PROBLEM-SOLVING FRAMEWORKS OF PROSPECTIVE SECONDARY MATHEMATICS TEACHERS

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This study explores the problem-solving frameworks of twenty prospective secondary mathematics teachers. Twenty prospective secondary mathematics teachers were asked to complete in writing the statement 'When I am given a mathematics problem to solve, this is what I do....' Prospective secondary mathematics teachers were found to rely on individual problem-solving frameworks to guide them when solving problems. The frameworks of prospective secondary mathematics teachers were very detailed and comprehensive.
Problem-Solving Frameworks of Prospective Secondary Mathematics Teachers

INTRODUCTION

Many educators in recent years have emphasised the importance of reflection in teaching and learning. Reflection allows an individual to practise or analyse their actions, decisions or product. Reflection can also help teachers and students in their teaching and learning. In addition, reflection through writing is an essential component of communication in the mathematics classroom. Research into metacognition has indicated that the probable value of equipping students is for them to reflect on and even take control of their learning (Garofalo & Lester, 1985). Metacognition is the ability to reflect on cognitive activity, to know what strategies are suitable in specific situations, create the awareness whether or not

one possesses the skills or capability to perform certain tasks. When carrying out metacognition, students can also express their emotions and attitudes to the cognitive activities that they are engaged in.

RATIONALE

The Singapore mathematics curriculum advocated that to develop metacognitive abilities in students, activities and opportunities should be provided for students to reflect in writing on their problem-solving experiences (Ministry of Education, 2001). Although there have been many discussions on the benefits of journal writing and reflection, Singapore teachers appear to have some doubts how these practices would truly benefit the students, given the time constraints to cover the mathematics syllabus in the classroom. Mathematics teachers will probably not use journal writing and reflection unless they have had the experience themselves of writing in relation to mathematics. As for the prospective mathematics teachers, not only do they need to have a good understanding of mathematical concepts and be knowledgeable in teaching mathematics, they should also understand that it is possible to implement journal writing in the mathematics classroom. The objective of this study was to explore prospective secondary mathematics teachers' problem-solving frameworks through writing. Through writing, prospective secondary mathematics teachers clarify their thoughts, learn to organise their ideas and finally build up meaningful connections among mathematics ideas for teaching and learning over time.

THE STUDY

Twenty prospective secondary mathematics teachers were asked to complete in writing the statement 'When I am given a mathematics problem to solve, this is what I do…'. The prospective secondary mathematics teachers were asked to respond to the statement by describing the ways they normally attempted to solve

mathematical problem and not necessarily the ways in which their school mathematics teacher might have shown them. The qualitative data arising from the responses were analysed. An analysis of their problem-solving frameworks is presented.

ANALYSIS AND RESULTS

Data Analysis Procedures

Twenty hand written responses to the statement "When I am given a mathematics problem to solve, this is what I do…" were analysed. The handwritten responses of the prospective secondary mathematics teachers were transported into online data with the help of a word processor and directly translated for coding and later recalled and analysed. The data were coded under the following phases of problem solving which were arrived at from a preliminary analysis of the data. In the preliminary analysis of the data, it was observed that the four phases concurred with Kaur (1995) study. It was decided that the four phases were appropriate for the present study.

Phase I - Understand/Represent the Problem (U)

Phase II - Find a Way to Solve the Problem (F)

Phase III - Solve the Problem (S)

Phase IV - Check the Solution (C)

The responses of individual prospective secondary mathematics teachers were further coded under the respective phases. The following examples show coded segments under the following four phases.

Phase 1 - Understand / Represent the Problem (U)

Teacher 5 - rearrange the information
- translate the information into pictorial form

Teacher 10 - read the question,
- underline the keywords and data

Phase 2 - Find a Way to Solve the Problem (F)

Teacher 5 - analyse the question
- choose the most suitable method

Teacher 18 - interpret the question
- think of the various methods that would be used to solve the question
- use the most confident method

Phase 3 - Solve the Problem (S)

Teacher 7 - solve my interpretation of question systematically
- arrived at the final solution.

Teacher 8 - show workings progressively
- obtain final answer

Phase 4 - Check the Solution (C)

Teacher 2 - use logical reasoning to check whether answer is correct

Teacher 19 - work backward again using the answer derived
- if wrong, go through the steps again

A CLASSIFICATION OF PROBLEM-SOLVING BEHAVIOURS

The coded segments at each phase for all twenty prospective secondary mathematics teachers were summarised and a classification of problem-solving behaviours developed. Table 1 shows the classification containing lists of behaviours categorised under the four suggested phases of problem solving which the prospective secondary mathematics teachers in the sample appeared to have adopted.

Table 1
Classification of Problem-Solving Behaviours

Phase I		Understand/Represent the Problem (U)
U1	-	read the problem
U2	-	mentally try to picture the problem/understand the problem
U3	-	highlight/note/underline the key points/list the given data
U4	-	rearrange the given information
U5	-	represent the information on the diagram
Phase II		Find a Way to Solve the Problem (F)
F1	-	analyse/look for the relationship between the givens and the goals
F2	-	look/try for ways to solve the problem/think of ways/methods/familiar method to solve the problem/plan out the steps/stages
F3	-	think of a shortest way/time to solve the problem
F4	-	think of most suitable method to solve the problem
F5	-	look for clues
F6	-	check my memory of doing similar problems
F7	-	look for ways to reduce the problem into smaller and simpler parts so that it is easier to solve

F8	-	try and solve the problem using: algebra, (b) logical reasoning (c) formula (d) diagrams, (e) guess and check, (f) work backward
F9	-	If in doubt, do it later
Phase 3		Solve the Problem (S)
S1	-	work/attempt/do the problem
S2	-	check workings step by step/do it carefully
S3	-	get the answer
Phase 4		Check the Solution (C)
C1	-	check solution by working backwards
C2	-	check the calculations to make sure answer is correct/double check
C3	-	check for mistakes/careless mistakes
C4	-	read the problem again and check the answer
C5	-	check and see if the answer is logical and or sible/reasonable
C6	-	if answer is wrong, try solving the problem in another way go through the steps again move on to the next problem.

Table 1 shows all the behaviours stated by the twenty prospective secondary mathematics teachers under the four phases. During Phase I, the prospective secondary mathematics teachers attempted to understand and represent the problem. The behaviours manifested by them during this phase (Understand/Represent the Problem) were "read the problem", "mentally try to picture the problem/understand the problem", "highlight/note/underline the key points/list the given data", "rearrange the given information" and "represent the information on the diagram." Having understood and represented the problem the prospective secondary mathematics teachers next searched for a means to solve the problem. The behaviours manifested by the prospective secondary mathematics teachers during this Phase II (Find a Way to Solve the Problem) were varied and ranged from very general to specific suggestions. The behaviours displayed were "analyse/look for relationship between the givens and the goals," "look/try for ways to solve the problem/think of ways/methods/familiar method to

solve the problem/plan out the steps/stages," "think of a shortest way/time to solve the problem," "think of a most suitable method to solve the problem," "look for clues," "check my memory of doing similar problems," "look for ways to reduce the problem into smaller and simpler parts so that it is easier to solve," "try to solve the problem using: algebra, logical reasoning, formula, diagrams, guess and check and work backward" as well as "if in doubt, do it later." From Phase II, the prospective secondary mathematics teachers moved to Phase III (Solve the Problem). During this phase, the behaviours manifested, or at times implied by them, were "work/attempt/do the problem," "check workings step by step/do it carefully" and "get the answer." Most prospective secondary mathematics teachers having obtained a solution then suggested checking it. The behaviours manifested during this Phase IV (Check the Solution) were mainly concerned with checking the result, and at times the solution, in a number of ways. These were "check the solution by working backwards," "check the calculations to make sure answer is correct/double check", "check for mistakes/careless mistakes," "read the problem again and check the answer," "check and see if the answer is logical or sensible/reasonable" and "if answer is wrong, try solving the problem in another way, go through the steps again and move on to the next problem."

PROBLEM-SOLVING FRAMEWORKS

The classification of problem-solving behaviours shown in Table 1 was used to outline the individual problem-solving frameworks of the prospective secondary mathematics teachers in the sample from their reflection (written responses to the statement). The example which follows shows how these frameworks were obtained.

Example

Teacher 11: Read the question U[1]. Note information given U[3] and make note of the solution required F[1]. Make logical deductions from the given information that may shorten working F[8b]. Plan the answer F[2] and find out what is the shortest and quickest way of doing this problem F[3] or exercise. Work out the solution S[1]. Check if the answer is logical C[5]. Check workings for mistakes C[3].

From this, the frameworks of the prospective secondary mathematics teacher with identification number 11 is:

U[1] – U[3] → F[1] – F[2] – F[3] – F[8b] → S[1] → C[3] – C[5]

which in condensed form reduces to:

U[1, 3] → F[1, 2, 3, 8b] → S[1] → C[3, 5]

Altogether twenty problem-solving frameworks for individual prospective secondary mathematics teacher in the sample were obtained and shown in Table 2.

Table 2
Mathematical Problem-Solving Frameworks of Twenty Prospective Secondary Mathematics Teachers

Teacher I. D.	Phase 1	Phase 2	Phase 3	Phase 4
01	U[1, 2]	F[2]	S[1]	C[2]
02	U[1]	F[2, 9]	S[1]	C[5]
03	U[1, 2,]	F[1]	S[1]	C[4]
04	U[1, 2]	F[1]	S[1]	-
05	U[4, 5]	F[1, 4]	S[1]	C[2, 5]
06	U[1, 2]	F[1, 6]	-	-
07	U[1, 2]	F[2, 8a]	S[1, 2, 3]	C[3]
08	U[2, 3]	F[1]	S[2, 3]	-
09	U[1, 2]	F[2, 4, 5]	S[1]	C[3,6a]
10	U[1, 3]	F[2, 3, 6, 8b]	S[1]	-
11	U[1, 3]	F[1, 2, 3, 8b]	S[1]	C[3, 5]
12	U[2]	F[1, 2]	S[2, 3]	C[2]
13	U[1, 2]	F[1]	S[1]	-
14	U[1, 2]	F[1, 2]	S[1]	C[2, 5, 6a]
15	U[2]	F[4, 7, 8a, 8c, 8d, 8e,8f]	S[1]	C[2]
16	U[1, 3]	F[2]	S[1]	C[2, 6a]
17	U[5]	F[2]	S[1]	C[6c]
18	U[1]	F[1, 2, 4]	S[1]	C[2]
19	U[1]	F[1, 2, 7]	S[1]	C[1, 6b]
20	U[1]	F[1, 2, 5, 7]	S[1]	C[6a]

DISCUSSION

Prospective secondary mathematics teachers were found to rely on individual problem-solving frameworks to guide them when solving problems. Of the twenty prospective mathematics teachers, fifteen of them had frameworks consisting of the four phases, Understand/Represent the Problem, Find a Way to Solve the Problem, Solve the Problem and Check the Solution which was similar, in some ways, to those of Polya's (1945) problem solving model. Four prospective secondary mathematics teachers had frameworks consisting of only three phases: Understand/Represent the Problem, Find a Way to Solve the Problem and Solve the Problem. Only one prospective secondary mathematics teacher had framework consisting of the first two phases: Understand/ Represent the Problem and Find a Way to Solve the Problem. The behaviours demonstrated during these phases varied among the prospective secondary mathematics teachers ranging from a minimum of one to a maximum of seven. The frameworks of twenty prospective secondary mathematics teachers were similar in the behaviours. Moreover, their frameworks were very detailed and comprehensive. In particular, the behaviours listed under the Check the Solution phase suggest that they were making an attempt to reflect on their solution to the problem.

CONCLUSION

Through this reflection, some prospective secondary mathematics teachers recognised the benefits of integrating writing activities along with other tasks and assignments in their future mathematics lessons. In addition, writing to reflect allows the prospective mathematics teachers to look back at their thoughts and problem-solving process. In fact, one prospective secondary mathematics teacher commented that she will make an attempt to implement journal writing and reflection on her future students. She indicated

that it would be a meaningful and relevant exercise and a good tool for feedback in a mathematics lesson.

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