Learning From the Implementers in a Design Experiment

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In a design experiment, the feedback from the teacher-implementer is crucial to the success of the innovation simply because the teacher is finally the one that brings the innovation to life in front of the students. We describe in this paper the feedback made by the teacher-implementer after teaching one cycle of the problem solving module in a mainstream school, and the modifications the researchers and the teacher-implementer have made in our design of the module to fit into the requirement of the school.

Mathematical Problem Solving for Everyone

Through our research project *Mathematical Problem Solving for Everyone* (MProSE), a module on problem solving has been developed for high school students (see Toh, Quek, Leong, Dindyal & Tay, 2011a). T he MProSE problem solving module takes as its theoretical foundation the problem solving model of Pólya (1945) coupled with insights from Schoenfeld (1985). M ProSE adopts the methodology of design experiment as it allows for the unique demands and constraints of the schools to be met, and, at the same time, the research imposes rigour onto the design. The methodology's advocacy of an implement-research-refine iterative approach to educational design appears to us to hold potential in dealing with the complexity of school-based innovations.

MProSE was first implemented in a Singapore secondary school specializing in Mathematical Sciences (hereafter, we shall call it the MProSE research school). After identifying the feasibility of implementing such a problem solving module and presenting the research findings in a seminar for Singapore mathematics teachers, we obtained the commitment of three mainstream schools to participate in the next phase of MProSE. The three mainstream schools have completed teacher professional development on mathematical problem solving and have taught at least one round of the problem solving module to one level of their lower secondary cohort of students since then, while the MProSE research school has completed three rounds of teaching the MProSE problem solving module to three cohorts of their students.

To ensure the sustainability of the problem solving module within the school curriculum, feedback from the teacher-implementers and the researchers' input based on their observations are crucial. As such, the researchers have met with the teachers from the four schools to discuss the modifications (adaptations and fine-tuning) that would be required to be made in the design of the module. Adaptations were made in view of the constraints of the school environment, and fine-tuning was made in consideration of improving the teaching of problem solving in the subsequent cohorts. This paper discusses the modifications that have been made in the design of the module in one mainstream school, and the evolution of the problem solving module in the MProSE research school.

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In the mainstream school Y, *all* Secondary Two (age 14) students from the express stream were involved in MProSE research project. Ten hours of curriculum time were allocated for the MProSE problem solving lessons spread over six months.

The Head of the Mathematics Department – Tommie (pseudonym) – took the lead in teaching the problem solving lessons, although she was not a Secondary Two resident teacher herself. Tommie adhered strictly to the proposed lesson plans and the problems. Tommie reported that the students' responses to the problems were more encouraging than expected: (1) many students were attempting to use algebraic notations to represent mathematical entities, and (2) some students were attempting to formulate a problem using algebraic equations as an alternative method to solving a given problem.

Assessment of Students' Work

The teachers headed by Tommie expressed their concern in the marking of students' solution of the problems. According to the researchers' original intention (see Toh et al, 2009), students are assessed on (1) Polya's stages, (2) use of heuristics and (3) checking and expanding the problem. Students who obtain full marks for (1) and (2), meaning that they manage to obtain correct solution of the problems, would then be awarded marks for (3) if they further demonstrate the ability to check their solution or expand the problem. In the event that they do not obtain the full score for (1) and (2), they would not be awarded marks for (3). However, Tommie felt that it was crucial for students to be encouraged to pursue the path of problem solving even though they might not obtain a completely correct solution. Consequently, it was decided that marks for (3) be awarded to students if they at least manage to obtain a partially correct solution.

Lesson Implementation

Although the MProSE handbook (Toh, et al, 2011a) provides the details of the lesson plans of the problem solving lessons, the pedagogy was not outlined. Adaptation was made by the teacher in her actual lesson implementation to enhance her students' learning. One crucial method observed by the researchers in this aspect of pedagogy is detailed below.

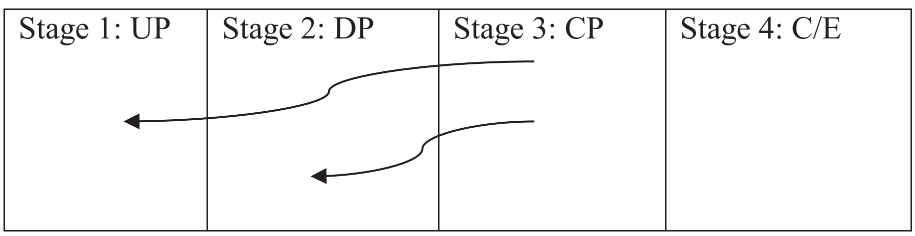

Figure 1. The use of the whiteboard in teaching problem solving

In order to present the four stages of Polya clearly to her students, Tommie effectively divided the whiteboard into four sections (see Figure 1). These four sections corresponded to the practical worksheet that the students were required to fill up. She modeled to the students how the practical worksheet was supposed to be used and also demonstrated clearly the flow of the entire thought processes in solving an entire problem through the use of Polya's model by moving from panel to panel, sometimes backwards to illustrate the backward loop of the model.

Tommie also provided the following feedback regarding Polya's Stage 4 as required by the problem solving lessons. Tommie expressed her concern that her students might have difficulty to have too many sophisticated extensions, generalizations or adaptations as proposed in the "Check and Expand" section. In Lesson 4, while adhering to the proposed lesson plans, she focused on the checking of the reasonableness of their own solutions rather than the "expand" or "generalize" part.

Choice of problems

The successful implementation of the problem solving module is highly dependent on the choice of *appropriate problems* (Dindyal, Tay, Toh, Leong & Quek, 2010). However, appropriateness may be different for different people. In the context of the MProSE problem solving research, we have first designed some problems on which we have collected some data from the students solving these problems when the study was first implemented in the first MProSE research school. In the scaling up of the study to the three mainstream schools, some of the initially selected problems were replaced so as to be better aligned to the school mathematics curriculum. In the context of a design experiment, it is permissible that a problem that was initially selected by the researchers be subsequently replaced by another problem that a participating school deems suitable, so long as it meets the original intention of the original problem (Quek, Dindyal, Toh, Leong & Tay, 2011).

Consider the following problem:

> On a Tuesday, Alice, Bernice and Carol and Dory, met for a movie. After the movie, they made plans for the next gathering. Alice, Bernice and Carol said that they could only go to the movies every 6, 3 and 4 days respectively, starting from that Tuesday. Dory said that she could go to the movies every day except on Fridays. After how many days would the four friends be able to meet again for a movie?

This problem was initially selected based on two reasons: (1) its content is aligned to the school mathematics curriculum at Secondary Two level; and (2) it could demonstrate the heuristics *Act it out*, *Draw a diagram*, *Restate the problem in another way* and *Aim for sub-goals*. However, as pointed out by Tommie, her students found this problem easy as they had solved many similar problems in their usual mathematics classrooms; thus, it had failed to illustrate the importance of the application of heuristics in solving the problem. Another problem that was a concern was the following:

> In a carpark, it was advertised as such - First hour parking: Free; Subsequent parking: $1 per hour or part thereof Sketch a graph for parking fee versus the number of hours of parking with axes as shown below.

This problem aimed to demonstrate the heuristics *Use suitable numbers* and *Divide into cases* in a real-world context grounded in the knowledge of sketching of graphs in the Cartesian coordinates. According to Tommie, most of her students were not able to plot the graph correctly or demonstrate any steps that could lead to the correct solution, since students at Secondary Two level have generally not mastered graph plotting and the Cartesian coordinate system. Furthermore, the context of this problem is not "real" to students (as they don't drive!). She suggested this problem be replaced by one which has a context more relevant to her students.

Conclusion

For the MProSE problem solving module to be sustainable within schools, it is crucial that the researchers and the teacher implementers constantly explore the relevance of the various components of the modules, which is highly dependent on many external factors and the constraints of reality. Although the participating schools generally find the concept of MProSE a sound and feasible model to be implemented in schools, modifications need to be made. In fact this report describes three changes that have come from the implementers themselves, they cover assessment, pedagogy and materials. Under the framework of design experiment, all these are permissible so long as the fundamental parameters of MProSE are not abandoned, i.e., that (1) problem solving is suitable for all students (not just the elite few); and (2) the processes (in addition to the product) of problem solving must be assessed.

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