WORKING MEMORY AND MATHEMATICAL WORD PROBLEM SOLVING

Kerry Lee, Ng Swee Fong and, Ng Ee Lynn
National Institute of Education
Nanyang Technological University
Singapore

In the third international mathematics and science study, Singapore's students obtained better results than did most of their peers in other countries. This is an enviable result and points to a positive educational and social environment. Despite this finding, a sizeable number of students do experience difficulties with mathematics. Quite often, such difficulties appear at an early age and remain for a number of years. For example, we were recently asked by a local school to assist with a number of pupils who were regarded as "slow learners". What became apparent was that many of these pupils started having problem with mathematics in the early primary years. When they reached upper primary school, their problems became more acute. One problem we identified was that mathematical problems were often presented to pupils using very complex language. Nevertheless, this was unlikely to be a main reason for the children's failure: their peers had to answer the same questions but most of them succeeded. In this study, we focused on several variables that might contribute to individual differences in mathematical performance.

Studies conducted overseas showed that there are significant inter-age and intra-age differences in mathematical abilities. Cockcroft (1982), for example, reported intra-age differences that varied by as much as the equivalence of a seven-year achievement range. Regarding intra-age difference, several variables have been identified as playing important roles: biological (see Geary, 1993, for review), motivational (e.g., Ashcraft, Kirk, & Hopko, 1998) and cognitive (e.g., Shalev, Manor, & Gross-Tsur, 1997). In this study, we focused on the influence of cognitive variables.

A number of cognitive variables have been studied in previous studies. Processing speed, for example, has been shown to be a reliable predictor of mathematics performance. Bull and Johnston (1997) found processing speed provided additional predictive value beyond that provided by reading ability, short-term memory span, and ability to utilise long-term memory. McLean and Hitch (1999) also found poorer general processing ability in children with poor mathematical abilities. In recent years, some researchers have focused on the relationship between mathematical performance and another aspect of cognitive ability: working memory. Working memory, first described by Baddeley and Hitch (1974), is a vital part of our memory network that allows for the representation and manipulation of information. It is responsible for short-term memory storage, reasoning, problem solving, and other cognitive tasks that require a consideration of past and present. For example, when one is presented with a problem, $13 + 7 = x$, the numbers and operation involved are first encoded into working memory. For adults, the encoded information will trigger retrieval of the solution from long-term memory. For young children who cannot directly retrieve the solution from long-term memory, the application of appropriate problem solving strategies occurs in working memory.

The latest version of Baddeley 's (2000) working memory model consists of four components: central executive, phonological loop, visuo-spatial sketchpad, and an episodic buffer. Both the phonological loop and the visuo-spatial sketchpad are short-term storage system. The former is responsible for storing and rehearsing auditory and articulatory information. The latter maintains and manipulates images. Exchange of information between these short-term stores and long-term memory is facilitated by the episodic buffer. The central executive is a resource manager. Like most cognitive accounts of thinking processes, Baddeley's model postulates an upper limit in attentional capacity. Performance deteriorates when task demands exceed this capacity. The central executive controls the allocation of such attentional resource. It is also understood to be involved in the execution of higher cognitive tasks (e.g., comprehension, reasoning, and decision making).

Recent findings on the relationship between working memory and arithmetic performance suggest the two are closely linked. Bull and Scerif (2001), for example, showed central executive measures predicted reliably to mathematical performance. Moreover, the contribution of these measures was independent of that attributable to reading and general intelligence measures. Others identified reliable differences between children with average versus poor mathematical abilities. Generally, children with poor mathematics abilities have difficulties with tasks designed to measure central executive function (Bull, Johnston, & Roy, 1999). However, deficits in phonological or visuo-spatial measures have also been found in some studies (Gathercole & Pickering, 2000; Hoard, Geary, & Hamson, 1999; McLean & Hitch, 1999). Of particular interest is the ability of working memory measures to predict to academic performance in mathematics. Using scores from the UK national curriculum assessment in mathematics, Gathercole and Pickering (2000) found working memory scores predicted to children's academic status – average versus below average -- with 83% accuracy.

The present study built on previous work and examined the relationship between mathematics performance, social variables, and cognitive abilities. Specifically, we examined the extent to which these variables predicted to performance on word problems. The proposed study differed from previous studies in several respects. First, previous studies had concentrated largely on basic skills such as counting, number knowledge, and basic arithmetic such as addition and subtraction. Such skills are fundamental building blocks, failure in which will likely result in further delay. In the present study, we looked one step further and focused on problem solving requiring algebraic thinking. Given Ostad's (1998) finding that there were significant differences in children's performances depending on whether questions were presented as number facts or word problems, findings based on fundamental mathematical abilities might not generalize to word problems.

On an applied level, findings from the present study may assist in identifying children's abilities. At present, decisions on streaming -- formal or informal -- are based on academic achievement. By providing information on children's underlying cognitive abilities, working memory measures may aid in decisions regarding children whose achievement is on the borderline. Previous studies suggest such measures provide more information than conventional measures of intelligence (Bull & Scerif, 2001).

Method

Participants

151 Primary-5 children participated in this study. All children were recruited from schools located in the western zone of Singapore and participated with parental consent. Because of school activities and unexpected absences, a number of children did not complete the whole battery of tests. The final sample contained 145 children with an average age of 10.7 years (*SD* = .65). A priori power analysis showed that this sample would yield approximately 95% power (statistical parameters: _ = .05, medium effect size f^2 = .15, 6 predictors).

Material and Procedure

Children were administered a number of tests: a mathematical word problem test devised by the authors, the Working Memory Test Battery (Pickering & Gathercole, 2001), vocabulary and block design (performance IQ) subtests from the Wechsler Intelligence Scale for Children, reading subtests from the Wechsler Objective Reading and Language Dimensions (Singapore), and a homework measure. Because of the length of each test, they were administered over several days.

Mathematical performance. The mathematical test contained ten word problems. Question selection was guided by the national school curriculum. Because testing was conducted at the middle of the academic year, we selected two questions from the primary 4 curriculum, six questions from the primary 5 curriculum, and two slightly harder items from the late primary 5 curriculum. Each question was selected on the basis that they could be solved by either the model method or the listing heuristics (an alternative method that is taught in many schools). Although some questions might have been perceived by the children as being more easily solved using one or another type of heuristics, questions containing obvious biases were not used. Three parallel versions were constructed. Questions in each version had the same mathematical structure, but differed in operand and context, e.g., "380 people attended a concert. There were four times as many boys as adults. There were 1/3 as many girls as boys. How many boys were there?" versus "A tank of water with 171 litres of water is divided into three containers, A, B and C. Container B has three times as much water as container A. Container C has _ as much water as container B. How much water is there in container B?" Different instructions were used for each version. Children were asked to use (a) only the model method, (b) any method but the model method, or (c) any method they could think of. The children were given one hour to answer the questions. Administration was spread over three weeks: one version per week. The sequence in which the three versions were administered was counter-balanced across schools.

Homework. To obtain a measure of the amount of homework performed by children, they were asked to complete a diary. Each page of the diary contained questions on the number of homework questions attempted, amount of time spent on home work, assistance received from care givers, and parallel questions regarding extra-curriculum tuition. Children were asked to fill out the diary for 18 days.

Reading. The Wechsler Objective Reading and Language Dimensions is a locally normed instrument designed to measure language attainment. Because we are primiarly concerned with children's ability to decode written information, only the reading component was used. This component has three subtests: .

Intelligence. To control for individual differences in intelligence, we administered the vocabulary and block design subtests from the Wechsler Intelligence Scale for Children. Previous work showed that amongst all subtests, these two tests have the highest correlations with the verbal and performance intelligence scores respectively (Sattler, 2001).

Results

The data showed moderate but reliable differences in test scores across the three mathematical tests, $F(2,298) = 7.66$, $p < .01$. Post hoc tests showed that children did better in the any method and any method but model conditions, $M_{log10\ transformed} = .50$, than they did in the model condition, $M_{log10\ transformed} = .45$. Regarding prediction to mathematical performance, data screening showed that the verbal IQ measure was highly correlated with the Wechsler language measures. To avoid problems of multicollinearity, a principal components analysis was conducted to derive a composite language ability score. Also, findings across the three versions of mathematical tests are similar; in the remainder of this report, we focus on the model method.

Pearson's correlations showed that the central executive, performance IQ, and language measures were all strongly and positively correlated with mathematical performance (see Table 1). On a bivariate level, the language ability, block design, and central executive component scores correlated most strongly with mathematical performance. Of interest was that the home-work measure was negatively correlated with mathematics performance. Because the homework measure was also negatively correlated with all other predictors, it is likely that the measure captured an aspect of problem solving speed. The slower an individual, the more likely he or she is to perform poorly.

The correlation analysis also showed that most predictors were highly inter-correlated. To examine the unique contribution of each variable, a standard regression analysis including all predictor variables was conducted. The predictors accounted for a large and reliable amount of variation in the mathematics test ($R^2 = .40$. $R^2_{adjusted} = .38$), $F(6,138) = 15.58$, $p < .01$. Inspection of the regression coefficients showed that only the central executive, performance IQ, and language measures predicted reliably to mathematics performance. The semi-partial correlations showed that these three predictors uniquely and respectively accounted for 3%, 6%, and 7% of variation in performance variation on the mathematics test.

Table 1

Intercorrelations Among the Study Variables

Variable	2	3	4	5	6	7
1. Phonological component	.20	.50	.29	.57	-.19	.34
2. Visuo-spatial component		.37	.41	.40	-.21	.28
3. Central executive component			.34	.56	-.18	.47
4. Block Design				.38	-.29	.48
5. Language ability					-.22	.53
6. Homework						-.23
7. Mathematics ability						

Note. all $p < .05$

Discussion

The findings showed that the children did not find the model method particularly easy. Although differences in test scores amongst the three tests were small, they were statistically reliable. They showed that children performed best when they were given a choice as to which strategy would best suit the problem at hand. At present, we are conducting more detailed analyses on the test scripts: paying particular attention to error patterns and the kinds of strategies children used when they were given a choice.

From a purely applied perspective, if one were to administer just one set of tests, the language ability tests accounted for more variance in mathematical performance than did other tests administered in this study. However, the data also showed that even when the effects of language and performance IQ were controlled, working memory measures still provided a moderate but reliable improvement to the prediction of mathematical performance. These finding suggest that the use of working memory tests may be useful for high stake examination.

Again, our data analysis is only at a preliminary stage. One issue that is receiving continuing attention is the theoretical relationships between the various predictor variables. For example, Baddeley, Gathercole, and Papagno (1998) argued that the phonological loop contributed to language development. Others have found language ability to be closely related to mathematical abilities (e.g., Bull et al., 1999). We intend to test these relationships in path models and to examine whether working memory measures contribute directly or indirectly, via language, to mathematical performance.

References

Ashcraft, M. H., Kirk, E. P., & Hopko, D. (1998). On the cognitive consequences of mathematics anxiety. In C. Donlan (Ed.), *The Development of Mathematical Skills* (pp. 175-196). Hove, England: The Psychology Press.

Baddeley, A. (2000). The episodic buffer: a new component of working memory? *Trends in Cognitive Sciences, 4,* 417-423.

Baddeley, A., Gathercole, S. E., & Papagno, C. (1998). The phonological loop as a language learning device. *Psychological Review, 105,* 158-173.

Baddeley, A. & Hitch, G. J. (1974). Working memory. In G. A. Bower (Ed.), *Recent Advances in Learning and Motivation* (pp. 47-90). New York: Academic Press.

Bull, R. & Johnston, R. S. (1997). Children's arithmetical difficulties: Contributions from processing speed, item identification, and short-term memory. *Journal of Experimental Child Psychology, 65 (1),* 1-24.

Bull, R., Johnston, R. S., & Roy, J. A. (1999). Exploring the roles of the visual-spatial sketch pad and central executive in children's arithmetical skills: Views from cognition and developmental neuropsychology. *Developmental Neuropsychology, 15 (3),* 421-442.

Bull, R. & Scerif, G. (2001). Executive functioning as a predictor of children's mathematics ability: Inhibition, switching, and working memory. *Developmental Neuropsychology, 19 (3),* 273-293.

Cockcroft, W. H. (1982). *Mathematics Counts: Report of the Committee of Inquiry into the Teaching of Mathematics in Schools Under the Chairmanship of W. H. Cockcroft.* London: Her Majesty's Stationery Office.

Gathercole, S. E. & Pickering, S. J. (2000). Working memory deficits in children with low achievements in the national curriculum at 7 years of age. *British Journal of Educational Psychology, 70 (2),* 177-194.

Geary, D. C. (1993). Mathematical Disabilities: Cognitive, Neuropsychological, and Genetic Components. *Psychological Bulletin, 114 (2),* 345-362.

Hoard, M. K., Geary, D. C., & Hamson, C. O. (1999). Numerical and arithmetical cognition: performance of low- and average-IQ children. *Mathematical Cognition, 5 (1),* 65-91.

McLean, J. F. & Hitch, G. J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. *Journal of Experimental Child Psychology, 74 (3),* 240-260.

Ostad, S. A. (1998). Developmental differences in solving simple arithmetic word problems and simple number-fact problems: A comparion of mathematically normal and mathematically disabled children. *Mathematical Cognition, 4 (1),* 1-19.

Pickering, S. J. & Gathercole, S. E. (2001). *Working Memory Test Battery for Children*. Kent: The Psychological Corporation.

Sattler, J. M. (2001). *AssessmentoOf Children: Cognitive Applications*. (4th ed ed.) San Diego: J.M. Sattler.

Shalev, R. S., Manor, O., & Gross-Tsur, V. (1997). Neuropsychological aspects of developmental dyscalculia. *Mathematical Cognition, 3 (2),* 105-120.