to be further studied.

Classification of problems

All the problems in the textbooks are first divided into two general categories – text problems, which are contained in the text part, and exercise problems, which are located in the exercises of all kinds in the textbooks. After that, different perspectives are employed to further classify all the problems into different categories according to the following classifications.

1. Routine problems versus non-routine problems

A non-routine problem is a situation that cannot be resolved by merely applying a standard algorithm, formula, or procedure, which is usually readily available to a problem solver. In contrast, a routine problem is one for which students can follow certain known algorithm, formula, or procedure to get the solution. Here is an example of a non-routine problem:

Find the two-digit number which has the sum of the cubes of its digits equal to three times itself (Teh & Looi, 1997a, p. 35).

2. Traditional problems versus non-traditional problems

In this study, non-traditional problems refer to one of the following four sub-types of problems. The first type is for problem-posing problems, which require students to create questions based on the given information for the problem situation. The second is for puzzle problems, which allows students to engage in potentially enriching recreational mathematics. The third is for project problems, which are tasks or a series of tasks for students to carry out using one or more of the following processes: gathering data, observing, looking for references, identifying, measuring, analyzing, determining patterns and/or relationships, graphing and communicating. A project usually requires students to take a substantial amount of time (e.g., a few days, weeks, or even months) to finish. The final type is for journal tasks asking students to write a piece of work through which the teacher can obtain useful information about mathematics learning and teaching.

Examples of such non-traditional problems from different school textbooks are given in Table 1.

Table 1. Examples of non-traditional problems

	Example
Problem-Posing Problem	Make up a question comparing two quantities which are increasing or decreasing, each at its own constant rate. Use substitution to answer your question. (McConnell, et al., 1996, p. 675)
Puzzle problem	Find the two-digit number which has the sum of the cubes of its digits equal to three times itself. (Teh & Looi, 1997a, p. 35)
Project	Conduct interviews with students in your school to find out their means of transport to school. (Teh & Looi, 1997b, p. 254)
Journal task	Write a brief report about what you have learned about triangles, their largest angles, and the lengths of their sides. (McConnell, et al., 1996, p. 61)

3. Open-ended problems versus close-ended problems

An open-ended problem has at least more than one possible answer. In addition, the solution is also open to the solver. Below is an example of an open-ended problem: Find two prime numbers whose sum is an odd number. Must one of the numbers be 2? (Teh & Looi, 1997a, p. 26)

In contrast, a close-ended problem is a problem whose solution is certain and fixed; there is usually only one answer to a close-ended problem.

4. Application problems versus non-application problems

A non-application problem is a problem unrelated to any practical background in everyday life or the real world. Correspondingly, an application problem is a problem related to or under the context of a real life situation.

Among the application problems, two sub-categories were further distinguished in this study. One is for fictitious application problems whose conditions and data are fictitiously made up by the author(s), and the other is for real application problems whose conditions and data are indeed from real situations or collected by students themselves. Sometimes, they are called "authentic" application problems. An example of a fictitious application problem is as follows:
Three bells toll at intervals of 8 min, 15 min and 24 min respectively. If they toll together at 3 p.m., what time will it be when they toll together again? (Teh & Looi, 1997a, p. 32)

Below is an example of an "authentic" application problem:

Here are the total number of votes (to the nearest million) cast for all major candidates in the presidential elections since 1940.

a. Graph the ordered pairs (year, number of votes).
b. Use the graph to predict how many votes will be cast for major candidates in the presidential election of 2000.

5. Single step problems, multiple countable step problems, and multiple uncountable step problems

Problems that can be solved by one direct operation are defined as "single step problems". Otherwise the problems are called "multiple step problems", or simply "multistep problems".

Furthermore, for a multistep problem, if the steps involved in the solution are countable, then it is "a multiple countable step problem". If the steps of solving a problem, such as a puzzle problem or an open-ended problem, are not countable, but more than one, then it is defined as "a multiple uncountable step problem",

6. Problems with just sufficient information, problems with extra information, and problems with insufficient information

If a problem contains more than enough information or conditions to solve, the problem is coded as "a problem with extra information". If the information provided in a problem is not enough to get the solution and it is not possible for the solver to know the needed information, then the problem is considered as "a problem with insufficient information". All the other problems are regarded as "problems with just sufficient information". Here is an example of a problem with extra information:

A boy scout in a jungle is heading south. He takes a right turn and walks for 40 m. Then he takes a left turn and walks again for a further 50 m. He then takes a left turn and walks for another 45 m. Finally, he takes a right turn. In which direction is he heading now? (Teh & Looi, 1997a, p. 47)

In solving this problem, the exact information about the distance that the boy walked through is not needed. In contrast, below is an example of a problem with insufficient information is given below:

How much will it cost to buy a 5-pound bag of dog food today if it cost $.20 less last week? (Hatfield, Edwards, & Bitter, 1997)

7. Problems in pure mathematical form, problems in verbal form, problems in visual form, and problems in a combined form.

This categorization is based on the stem of the problem that describes the setting and presents the data for the questions. If the stem only includes mathematical expressions, then the problem will be classified into the category of "problems (presented) in mathematical form". If the stem is entirely verbal, namely in written words only, then it is coded into the category of "problems in verbal form". If the stem simply consists of figures, pictures, graphs, charts, tables, diagrams, maps, etc., then such a problem is classified into "problems in visual form". The rest are "problems in a combined form", presented in a combination of the two or three of the above forms.

The above classifications provide a framework for us to examine the quantitative distribution of different types of problems. Researchers have argued that the quantitative patterns implies the frequencies with which students are exposed to different kinds of problems and therefore might have substantial influence on students' learning and their performance in problem solving (Fan, 1999; Stigler, Fuson, Han, & Kim, 1986).

Problem Solving Procedures

The problem solving procedures were examined based on Pólya's problem solving four-stage model and the Singapore Mathematics Syllabus (Lower Secondary) (Ministry of Education, 1990, 2000). We first reviewed whether the solutions presented in the textbook displayed the following stages (Pólya, 1957):

1. **Understanding the problem**. This stage includes extracting and assimilating the relevant and valuable information from the given, determining the goal of the problem, recoustructing the problem if necessary, and introducing suitable notations whenever possible for easy reference and manipulation.

2. **Devising a plan**. This stage is to make a general plan and select relevant methods, or more appropriately, heuristics, that might be useful for solving the problem based on the understanding of the problem at the first stage.

3. **Carrying out the plan**. This stage is to carry out the plan, which has been decided at the preceding stage, and to keep the track to obtain the answer.

4. **Looking back**. This stage includes checking the correctness of the solutions, reflecting on key ideas and processes of problem solutions, and generalizing or extending the methods or the results.

The above four stages are termed general strategies in this study. Under these stages, a framework of specific heuristics, briefly explained in Table 2, were used to examine the two textbooks. The framework is basically based on the Singapore Mathematics Syllabus (Lower Secondary) (Ministry of Education, 1990, 2000). Readers can refer to the syllabus for examples of the problems that can be solved using the various heuristics,

Table 2. A list of heuristics of problem solving

Heuristic		Brief explanation
1.	Act it out	Use people, objects, or items to physically show what is exactly described in the problem.
2	Change point of view	Approach the problem from a different angle when the previous way, often a conventional one, is not effective.
3	Draw a diagram	Draw a graph based on the information to visualize the problem.
4	Guess and check	Make a reasonable guess of the answer, and check the result against the conditions of the problem to see if it is the answer.
5	Look for a pattern	Observe common characteristics, variations, or differences about numbers, shapes, etc. in the problems to find the solution.
6	Make suppositions	Make a hypothesis and, based on the given and hypothesis, find out the relationship between the known and unknown.
7	Make a systematic list	Construct an organized list containing all the possibilities for a given situation and find the answer.
8	Make a table	Organize data into a table; then use it to solve the problem.
9	Solve part of the problem	Divide the problem into several sub-questions, then to solve them one by one, and finally to solve the original problem completely.
10	Simplify the problem	Change the complex numbers or situations in the problem into simpler ones without altering the problem mathematically.
11	Use a model	Use physical objects or drawings to help solve problems.
12	Use an equation	Use letters as variables to represent unknown quantities, and then establish and solve equation or inequality to get the answer.
13	Use before-after concept	Observe the change from one situation (before) to another situation (after) to find the solution.
14	Work backwards	Attack the problem from the outcomes or conclusions backwards to find what conditions they eventually need.

Procedure

Using the above conceptual framework and classifications, we examined and coded all the problems in the textbooks selected. However, when looking into the issue of how the textbooks represent different problem solving procedures, we only included the text problems, that is, the problems presented in the text part. The main reason for this decision is that problem solving procedures are not provided in the textbooks for the exercise problems, though the answers to almost all exercise problems are provided at the back of the textbooks.

Results and Discussion

An overall picture

The *New Syllabus D Mathematics 1* (Book 1) contains 17 chapters, covering the following topics: "whole numbers", "factors and multiples", "number sequences", "fractions and decimals", "real numbers", "estimation and approximation", "algebraic expressions and formula", "algebraic equations", "perimeter and area", "volume and surface area", "ratio, rate and proportion", "arithmetic problems", "basic geometrical ideas and properties", "angle properties of polygons", "similarity and congruence", "scales and maps", and "symmetry". *The New Syllabus D Mathematics 2* (Book 2) contains 14 chapters, covering the following topics: "arithmetic problems", "indices and standard form", "quadratic equations", "algebraic manipulation and formula", "simultaneous linear equations", "linear inequalities", "linear and quadratic graphs", "graphs in practical situations", "mensuration", "congruent and similar triangles", "area and volume of similar figures and solids", "statistics", "motion geometry", and "Pythagoras' theorem and trigonometrical ratios". Book 1 has 368 pages excluding preface and content list, while Book 2 has 373 pages. Answers to the problems in all the exercises are provided in the last 18 pages of Book 1 and the last 20 pages of Book 2.

There are 4471 problems in Book 1 and 4414 problems in Book 2. On the average, there are about 13 problems on each page in each book. In Book 1, 13 pages, including the first pages of six chapters, do not contain any problem; while the number in Book 2 is 5. It means that the openings of eleven chapters in Book 1 and all fourteen chapters in Book 2 are led by problems.

Both texts and exercises contain problems. As we can easily see, there are many more problems in the exercises than in the texts. In fact, there are 835 text problems and 3636 exercise problems in Book 1 and 645 text problems and 3769 exercise problems in Book 2.

The main results of the study are reported below in two sections. The first section is about how different types of problems are represented in the two textbooks, and the other is about how different problem solving procedures, namely, general strategies and specific heuristics, are represented in the books.

How different types of problems are represented?

Based on different placements and different purposes of problems provided in the textbook, we below discuss the types of problems represented in texts and those represented in exercises respectively, using the same framework described above.

Representation of the text problems

There are two kinds of problems in text according to the locations of the problems. One is located in main text, including example problems, class activity problems, and other problems which are given in text but not listed explicitly as example problems or class activity problems. The other is located in the margin, called marginal problems. The purpose of marginal problems, as stated in the textbooks' preface, is to provide activities and interesting information (Teh & Looi, 1997a, 1997b). Therefore, many of them are non-traditional problems. In Book 1, out of the 171 marginal problems, 60 problems are puzzle problems and one requires students to pose problems by themselves. In Book 2, there are 157 problems in the margins of the textbooks and 89 of them are puzzle problems. The percentage of non-traditional problems in all the problems contained in the margin is 45%. In contrast, all but four of the example problems are traditional problems and about 90% of the class activity problems are traditional problems in the two books. Table 3 shows the distribution of traditional and non-traditional problems of the text problems.

Table 3. Distribution of traditional and non-traditional problems in the text problems

	Book 1				Book 2			
	Main Text			Marginal Text	Main text			Marginal text
	EP	CAP	Other		EP	CAP	Other	
Traditional problem (TP)	384	153	121	110	329	93	57	89
Non-traditional problem (NTP)	2	4	0	61	2	3	4	68
Ratio of NTP:TP	0.5%	2.6%	0	55.5%	0.6%	3.2%	7.0%	76.4%

Note: EP = example problems; CAP = class activity problems.

The percentage of the marginal problems in all the text problems is around 22%. Except for two problems, all the other problems are supplied with no answers in the textbooks. These problems are designed for instructional enrichment, and hence present more challenges to students. They could be skipped in teaching especially when teachers do not have enough time in class. In that case, it is up to individual students whether or not they want to solve these problems at their own pace and time.

There are 386 example problems in Book 1 and 331 in Book 2. They number near half of the text problems (48%). Almost all the example problems are traditional problems in the two books; except four examples are puzzle problems and they are posed for the purpose of introducing two specific heuristics: "change your point of view" and "look for a pattern", respectively. In most cases, example problems are given to explain certain algorithms or heuristics, which are introduced just before the problems, so those example problems can be considered to be routine problems. Less than 2% of the example problems are non-routine problems.

Although there are more routine and traditional problems in examples, the majority of example problems need more than one step to solve. As solving multiple-step problems is usually more challenging than solving one-step problems, it implies that these example problems are not necessarily easy to solve.

In the two books, there are 37% of the example problems presented simply using pure mathematical expressions, 40% of the problems in written words only, and 20% of the problems using visual information (i.e., figures, tables, etc.). The remaining are presented with a combination of the above two or three forms.

As to the content of the example problems, only 27% of them are application problems. The situations used in those application problems are more fictitious than "authentic". In fact, out of the 4471 problems including both text and exercise problems in Book 1, only 73 problems are really related to real life situations, and the corresponding number is 54 in Book 2. The total percentage is around 1%. The quantity of the application problems in the two books is also small and they only cover 21% of all the problems.

About 17% of the text problems are under the sections of "class activity" in the two textbooks. Four of them are project problems: one in Book 1 and three in Book 2; and they are also the only project problems in the two books. All the other problems in "class activity" sections show actually no significant difference from the other text problems. Most of them are routine (88%) and traditional problems (97%). Only 27% of the problems are related to real life situations.

The class activity problems usually require students to do mathematics by themselves in classroom. They provide more chances than other kinds of problems such as examples to let students be involved in doing mathematics. Many of them need to be solved in multiple steps and in most cases (nearly 96%) the textbooks do not provide answers to them. Therefore, those problems provide good opportunities for teachers and students to exercise their creativity and high-order thinking skills; and to solve them usually requires more time.

In short, most text problems in the textbooks are routine and traditional problems. Overall the percentages of these two kinds of problems are 80% and 89%, respectively. Relatively, more non-routine and non-traditional problems are provided in the marginal text.

Table 4 shows the information on how the text problems are presented in the books. As we can see from the table, near half of the text problems are verbalized, 23% are given using pure mathematical expressions, 14% are presented in visual form using figures, tables, etc., and the other 18% are posed in a combination of the above two or three forms. The variety of representation forms requires students to possess solid reading, interpreting, and understanding abilities, which are important for students to develop their communication skills.

Table 4. Distribution of problems by different forms in which the text problems are presented

	Book 1				Book 2				Total
	Main Text			Marginal Text	Main Text			Marginal Text	
	EP	CAP	Text		EP	CAP	Text		
Mathematical	141	16	21	8	126	23	6	6	347
Verbal	203	61	65	111	86	22	25	91	664
Visual	19	42	21	22	39	34	22	5	204
Combined	23	38	14	30	80	17	8	55	265

Note: EP = example problems; CAP = class activity problems.

Table 5 presents the data on the frequency of problems in terms of the number of steps required for solving them. It indicates that the percentage of single step problems is slightly higher than that of multiple countable step problems (42% to 40%). In total, there are around 58% of the text problems requiring more than one step to solve.

Table 5: Distribution of problems by the number of steps needed for their solutions

	Book 1				Book 2				Total
	Main Text			Marginal Text	Main Text			Marginal Text	
	EP	CAP	Text		EP	CAP	Text		
Single Step	161	97	75	35	142	63	24	26	623
Multistep (C)	222	49	33	34	189	26	22	20	595
Multistep (U)	3	11	13	102	0	7	15	111	262

Note: EP = example problems; CAP = class activity problems. C = countable; U = uncountable.

Almost all the text problems provide exactly sufficient information to the students. Only eleven of them contain extra conditions: six in Book 1 and five in Book 2, and four problems do not contain enough information. According to the textbooks, students might potentially have little exposure to problems with insufficient or extra information, therefore come to an impression that a problem always has just enough information, and to solve a problem is equal to use all the information provided in the problem. But such an impression could be unrealistic in solving real life problems where students need to actively gather, judge, and select information that are needed.

In addition, most of the text problems are close-ended. The percentage of open-ended problems in all the problems is only 7.5%. Particularly, only four examples are open ended and all of them are in Book 2.

Technology is not heavily used in the textbooks in solving problems. Among the 1480 text problems, only 56 problems involve the use of calculators, the only way of using technology in the books. It seems that the textbook authors emphasized more on students' theoretical thinking skills and computational skills without a calculating aid. As we know, the latest Singapore mathematics syllabus (Lower Secondary) (Ministry of Education, 2000) stresses the importance of integrating information technology (IT) in mathematics learning and teaching including problem solving. We believe that more attention needs to be paid to the integration of using IT in problem solving in current textbook reforms.

Representation of the exercise problems

As said earlier, there are 7405 exercise problems in the two textbooks. The ratio of the exercise problems to the example problems is 1:10. In general, the exercise problems are quite similar to example problems.

Almost all the exercise problems are routine and traditional. Less than 20% of the problems are related to real life situations. In addition, almost all the exercise problems have just sufficient conditions and close-ended solutions.

Most exercise problems are presented either using mathematical expressions or in verbal form only, with the percentages being 45% and 33%, respectively. Only 4% of the problems entail the use of calculators. Answers to all the exercise problems except those requiring drawing or proving (8%) and those under "challenge yourself" (3%) are given in the back of the textbooks.

The section of "challenge yourself" in the exercises is special. This section is intended to provide more challenges than the other general exercise problems for students (especially for high-performing students) to further develop their critical thinking skills and creativity. About 26% of the problems within this section are non-routine problems, and 25% are non-traditional problems, though all of them are puzzles. The percentages are much higher than those found with the problems in other parts of the textbooks such as example problems and other general exercise problems. In addition, about 89% of these problems under this section are multistep problems. Nevertheless, nearly 90% of the problems are close-ended problems.

Besides, about 2.5% of the problems are given with hints, and most of them are very challenging.

Representation of both the text and the exercise problems

Taking problems both in the text part and in the exercise part together, we can see that overall there are 8885 problems in the two books, 97% of the problems are traditional problems, and 96% are routine problems. Among the non-traditional problems, the majority (98%) are puzzle problems. There are only four project problems and one problem asking students to pose a problem based on the given condition. The non-traditional problems are mainly given in the marginal text or in the "challenge yourself" section in exercises.

While the problems in both the text and exercise parts usually contain just sufficient conditions and are close-ended, they are presented in a variety of ways in the textbooks. About 35% of all the problems are presented verbally, 41% are presented with pure mathematical expressions, and 22% are in visual form. In addition, nearly 60% of the problems need more than one step to solve. As to the problem contents, the problems with practical background or related to real life situations are relatively few – only 21%. Particularly, less than 2% of the problems are "authentic" application problems, that is, the context and data are really from real life. Finally, the percentage of the problems involving the use of technology is relatively low (4%) and the calculator is the only tool of technology used in the books.

How Different Problem Solving Procedures are Represented?

The majority of the problems in the two textbooks are solved in detail or at least supplied with the final answers. There are two main kinds of problems without answers – text problems in the margin and exercise problems under the "challenge yourself" section. As pointed out earlier, to analyze the procedures of problem solving represented in the books, we only focused on the problems with detailed solving procedures. The problems in all the exercises are not included, because no problem solving procedures are explicitly given in the textbooks to them, though the final answers are given to most of them in the back of the textbooks, as aforementioned.

There are 789 problems solved in the text in the two books – 452 in Book 1 and 337 in Book 2. Nearly 89% of them are presented with detailed solution procedures and the other 11% are only given the final answers. Furthermore, 291 problems are solved with detailed word explanations and 10% of them are accompanied with diagrams so that the illustration of the solving procedures is visualized. It seems to us that the use of these means can help students to understand how the problems are solved thoroughly.

General strategies

According to Pólya's problem-solving model, there are four main stages in problem solving procedures – understanding the problem (U), devising a plan (D), carrying out the plan (C), and looking back (L). Based on this general framework, all the solutions to the 789 problems were examined and coded. The general results are shown in Table 6.

Table 6. Distribution of problems by the numbers of stages based on Pólya's model displayed in their solutions in the solved text problems

		Book 1	Book 2	Total
One Stage	C	290	206	496
Two Stages	U + C	65	22	87
	D + C	11	15	26
	C + L	55	59	114
Three Stages	U + D + C	5	17	22
	U + C + L	19	2	21
	D + C + L	7	14	21
Four Stages	U + D + C + L	0	2	2

Note: U = understanding the problem, D = devising a plan,
C = carrying out the plan, and L = looking back.

As we can see from Table 6, in all the 789 problems, only two problems are solved with all the four stages. The solutions to 37% of the problems include two or three stages, and 63% of the problems are solved with just using the third stage. Moreover, 132 problems are solved with both the first and the third stages; 24 of them further include the second stage, and 23 of them further include the fourth stage. Seventy-four problems are solved involving both the second and the third stages and 23 of them further include the fourth stage. There are 158 problems solved with both the third and the fourth stages.

About 17% of the 789 problems are solved with the explanation of the "understanding the problem" stage, 9% with "devising a plan" stage, 20% with "looking back" stage, and overall only 37% of the problems are solved with more than one stage, the third stage. It implies that the students are exposed to more about the procedures of how the problems are solved but less on how to understand, approach, and extend and reflect on the problems and the problem solving process. The procedures what are showed in the textbooks are usually routine, easy to figure out, and less of the nature of exploration. It seems that such work needs teachers to complement in actual teaching.

Specific heuristics

In the instruction of problem solving, teaching students heuristics is one major content (Ministry of Education, 1990, 2000). It seems to us that good attention was paid to this aspect in the two books. In fact, one whole chapter, namely chapter 3, in Book 1 is specifically devoted to problem solving and introducing heuristics to students.

In total, 14 specific heuristics are introduced in the two books. They are "act it out" (1), "change your point of view" (2), "draw a diagram" (53), "guess and check" (1), "look for a pattern" (6), "make suppositions" (5), "make a systematic list" (9), "make a table" (5), "solve part of the problem" (1), "simplify the problem" (9), "use a model" (3), "use an equation" (43), "use before-and-after concept" (2), and "work backwards" (3). The number in the each pair of parentheses shows the number of times the corresponding heuristic is used in the two books.

However, compared to the number of the problems that are solved in detail in the textbooks, the frequency of using heuristics is relatively low. The distribution of these heuristics is also relatively concentrated. Usually in several consecutive pages, many heuristics are introduced and used; but in many other pages few heuristics are used. It appears that the heuristics are only treated as a specific mathematics topic and do not penetrate the whole textbooks. Therefore, we believe the distribution of these heuristics could be further improved.

Some other results of problem solving procedures

The data we obtained in the study also show that most of the solved problems in the textbooks are solved in one way. There are only 59 problems (7%) solved in more than one way. Nine of them are solved in three ways. Only two problems are solved in four different ways and each book has one such problem.

It should be pointed out that there is a special feature of the two textbooks. Namely, unlike in traditional textbooks we can often find that solving a problem is to apply or practice a certain concept or algorithm, which has been introduced in advance, in the two books a considerable number of concepts and algorithms are introduced through solving problems. In fact, there are 93 problems in the textbooks leading to new concepts and 153 problems leading to new algorithms or theorems. Such an approach, learning mathematics from solving problems, reflects the textbook developers' philosophy of using problem solving as a vehicle for students to learn mathematics.

Summary and Conclusions

This study examined one of the most commonly used series of Singaporean mathematics textbooks at lower secondary level, *New Syllabus D Mathematics*. The purpose of the study is not to make an overall evaluation of the quality of the textbooks but to investigate how the textbooks represent problem solving. Moreover, as we pointed out earlier, an analysis of textbooks is a necessary but not a sufficient condition to understand what really happen in actual classroom teaching.

In summary, there are 8885 problems in these two textbooks, with 96% of the problems being routine problems and 97% being traditional problems. The non-routine and non-traditional problems are mainly represented in the marginal text and in the "challenge yourself" section in the exercises. Almost all the non-traditional problems are puzzle problems. There are only four project problems and only one problem-posing problem. Moreover, 98% of the problems are close-ended.

About 35% of the problems are presented verbally, 41% are given using pure mathematical expressions, and nearly 22% of the problems are posed in visual form. There are more multistep problems than one-step problems (5248 to 3637) in the books. Many problems designed in these two textbooks are very challenging.

Around 21% of the problems are related to real life situations application problems, but only less than 2% of the problems are "authentic" application problems, and only one of them is an example problem, which is found in book 2. Almost all the problems contain just sufficient information, only 61 problems provide extra conditions, and five problems supply less than enough information.

Among 1480 text problems, 789 problems are provided with complete solutions including the final answers. However, the solutions to most problems displayed in the textbook just show how to carry out the plan, which in Pólya's model is the third stage. In addition, about 92.5% of the solved problems are solved in only one way.

Totally 14 specific heuristics are presented in this textbook. One whole chapter in Book 1 is especially devoted to introducing heuristics. However, the frequency of using these heuristics is relatively low and the distribution of these heuristics is concentrated.

A special feature of the two textbooks is that the introduction of many new concepts and new algorithms is through solving particular problems. We believe such as approach can facilitate students' learning new knowledge through their own exploration and enrich the teaching and learning of mathematics.

In conclusion, we think the two textbooks present a good foundation for students to develop their abilities in problem solving, and are particularly strong in many aspects such as applying fundamental/theoretical knowledge to solve problems, developing logical and higher-order thinking skills through solving multistep and challenging problems, exposing students to a variety of heuristics, and learning new concepts and algorithms by problem solving. Nevertheless, like many other textbooks, they can be also improved in some other aspects. In particular, students could be exposed to more non-routine problems, non-traditional problems (e.g., projects), open-ended problems and application problems (especially using "authentic" real-life situations). In addition, in problem solving procedures, students could be more exposed to the general strategies of Pólya's four-stage model, various heuristics could be used more frequently, and the distribution of the heuristics could be more balanced.

References

Beaton, A. E., Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Kelly, D. L., & Smith, T. A. (1996). *Mathematics achievement in middle school years: IEA's Third International Mathematics and Science Study (TIMSS)*. Boston, MA: Center for the Study of Testing, Evaluation, and Educational Policy, Boston College.

Cheung, J., & Chong, T. H. (1993). Towards mathematics excellence in Singapore. In G. Bell (Ed.), *Asian perspectives on mathematics education* (pp. 122-135). North Rivers Mathematical Association.

Dacey, J. S. (1989). Peak periods of creative growth across the lifespan. *Journal of Creative Behavior, 23*(4), 224-47.

Days, H. C., Wheatley, G. H., & Kulm, G. (1979). Problem structure, cognitive level, and problem-solving performance. *Journal for Research in Mathematics Education, 10*(2), 135-146.

Fan, L. (1999). The sources of teachers' knowledge of instructional materials. *The Mathematics Educator, 4*(2), 42-69.

Fan, L. (1999). Applications of arithmetic in US and Chinese textbooks: a comparative study. In G. Kaiser, E. Luna, & I. Huntley (Eds.), *International comparisons in mathematics education* (pp. 151-162). London: Falmer Press.

Fan, L., & Kaeley, G. S. (2000). The influences of textbooks on teaching strategies: An empirical study. *Mid-Western Educational Researcher, 11*(2), in press.

Fong, H. K. (1996). An integrated approach for teaching mathematical problem solving. *The Mathematics Educator, 1*(1), 36-44.

Graybeal, S. S. (1988). *A study of instructional suggestions in fifth-grade mathematics and social studies teacher's guides and textbooks.* Unpublished doctoral dissertation, University of Chicago.

Hatfield, M. M., Edwards, N. T., & Bitter, G. G. (1997). *Mathematics methods for elementary and middle school teachers* (3rd ed.). Boston: Alley & Bacon.

Hembree, R. (1992). Experiments relational studies in problem solving: a meta-analysis. *Journal for Research in Mathematics Education, 23*(3), 242-273.

Holmes, E. E. (1995). *New directions in elementary school mathematics.* Englewood Cliffs, NJ: Prentice-Hall.

Howson, A. G. (Ed.) (1995). *Mathematics textbooks: A comparative study of Grade 8 Texts.* Vancouver, Canada: Pacific Educational Press.

Keys, W., Harris, S., & Fernandes, C. (1996). *Third international mathematics and science study: First national report (Part I).* Berkshire, U.K.: National Foundation for Educational Research.

Keys, W., Harris, S., & Fernandes, C. (1997). *Third international mathematics and science study: First national report (Part II).* Berkshire, U.K.: National Foundation for Educational Research.

Krammer, H. P. M. (1985). The textbooks as classroom context variable. *Teaching & Teacher Education, 1*(4), 273-278.

Kilpatrick, J. (1985). A retrospective account of the past 25 years of research on teaching mathematical problem solving. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving multiple research perspectives*. Hillsdale, NJ: Erlbaum.

Krulik, S., & Rudnick, J. A. (1987). *Problem solving: A handbook for teachers* (2nd ed.). Boston: Allyn and Bacon.

McConnell, J. W., Brown, S. Usiskin, Z., Senk, S. L., Widerski, T., Anderson, S., Eddins, S., Feldman, C. H., Flanders, J., Hackworth, M., Hirschhorn, D., Polonsky, L., Sachs, L., Woodward, E. (1996). UCSMP *Algebra* (2nd ed.). Glenview, IL: ScottForesman.

Ministry of Education. (1990). *Mathematics Syllabus (Lower Secondary).* Singapore: Ministry of Education,

Ministry of Education. (1999). *Education Statistics Digest 1999.* Singapore: Ministry of Education,

Ministry of Education. (2000). *Mathematics Syllabus (Lower Secondary).* Singapore: Ministry of Education.

Moely, B. E. (1977). Organization in memory. In R. V. Kail, Jr., & J. W. Hagen (Eds.), *Perspectives on the Development of Memory and Cognition* (chapter 7). Hillsdale, NJ: Erlbaum.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: NCTM.

Ornstein, P. A., & Charles, L. (1973). *Rehearsal Processes in Children's Memory.* Paper presented at the biennial meeting of the society for research in child development. Philadelphia, PA. (ERIC Document Reproduction Service No. ED 085 077).

Quek, T. (2000, April 8). US education chief praises Singapore way. *The Straits Times*, p. 8. Singapore.

Pólya, G. (1957). *How to solve it: A new aspect of mathematical method* (2nd ed.). Princeton, NJ: Princeton University Press.

Robitaille, D. F., & Garden, R. A. (1988). *IEA study of mathematics II: contexts and outcomes of school mathematics*. New York: Pergamon.

Robitaille, D. F., & Travers, K. J. (1992): International studies of achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 687-709). New York: Macmillan.

Song, M., & Ginsburg, H. P. (1987). The development of informal and formal mathematical thinking in Korean and U.S. children. *Child Development, 58*(5), 1286-1296.

Sosniak, L. A., & Stodolsky, S. S. (1993). Teachers and textbooks: materials use in four fourth-grade classrooms. *The Elementary School Journal, 93*(3), 249-275.

Stevenson, H. W., Lee, S., Chen, C., Lummis, M., Stigler, J., Fan, L., & Ge, F. (1990). Mathematics achievement of children in China and the United States. *Child Development, 61*(4), 1053-1066.

Stevenson, H. W., Stigler, J. W., Lucker, G. W., Lee, S., Hsu, C. C., & Kitamura, S. (1986). Classroom behavior and achievement of Japanese, Chinese, and American children. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 3). Hillsdale, NJ: Erlbaum.

Stigler, J. W., Fuson, K. C., Ham, M., and Kim, M. S. (1986). An analysis of addition and subtraction word problems in American and Soviet elementary mathematics textbooks. *Cognition and Instruction, 3*(3), 153-171.

Stigler, J. W., Lee, S., Lucker, W., & Stevenson, H. W. (1982). Curriculum and achievement in mathematics: a study of elementary school children in Japan, Taiwan, and the United States. *Journal of Educational Psychology, 74*(3), 315-322.

Stodolsky, S. S. (1989). Is teaching really by the textbook? In P. W. Jackson & S. Haroutunian-Gordon (Eds.), *From Socrates to Software: The teachers as text and the text as teacher: Eighty-ninth yearbook of the National Society for the Study of Education* (Part I). Chicago: University of Chicago Press.

Teh, K. S., & Looi, C. K. (1997a). *New Syllabus D Mathematics 1* (General editor: Lee, P. Y.) (4th ed.). Singapore: Shing Lee Publishers.

Teh, K. S., & Looi, C. K. (1997b). *New Syllabus D Mathematics 2* (General editor: Lee, P. Y.) (4th ed.). Singapore: Shing Lee Publishers.

Viadero, D. (2000). Expert exports. *Education Week, 12*(3), 18-19. (Also see online http://www.edweek.org/tm/tm-printngstory.cfm?slug=03singapore.h12)

Yudin, L., & Kates, S. L. (1963). Concept attainment and adolescent development. *Journal of Educational Psychology, 54*(4), 177-182.