Utilizing A Collaborative "Cross Number Puzzle" Game on Group Scribbles to Develop Students' Computing Ability of Addition and Subtraction

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Abstract: While addition and subtraction is a key mathematical skill for young children, a typical activity for them in classrooms involves doing repetitive arithmetic calculation exercises. In this study, we explore a collaborative way for students to learn these skills. In our study, 52 students in Grade 4 (ages 10 or 11) participated in the study and were asked to solve arithmetic problems collaboratively as a group. This "Cross Number Puzzle" was also designed with the "feedback" mechanism to assist students' problem solving. In the two classes we studies, one class had the students played the game individually and the other class had the students play the game collaboratively. The low-ability students in the collaborative class were found to have made the most significant progress in arithmetic skills through playing this game.

Introduction

Computing addition and subtraction is important mathematical ability in our curriculum. In seeking to design more interesting learning experiences for children to learn math, we designed and implemented a game. The aim of this game is to promote the concept of flexible use of addition and subtraction, and to enhance children's capacity to build up their arithmetic skills progressively. The purpose of this study is also to explore different collaborative learning patterns that involve students working together on arithmetic problems. We also examine differences between individual learning and collaborative learning collaboratively.

A variety of educators have classified operating addition and subtraction problems into four problem types: change, combine, compare and equalizer (Carpenter, Hiebrt, and Moser, 1981; Fuson, 1992; Gustein and Romberg, 1995). English (1998) pointed out that change and combine are easier while take-away and compare are more difficult challenges for elementary school students. In an arithmetic equation, any of the three numbers could be the unknown number. We adopted this widely method in our study. Fuson (1992) defines these three types of "change" (placeholder) as: Missing End, Missing Change, and Missing Start. Van de Walle (2001) also classified the type of "change" into three types: result-unknown, change-unknown and initial-unknown. The three types of problems present different levels of difficulty to the students. If the student applied the direct modeling strategy by using counters or tally marks to model directly the action or relationships described in the problem, he or she always does not know how many counters to be put down to begin with. Table 1 below illustrates the three levels of change types in problem. Level I is when the result number is unknown. Level II is when the change number is unknown, Level III is when the initial number is unknown (Peterson, Fennema et al. 1991).

Table 1: Three levels of "Change" types in problems.

Change Types	Join(Add to)	Separate(Take from)
Result number unknown	Standard sentence: A + B = □	Standard sentence: A – B = □
Change number unknown	Standard sentence: A + □ = B	Standard sentence: A - □ = B
Initial number unknown	Standard sentence: □ + A = B	Standard sentence: □ – A = B

With these three levels in Table 1 in mind, we design our system by having five stages of problem to pose to the students (Table 2):

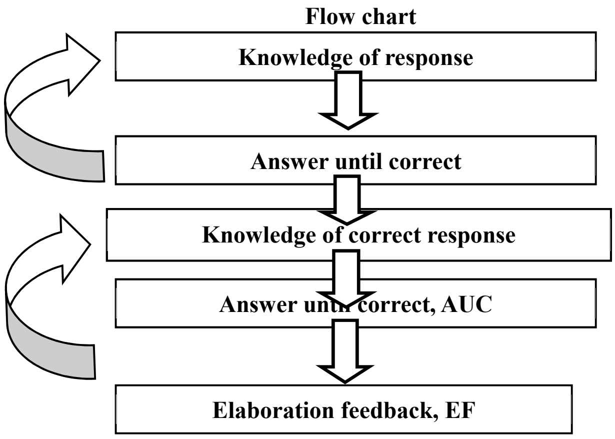
Table 2: Level of difficulty design.

Level of difficulty	Description	Example
Level 1	Result number unknown — basic skill practice	A ± B =□
Level 2	Remove operator — between basic skill practice and comprehension application	A □ B = C
Level 3	Change number unknown add-to or subtraction — comprehension application	A ± □ = B

Level 4	Initial number unknown add-to and subtraction — comprehension application	□ ± A = B
Level 5	Change number unknown and Initial number unknown, addend or summand type — the most difficult level	□ ± □ = A

Design of the "Feedback" System

A 'feedback' mechanism was introduced to the game design in this study. Feedback is considered to have strong impacts on learning process and result (Bangert-Drowns, Kulick, Kulik and Morgan, 1991). Appropriate feedback can lead the learners to focus on key elements of learning. The learner can always adjust their learning strategies to try to close the gap between their actual performance and the goal. They reflect on their learning by a self-monitoring feedback loop. Hence, they can change their learning strategies in the follow-up learning and seek a better way of learning (Alexander and Shin, 2000). Collins (Collins, Carnine, & Gersten, 1987) pointed out three levels of feedback messages: little feedback; just show the answer is right or wrong. basic feedback; descriptive feedback; give some hints to learner, to drive right answer. Descriptive feedback can promote the motivation to challenge new tasks and new problem. Feedback mechanism provided by software systems mainly involves five levels that can get right answer or direction of goals (Sales, 1998) summarized. These five levels are: no feedback, knowledge of response, knowledge of correct response, answer until correct, and elaboration feedback. In our research, the feedback mechanism is designed as follows as figure 1



Description

Feedback to the student whether her response is correct or incorrect.

Set the upper limit of number of incorrect attempts before providing a give new clue.

Point out what should be amended

Set the upper limit of number of incorrect attempts before providing a give new clue.

Provide the detailed answer, explain and elaborate the answer.

Figure 1. Five-level Feedback Mechanism.

Methodology

Fifty two students in Grade 4 (ages 10 or 11) participated in our study. We had two experimental classes: students in Class A played the 'Cross number puzzle' game in small groups, and students in Class B played the game individually. All students were grouped according to their mean scores of the previous three tests in this term.: high-math achievers; medium-math achievers, and low math achievement. Students in class A were divided into homogeneous groups with three per group.We utilized Group Scribble as the platform for the game, and conducted analysis of the collaborative work within these groups. Group Scribbles (GS) is a computer-supported collaborative learning system developed by SRI International to conduct small-group collaborative concept mapping activities. Each student has a mobile device, and sees a screen divided into upper and lower frames. The lower frame is the Private Board that the student scribbles or types her answer individually. The upper frame is the Public Board in which the students show all of their individual answers, and work together as a group. The teacher can monitor their process of learning and provide appropriate guidance.

Questions designed ranging from the easy to the difficult in terms of the five levels of difficulty. When the students complete the calculation, they can fill in the answer box and press OK button below question area to submit. If the answer is correct, there will be a brief description of the key points. If the answer is wrong, the system will execute a step-by-step hints based on the number of errors from the user inputs. The action repeats until the maximum number of errors reaches the upper limit. Then the system will show the correct answer and the methods of problem-solving. The following table shows four different types of questions in the "Cross Number Puzzle".

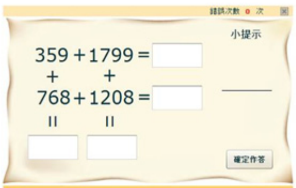


Figure 2. Question Type 1.

Figure 3. Question Type 2.

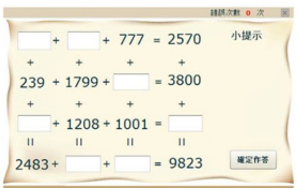


Figure 4. Question Type 3.

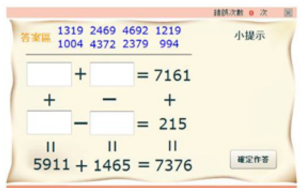


Figure 5. Question Type 4.

Findings

The mean score of Class B is 4.17 higher in the post-test (57.21) than in the pre-test (53.04) a significance level of .026 ($p<0.05$). This indicates learners made progress through playing the game. Table 3 shows the pre/post-test results of Class A. Students in Class A also have higher average score in the post-test. Their mean score had a great increase by 13.00, from 50.29 in pre-test to 63.29 in post-test ($p=0.002<0.01$). This indicates students in Class A made greater progress than those students in Class B through playing the game collaboratively. Further observation of these collaborative groups implied that the low math-achiever students made the most significant progress, as can be easily gathered from the following table.

Table 3: T-test of pre and post tests in collaborative group.

Number of Participants (N=24)	t-test of collaborative group							
Tests	Participant Number	Min.	Max.	Mean	SD	Progress	t	p
Pre-test of High-achiever	6	63.34	76.68	70.56	3.141	7.22	1.308	.248
Post-test of High -achiever		48.34	100	77.78	10.576			
Pre-test of Medium achiever	9	45.01	66.68	56.48	5.182	6.47	.880	.404
Post-test of medium-achiever		40.01	98.35	62.96	14.454			
Pre-test of low-achiever	9	6.67	55.01	30.56	10.111	23.33	4.834	.001**
Post-test of low-achiever		16.67	76.68	53.89	9.874			

The low-achiever groups in Class A were found the highest increase in post-test scores with high level of significance (*P=0.001*). This indicates that low-achievers of these collaborative groups derived the most benefits in this study. Table 4 showed further analyses conducted on three different types of test questions on "addition and subtraction". Students had better scores in all three types of questions in the post-test. But the low-achiever groups achieved significantly highest improvement in questions of "basic computing", "unknown constant" and "Cross Number Puzzle" with the increase of mean score 9.63, 7.38 and 6.32 respectively. This suggests that these low-achievers benefited the most from the "Cross Number Puzzle" in improving their basic arithmetic skills.

Table 4: Low achieved students' progress in pre and post tests.

Low achievers in Class A (N=9)		Pre -test	Post -test	Average increased scores	Ratio of progress in different questions
Basic computing skills	Score of question 1 to 5（33.33）	18.52	28.15	9.63	41.3%
Unknown constant	Score of question 6-12（46.67）	8.90	16.28	7.38	31.7%
Cross number puzzle	Score of question 13 to 15（20.00）	3.14	9.46	6.32	27.0%

Collaboration

Questionnaire results illustrated 85% students tried to do cooperation and discussion before they submitted the answer when they play the "Cross number puzzle" game. There was one high-achiever student who did not discuss with others when he did his calculations. He explained in the follow-up interview that he was quite confident and only shared his results with others when he completed all his calculations. 87.5% students claimed that it was much easier to complete the calculations with collaboration than to have to do it individually. Those students without confidence in mathematics found it easier to share their own ideas with others and co-complete the calculation. All students agreed that they derive benefits from discussion with other classmates.

"Hints" Usage in Class A and Class B

As we mentioned before, students in Class A play the game collaboratively in groups while students in Class B completed the game totally individually. We can easily conclude from Table 5 below that feedbacks in the form of "Hints" were much more frequently used in Class B than in Class A. It suggests that when students encounter problems and difficulties but without other people's help, he or she would search help from the "feedback" system. On the other hand, students in Class A would discuss their strategies to solve the problem within a group first, allocating cooperative work among group members. They only referred to the "feedback" system when all students in the group were uncertain or in a dilemma. They used the "Hints" less often then students in Class B. However either in Class A or Class B, high-achiever students seemed to have used the "Hints" far less than low-achiever students. Low-achieved students relied more on "Hints".

Table 5: "Hints" usage in Class A and Class B.

Number of use in different group	Class A (N=24)	Class B (N=28)
High-achiever	0.54	0.84
Medium-achiever	0.71	1.31
Low-achiever	1.25	2.09
Average usage	0.86	1.40

Activity 1: Putting the Operator

Four different patterns of collaborative problem solving were found in their activities of "putting the operator": whole-group-deciding, two-member- deciding, leader-deciding and individual deciding. Group 6 made the decision by all group members. Three groups, Group 1, Group 3 and Group7 decided the answer individually. Two groups took the two-member deciding pattern and the rest two groups took leader-deciding pattern. The following figures (Figure 6 to Figure 9) shows different layout of the game in different collaborative methods. For example, in figure 12, three students in group 6 (one student in one color of "+") post their answer as 4777 +++ 4611 +++ 1799 = 11154, six "+" and one "=". All these three students operate the addition correctly. Therefore we could judge that this group's answer was decided by the whole group. In figure 14, only one answer was pasted, checking the video recording, obviously, this group was leader-deciding.

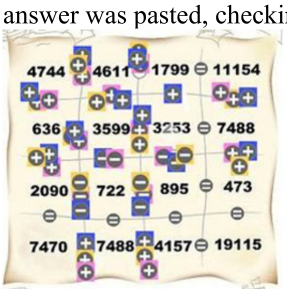


Figure 6. Whole-group-deciding.

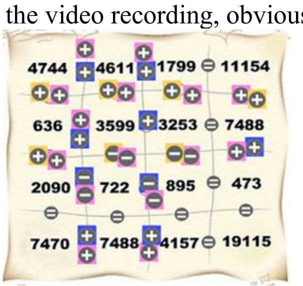


Figure 7. Two-member-deciding.

Figure 8. Leader-deciding.

Figure 9. Individual Deciding.

Activity 2: Fill in the Figure in the Formula Sentence

To enable learners get the unknown number in the puzzle by observing, calculating those given numbers and estimating the result, for example , A ± □ = B & □ ± A = B, tasks division and coordination were necessary in one group. From the procedural layouts of the game on the screen we got some insights of methods of students' collaboration and their strategies to complete the calculation. The results were shown in Table 6.

Table 6: Methods of collaboration in Class A (8 groups).

Methods	Description	Group	Ratio of different method used
Individual calculation	Group members did the calculation by themselves individually. Little collaboration occurred.	G1 G2	25.0%
Comparison	Started from different thread and compare each other's result at the intersection	G3 G5	25.0%
Relay	One finish one section and another take over to continue calculating	G4	12.5%
Assisted calculation	One of the group members is in charge of all calculation and other members checking his/her calculating process	G6	12.5%
Through-out calculation	Some members calculate from the beginning to the end and other members calculate from the end to the beginning then they compare at the intersection.	G7 G8	25.0%

Interactive Patterns

Milson (1973) identified seven frequently occurring interactive patterns within small learning groups, namely: *Unresponsive, Unsocial, Dominant leader, Tete-a-tete, Fragmented cliquish, Stilted, Ideal* Three of Milson's interactive patterns were identified in our study; the ideal interaction occurred most often. The students all did well in their collaboration. The groups doing fragment and unresponsive interaction were not as interactive as the ideal groups. They had fewer communication and little cooperation.

Conclusion and Discussion

Our observations and investigations of the two classes who played the game individually and collaboratively respectively showed some interesting differences. The collaborative learning groups (Class A) were found to have made greater progress than individual learning groups (Class B). It suggests that collaborative learning may have enhanced learning effectiveness. From the statistics, we can conclude the low-achieved students benefited the most in this "cross number game". Collaboration also plays an important role in enhancing learning in Class A with the incorporation of the "feedback system" and collaboration strategies. In both classes, the low-achieving students accessed the "Hints" most often while the high-achiever the least. The individual learning groups in Class B had much higher frequency of access to "Hints". The low-achieving students had the highest demand for "Hints" for help. Students in collaborative learning groups presented four different methods of problem solving in their activities of "removing the operator": whole-group-deciding, two-member-deciding, leader-deciding and individual deciding. In the activities of "fill in the figure in the expression," the students had five methods of calculations: *individual calculation, comparison, relay, assisted calculation* and *through-out calculation*. Students also showed four different ways of calculation: *free calculation, calculate from the top, calculate from the bottom* and *calculate from both the top and bottom.*

Future Work

Based on the findings in this study, we make some recommendations for future research. *More time on playing the "Cross Number Puzzle":* Our study has a limitation on time and scale. To make the cross number puzzle more applicable we may need more experiments and expand the users. *Adaptive feedback:* We only offered phased hints to students in this "Feedback system". The feedback only includes the general direction of calculation concept and the problem solving process. If we can diagnose and evaluate the individual student's errors, we can provide each student with the individual corresponding solutions or suggestions to fit his skills.

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