EXPLORING THE RELATIONSHIP BETWEEN MATHEMATICAL PROBLEM POSING AND PROBLEM SOLVING, TYPE OF TASK AND GRADE LEVEL

Yeap Ban Har & Berinderjeet Kaur
National Institute of Education
Nanyang Technological University
Singapore

Abstract: This paper focuses on one aspect of an investigation into children's mathematical problem posing. All the children in Primary 3 and Primary 5 in three primary schools were asked to attempt a problem-solving test as well as to write mathematics word problems. The children's responses to the problem-posing test were initially classified as solvable and unsolvable problems. Subsequently, the solvable problems were further analyzed to obtain an index of problem complexity. The framework of analysis is modified from a previously suggested one (Silver & Cai, 1996) based on possible situations in arithmetic word problems (Marshall, 1995). This paper aims to investigate the complexity of problems posed by children of different ages and problem-solving ability when given different problem-posing tasks.

Introduction

Mathematical problem posing is defined as the generation of new problems or the re-formulating of existing ones (Silver, 1994). It has been identified as an essential mathematical activity (NCTM, 1989) and a companion to mathematical problem solving (Kilpatrick, 1987). Kilpatrick (1987) also suggested that problem posing is becoming increasingly important in the information age, where there is a shift from memorization of routine algorithms towards conceptual understanding of various procedures and their uses. Silver (1994), among other perspectives, views problem posing as a feature of creativity.

Mathematics curriculum around the world calls for opportunities for pupils to generate problems in the mathematics class. In the United States of America, the *Professional Standards for Teaching Mathematics* (NCTM, 1991) states that "[s]tudents should be given opportunities to formulate problems from given situations and create new problems by modifying the conditions of a given problem" (p. 95). This echoed a similar call made previously in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) which identified problem posing as "an activity that is at the heart of doing mathematics" (p. 138). In Australia, the *National Statement on Mathematics for Australian Schools* (Australian Educational Council, 1991) advocates the use of problem posing in the teaching and learning of mathematics. In Singapore, the revised mathematics curriculum calls for opportunities for pupils "to extend and generate problems" (p. 17, Ministry of Education, 2000).

Kilpatrick (1987) observed that while there is much research literature on problem solving, the same cannot be said for problem posing. Although in recent years, there has been some significant investigations into mathematical problem posing (Winograd, 1990; Leung, 1993; Stoyanova, 1995; Silver & Cai, 1996; Silver, Mamona-Downs, Leung & Kenney, 1996; English, 1997, 1998), Silver (1994), in a landmark paper, calls for more systematic research various aspects of problem posing.

This paper reports on an on-going investigation into children posing mathematics problems. The investigation involves all the pupils in Primary 3 and Primary 5 in three primary schools. In this paper, we explore possible relationship between problem posing and problem solving, grade level and type of task. Silver (1994) describes the relationship between problem-solving and problem-posing as complex. He suggests further exploration into the relation of problem-posing to different aspects of mathematical knowing and performance, including problem-solving. In his review of studies that explored the relationship between problem-posing and problem-solving, he concluded that there is no clear, simple links between the two.

The Study

Sample

273 children in one of the schools were involved in the study reported in this paper. There were 140 Primary 3 pupils and 133 Primary 5 pupils in the EM1, EM2 and EM3 streams. There was about the same number of boys and girls.

Data Collection

All the children in Primary 3 and Primary 5 in three primary schools were asked to complete a problem-solving test as well as to write mathematics word problems on two different days. The problem-solving test comprises six items. All the items, except the first, were non-routine to the children tested. The problem posing test comprised 5 tasks. Four of the tasks contain numerical information in the symbolic form (Yeap, 2000). The analysis for this paper was done on responses from three of the tasks. The three tasks are shown in Figure 1.

Task 1
Write a mathematics word problem.

Task 2
Write a mathematics word problem for a friend to solve. Your problem must have the numbers 3, 5 and 36. You can use more numbers, if you like.

Task 4
Write a mathematics word problem for a friend to solve. The answer to the question in the problem must be 10.

Figure 1: Three of the five tasks in the problem-posing test

Data Coding

The solutions to the problem-solving test were coded as successful (score = 2), partially successful (score = 1) or unsuccessful (score = 0). Successful responses were those that obtained a correct answer with a correct method. Partially successful responses were those that did not obtain a correct answer but have a correct method. Unsuccessful responses were those without a correct method. A score for each child that ranged from 0 to 12 was obtained. Based on this score, the children were put into four quarters. The top quarter formed the Good Solvers. The bottom quarter formed the Poor Solvers. The middle half formed the Average Solvers.

The problems posed by the children in the problem posing test were initially classified as solvable or unsolvable problems. Unsolvable problems include problems with insufficient information and problems with inconsistent information. The following categories of responses were also excluded from the next stage of analysis: (1) computations or word problems that are essentially computations, (2) responses that did not fit the task requirements, (3) non-arithmetic problems, and (4) no response. Figure 2 show some examples from each category.

<u>Problem with insufficient information</u>
One day, Susan bought 20 apples, when she went home. She found that 4 were rotten, She than gave her friend seme apples. How many apples did her friend get? [R5006]

<u>Problems with inconsistent information</u>
Cobbler A has mend 36 shoes. Cobbler B has mend 3 shoes less than cobbler C. Cobbler C has mend 5 shoes more than cobbler A. How many more shoes does cobbler B mend than cobbler C? [R5001]

<u>Problem that is essentially computation</u>
There are 3 numbers. 3, 5 and 36. Solve this numbers to get 17. You can use add, subtract, divide or times. [R5020]

<u>Problems that do not fit task requirements</u>
There were 10 frogs in my pond. He has 10 more times frogs than me in his pond. How many frogs were there in his pond? [R3027]

Figure 2: Examples of responses that are not further analyzed

Subsequently, the solvable problems were further analyzed to obtain an index of problem complexity. The framework of analysis is modified from a previously suggested one (Silver & Cai, 1996) based on possible situations in arithmetic word problems (Marshall, 1995). Each solvable response was given a complexity index that ranged between 0 and 5. A problem posing score was obtained by adding the complexity indices for the three responses. A score that ranged from 0 to 15 was obtained. Based on this score, the children were put into four quarters. The top quarter formed the Good Posers. The bottom quarter formed the Poor Posers. The middle half formed the Average Posers. Figure 3 shows examples of responses with different complexity indices. In our investigation, we have defined a more complex problem as one that is more difficult for children in the age group as the poser.

Complexity Index = 0
I have $20. Mary have $10. How many did Mary have? [R3080]

Complexity Index = 1
Aaron have 5 books. Mary have 3 books. John have 36 books. How many books did they have altogether? [R3086]

Complexity Index = 2
They are 36 marbles in 5 bags altogether [Group]. If each bags has 3 marbles [Vary], how many marbles are left? [R3021]

There are 36 people taking part in a running contest.[Group] There are 6 more boys than girls.[Restate] There are also 5 woman and 3 man. How many boys are there? [R5039]

Complexity Index = 3
Ail have $36 for new year. Ail buy a book for $15. [Change] Some sweet for $3 [Change] and a cdroom [Change] that $5 more [Restate] than this two thing [Group]. How many money has she new? [R3055]

Sally and Barney both share 158 cards. [Group] Barney have 58 more cards then Sally. [Restate] How many cards will Sally have if she gave away 40 cards? [Change] [R5060]

Figure 3: Examples of responses with different complexity indices

Results

Three research questions are explored in this paper.

1. Is there a relationship between children's mathematics problem posing ability and grade level?
2. Is there a relationship between children's mathematics problem posing ability and problem solving ability?
3. Is there a difference in complexity among children's responses to different problem posing tasks?

To investigate possible relationship between grade level and problem posing ability, all the 273 children were put into one of four groups based on the total of complexity indices of the three problems they posed.

Chi-square test revealed that Primary 5 children posed significantly more complex problems than Primary 3 children, χ^2 (3, n = 273) = 34.95 (>7.81), $p < .05$.

Table 1
Problem Posing Ability of Children in Different Grade Levels.

Problem Posing Score	Primary 3	Primary 5	
0 - 1	46	19	65
2 - 3	79	62	141
4 - 5	14	40	54
> 6	1	12	13
	140	133	

To investigate possible relationship between problem posing ability and problem solving ability, all primary 5 pupils who completed both the problem posing test and problem solving test were classified as Good Solver, Average Solver and Poor Solver as well as Poor Poser, Average Poser and Poor Poser as explained previously.

Table 2
Problem Posing Ability of Children with Different Problem Solving Profiles.

	Poor solver	Average solver	Good solver	
Poor poser	14	14	3	31
Average poser	16	37	13	66
Good poser	6	14	10	30
	36	65	26	127

The chi-square test revealed that the null hypothesis of there is no relationship between problem posing ability and problem solving ability cannot be rejected, χ^2 (4, n = 127) = 9.00 (<9.49), p < .05.

Two of the three tasks used in the problem posing tests contain numerical information in symbolic form (Yeap, 2000). In Task 2, children are given three numbers and asked to pose a word problem. In Task 4, children are told that answer to the problem and asked to pose a word problem. In Task 1, children were simply asked to pose a word problem. We are postulating that some tasks can elicit more complex problems than others.

Table 3
Complexity of Problems Posed in Different Symbolic Tasks.

	Task 1	Task 2	Task 4
Unable to respond	53	93	116
Complexity Index < 2	164	123	137
Complexity Index = 2	49	46	17
Complexity Index > 2	7	11	3
	273	273	273

Table 3 seems to suggest that Task 2 was more able to elicit more complex problems from children than Task 4.

Discussion

The framework that we used to analyze the problems posed by children in the present study considers a more complex arithmetic word problem to be one that contain more types of situations as described by Marshall (1995). Previously, Silver & Cai (1996) have suggested to use the number of situations as a measure of a more complex problem. In the present investigation we have defined a more complex problem as one that is more difficult for children in the age group as the poser. In a pilot study (Yeap & Kaur, 2000), we have some tentative evidence that problems with more types of situation tend to be more difficult. A study to validate this framework is presently in progress. A word problem test comprising problems posed by children in the present study will be administered to Primary 3 and Primary 5 children. The data will be used to investigate if the number of situation types makes a problem more difficult.

There are some weaknesses of the framework employed in the present investigation. One of our main concerns is that non-arithmetic responses cannot be analyzed. However, children in the sample rarely posed problems that are non-arithmetic and non-situational. Very few responses were excluded from this analysis as a result. For our purpose, the framework is adequate.

The finding that Primary 5 children were able to pose more complex problems is hardly surprising. The finding that problem solving is not significantly related to problem posing suggests that these are mathematical activities which, although are not entirely unrelated, involve some processes that are unique to each. Analysis of data from the entire sample will provide a clearer picture. Also, a clinical interview study, that is the next stage of investigation, on a purposive sample of children with different problem posing and problem solving profile will provide data to investigate the type of cognitive (and perhaps, non-cognitive) processes involve in problem posing.

Although inferential statistical techniques have not been used to show conclusively that different types of tasks elicit from children problems of different complexities, there was some indication that this is so. Further analysis of the data and the clinical interview study will provide further understanding on this issue.

Conclusion

We conclude by outlining the other parts of the on-going investigation on children's mathematical problem posing and recommending some other ways to use the data we have collected.

The other parts of the on-going investigation on children posing mathematics word problems include, (1) validation of framework to analyze problems posed by children; (2) validation the framework we propose to classify problem posing tasks (which is in terms on the nature of numerical information in the task, and the cognitive processes the task can elicit); and (3) identification of cognitive and non-cognitive processes involved in problem posing.

The data collected in the on-going investigation can be used in other ways to answer questions in mathematics education. (1) Analysis of meanings children attach to numerical information and arithmetic operations. (2) Analysis of situations children encounter in their mathematics learning and its impact on the problems they pose.

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