Learning Experiences: Connecting A-Level Mathematics With Mathematics Used In The Real World Via Machine Learning

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Abstract

One of the aims of the current Advanced Level H2 Mathematics Syllabus in Singapore is for students to connect ideas within mathematics and apply mathematics in the contexts of sciences, engineering and other related disciplines. A practising classroom teacher will find this very hard to achieve. On one hand, the mathematics that supports real scientific and engineering applications is often too sophisticated and lies beyond the reach of classroom mathematics – at least as perceived by the teachers. On the other hand, textbook examples that bear some semblance of a real-life application often appear contrived. The teacher's challenge is to find middle ground that caters for both accessibility and authenticity. This paper situates Gradient Descent, an elementary concept/technique commonly featured in Machine Learning, to create meaningful learning experiences with the aim of connecting topics within mathematics, and with the actual mathematics used to solve real world problems.

1 Introduction

In recent years, two types of connections in the field of mathematics education have increasingly gained research traction: one internal and the other external.

The internal connection is about linking topics *within* school mathematics through Big Mathematical Ideas. Charles defines a *Big Idea in Mathematics* (Big Ideas, for short) to be "a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" ([4]). Expectant of mathematics teachers to see mathematics as a coherent and connected whole body of knowledge ([16]), Charles [4] advocates the grounding of subject content knowledge and teaching practices around Big Ideas. For students, through Big Ideas they link and see purposes in classroom activities, and construct richly linked understandings of mathematics ([9]).

The external connection *beyond* school mathematics concerns with the connection of school mathematics with the mathematics actually used in real-world contexts. Globally, mathematics is valued as an important subject in school curriculum because of its usefulness as a tool to be used by students to solve problems within personal, occupational, societal, and scientific contexts ([12]). Since the 1990s, there have been repeated calls, ranging from both educators to professional associations, for teachers to make explicit connections between school mathematics and its applications to relevant real-world situations ([11, 5]). Such demands grew out of genuine needs of both teachers and students. Studies have reported that certain middle-grade mathematics teachers valued real-world connections because these helped engage ([8]) and motivate students ([6]). Elsewhere (e.g., [2]), students have also expressed their need for a better understanding of how school mathematics connected with contexts and situations relevant to their lives.

Becoming cognizant of the importance of the two connections in mathematics education, many national mathematics curricula over the last five years responded by including official documents the need for mathematics teachers to exploit Big Ideas to present a coherent view of mathematics for their learners and to design tasks and activities, especially those that involve mathematical modeling, to link school mathematics to mathematics used in real-world contexts ([13]). Notably, textbooks in several countries (e.g., in the Netherlands, see [17]) have responded to this recent mathematics education reform embracing the so-called *Realistic Mathematics Education* (RME) —- one that emphasizes the connection of school mathematics with its applications to real-world contexts.

Despite such curricular requirements, interpreting the written curricula as intended curricula and then implementing this in enacted form are admittedly challenging processes as expressed by Stein and her colleagues in [15]:

> *Between* the written and intended phases, teachers bring their prior understandings, beliefs, and goals to bear on the written curriculum and, in the process, transform it into a form that they believe will be workable in the classroom. *Within* the enacted phase, the teacher and the students, in their interaction with each other, create something different from what exists on the pages of the book or in the teacher's mind or lesson plan.

The reality of the classroom enactment is far from being ideal: research findings reported gaps between teachers' vision of making connections and their ability to enact the vision in practice ([7]). Meaningful real-world connections were not often occurring in classrooms, especially at the middle school level. Even if these connections had occurred, they tended to be cursory and insufficient to engage students ([5, 7]). It therefore comes as no surprise that many middle-grade mathematics teachers found it challenging to first interpret the written requirement for them to establish the two kinds of connections and then to enact those interpretations into actual lessons in the classrooms. For example, teachers found themselves entrapped in several kinds of dilemma in their attempts to incorporate real-life applications of school mathematics into their lessons ([14]).

An extensive literature review of the potentialities and challenges of making connections within mathematics and with real-world applications in classrooms returned research works performed in elementary and middle school levels. There is a paucity of literature in this area at the pre-university level. Does this suggest that mathematics teachers at the pre-university level do not face those challenges that their elementary and middle school counterparts encountered? Perhaps, the mathematics content at the pre-university level is sophisticated enough that makes it easy to connect with the advanced mathematics employed in real-world contexts. These questions do not seem to have ready answers and thus pique our interest. A small step towards understanding the situation at the pre-university level will be for the authors to start our investigation locally in their home country, Singapore.

Two research questions, which will be addressed in this paper are: (RQ1) To what the extent are Singapore pre-university mathematics teachers designing and implementing lessons that help students make connections within the topics taught in the syllabus and with the (presumably, more advanced) mathematics actually applied to solve real-world problems? (RQ2) What are the challenges that these teachers face when they attempt to do this? Additionally, we make some recommendations that pre-university mathematics teachers can use to create meaningful learning experiences that crucially help students make connections internally and externally.

We organize this paper as such. In Section 2, we initiate a case study in Singapore to understand (1) how mathematics teachers incorporate making connections *internally* and *externally* in teaching H2 Math to their students, and find out (2) what difficulties and challenges pre-university mathematics teachers encounter when they attempt to do so. A small sample of Junior College H2 Math teachers volunteered to complete an online survey designed to find out more about (1) and (2). Carrying these teachers' concerns and challenges, we revisit Tyler's curriculum model and apply it to resolve some of these issues the teachers raised in In Section 3. Then, in Section 4, we outline three learning experiences to help H2 Math students make internal connections within the mathematics they learn in the syllabus, and also external connections between H2 Math topics with the mathematics applied in real-world contexts. Specifically, we propose the Gradient Descent Method, a commonly used technique in Machine Learning, as a bridge that makes these connections. Finally, in Section 5, we make some concluding remarks and present some ideas for future work.

2 A case study in Singapore

2.1 Background information

In Singapore, Grades 11 and 12 students receive pre-university education, situated between secondary and university education. In the 2023 Joint Admissions Exercise (JAE), which involved 19,600 students, 52% of them were posted to a polytechnic, 39% to Junior Colleges or Millennia Institute, and 9% to the Institute of Technical Education. Being a pre-requisite to many science-related and engineering courses at universities, Higher 2 Mathematics (H2 Math, for short) is chosen by Junior College students who would sit for a final national A-Level examination at the end of their second year. Our paper is about H2 Math syllabus, the aims of which in [10] are:

1. acquire mathematical concepts and skills to prepare for their tertiary studies in mathematics, sciences, engineering and other related disciplines;

2. develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving;

3. connect ideas within mathematics and apply mathematics in the contexts of sciences, engineering and other related disciplines;

4. experience and appreciate the nature and beauty of mathematics and its value in life and other disciplines.

(3) and (4) show that the Ministry of Education, Singapore (MOE, for short) places strong emphasis on *both* the *internal* connection within the topics of mathematics in the national curriculum and the *external* one between school mathematics and the mathematics applied to real-world contexts. However, (3) and (4) – being generically phrases – do not provide explicit ways for H2 Math teachers to plan plan and enact their lessons to achieve them. It is also unknown to us what difficulties and challenges teacher face when they think of or attempt to achieve (3) or (4) in their lesson plan and enactment.

2.2 A small sample of responses to an online survey

Four H2 Math teachers (of varying lengths of teaching experience) participated voluntarily in a short survey (see Appendix A), designed to shed some insight to RQs 1 and 2. Given that this small sample gives some 'ground-sensing', the authors are aware of its limitations. In what follows, we report some of the responses to this survey, and draw some understanding towards our research questions.

2.2.1 Internal connection

All the respondents indicated that they teach towards Big Ideas as often as they appear in the Scheme of Work[1] When asked to give an example, we received Big Ideas like 'Functions', 'Vectors', 'Limits', and themes such as 'Extensions'; these being key words that appear in the H2 Math Syllabus document ([10]). None provided further descriptions. One teacher mentioned the sum to infinity for convergent geometric sequences, and offered no further explanations. Another commented that Big Ideas were useful for students to gain a deeper understanding of mathematics by saying that "The idea of function as a mapping is pervasive in math." In illustrating how students view and understand 'cross-multiplication' when performing algebraic manipulations in equations and inequalities, there is a comment: "students get to see the different scenarios and determine the conditions that are required to apply the cross multiplication, especially in inequalities." When it comes to the challenges, the most frequently occurring response is that time is a constraint. One teacher, in particular, was worried of "lower ability students not being able to draw the link as their foundation in the topic is weak". Also, one teacher mentioned about the lack of mathematical maturity among the students, and the presence of 'critics' who were fixated on students' academic performance. Another problem centres around sequencing of topics, i.e., "some chapters taught in front might be linked to chapters taught behind; students at the earlier stage might not be able to understand terminologies used in later chapters".

[1]Here, the Scheme of Work refers to a document drafted out by teachers to chart out all the topics to be taught within a given time-frame (e.g., a school term), along with pedagogical remarks and teaching tips.

2.2.2 External connection

75% of the respondents indicated that they made use of applications of mathematics in real-world contexts during their lessons as often as were indicated in the Scheme of Work. Two teachers used the examples involving the phenomenon of rainfall: "Use the example of rain appearing slanted when travelling in a bus or train to illustrate vector addition", and "Using the idea of holding an umbrella against raindrops to illustrate how the human mind uses projection to determine the direction of raindrops." Linear transformations and Magnetic Resonance Imaging (MRI) in medicine, as one teacher pointed out, are related, while bank study-loan repayment packages were used to illustrate the use of geometric progression in real life.

Interestingly, all the teachers agreed that the examples they used seemed authentic to most students, and even more, their students realized that applications of mathematics are "innate" in many aspects of life because they saw "mathematics as something that connects to real world experiences". The teacher who used the example of study loan repayment said that "study loan is going to be something that students are going to face real soon. They get to understand what the real world offers and think about questions like the assumptions we have made in our calculations (e.g., questioning when the interest rate will charged or when the repayment will be made)." When asked to state two challenges that the teachers when attempting to incorporate mathematics applied to real world contexts to externally connect H2 Math topics with the real world, all teachers mentioned about time constraints again. One teacher explained that "more time is required for a meaningful discussion. Usually we can do it for faster and better classes", while another pointed out that "while there are quite a lot of real life applications around us, we need to choose a suitable one that relates to the students and the one that we can "simplify" enough to apply the current knowledge that the students have."

2.3 Findings

Those H2 Math teachers took every available opportunity to bring in internal and external connections whenever the Scheme of Work requires of them. However, many of the Big Ideas used to establish internal connections appeared sketchy. Big Ideas that were used by these teachers, e.g., Functions, were taken directly from the H2 Math Syllabus document are different from those listed in [4]. When the teacher mentions that "function as mapping is pervasive in mathematics", such pervasiveness cannot be demonstrated in class. This same teacher attempted to link the concept of sequence with function by pointing out that a sequence is just a function of the form $f : \mathbb{N} \longrightarrow \mathbb{R}$. However, this is at best a formal definition of a real-valued sequence and does not seem at all convincing in demonstrating the pervasiveness of functions.

As for making external connections, examples mentioned by the teachers at best provide some believable links between mathematics and real world phenomenon, e.g., the rainfall looks slanted when viewed from moving vehicles. The authenticity of the real world contexts is not strongly conveyed through the mathematics. Worth mentioning is the example of the bank study-loan repayment scheme, where the teacher made it relatable to the students because these students would soon be enrolled for university education and might need to make such bank loans. From this teacher's description, one gathers that this example had been incorporated into some class activity in which students employed geometric progression to model the repayment scheme. Even in this example, the teacher was aware of the limitations of his approach because several assumptions had to be made to 'simplify' the mathematics so that it becomes accessible to the students. This then begs the question of how authentic can such classroom examples be.

Finally, one common problem that plagued these H2 Math teachers is the lack of time to implement such enriching learning experiences. They opined that "more time is required for a meaningful discussion". The lack of mathematical maturity for the lower ability students who struggled with the basics presents yet another major deterrent to teachers who wish to enact lessons that make internal and external connections.

3 Making connections through learning experiences

In order for teachers to circumvent some of the obstacles presented in Section 2.3, it is perhaps instructive to revisit some fundamentals concerning the creation of learning experiences. For this, we look no further than the Ralph Tyler's original curriculum model, often termed as *Tyler's rationale*:

Identifying objectives. This involves defining what the learners should know or be able to do at the end of the course. The objectives should take into account the needs of the learners, societal values and aims, and the philosophy of the school.

Selecting learning experiences. This step involves identifying the educational experiences that can help learners achieve the defined objectives. The experiences should be relevant and meaningful to the learners.

Organizing learning experiences. The selected experiences are then organized in a logical and effective sequence. This organization facilitates the learning process and helps learners make connections between different parts of the curriculum.

Evaluating the curriculum. The final step involves assessing whether the objectives of the curriculum have been achieved. This evaluation helps in understanding the effectiveness of the curriculum and provides insights for future improvements.

Specializing Tyler's rationale to this case study, the above processes now read:

1. Identifying objectives:
   (a) Students demonstrate relevant mathematical competencies in connecting two (or more) seemingly different concepts and use this connection to create deeper understanding of the H2 Math topics.
   (b) Students use their knowledge of H2 Math concepts to access more advanced mathematics which are deployed in authentic real world problems, and thus appreciate and see the value of H2 Math learnt in school with life.
2. Selecting learning experiences: When teaching a certain H2 Math topic, the teacher selects an authentic real world context that involves a mathematics concept beyond the H2 Math syllabus. The teacher then designs lesson examples, tasks and activities that made advanced mathematics and authentic real world applications accessible to students, tapping on their prior knowledge of H2 Math topics.

3. Organizing learning experiences: The teacher creates a logical, effective and workable sequence of learning experiences within a given time-frame. Crucially, the teacher facilitates the learning process and helps the students make internal and external connections between different parts of the H2 Math syllabus.

4. Evaluating the curriculum: The teacher designs assessment tools to test whether the students have acquired the ability to make internal and external connections and exploit this ability to solve real world problems. This evaluative part of the assessment helps the teacher judge the effectiveness of the learning experience and make further improvements for subsequent implementations.

4 Learning experiences leveraging on ML

Both existing literature and the responses given by the teachers in the survey have informed us that teachers need to (but have difficulties) (a) select an authentic real world problem that (b) invokes mathematics beyond H2 Math syllabus that still stays within reach of these students and (c) plan and enact learning experiences within a workable, preferably short, time-frame. One likens this situation to building a bridge that connects an island to the mainland: the obvious optimal solution is to choose a pair of closest points and build in the shortest time! Translating this to the organization of learning activities, it would mean that the teacher locates an 'advanced' mathematics topic that is relatable to many topics in H2 Math syllabus, preferably those which are taught during the first year of the two-year program. Some of these topics include (1) Sequences and Series, (2) Vectors, and (3) Calculus (see Table 1 for the relevant extraction from [10]).

Backed by his background in Machine Learning (ML), the second author recognizes certain H2 topics such as Sequences and Series, Vectors, Calculus and Linear Regression as prerequisites in Machine Learning courses offered at the tertiary level. This observation gives rise to the proposition that Machine Learning be used as the suitable advanced mathematics topic (outside H2 Math Syllabus) because of its links with many H2 Math topics. Furthermore, Machine Learning is a field of study in Artificial Intelligence (AI) that finds many applications in real world contexts, such as natural language processing, computer vision, speech recognition, business, medicine. The mathematical foundations of ML are provided by mathematical optimization methods, such as Gradient Descent Method, Linear Regression, etc. ML concerns with the development and study of statistical algorithms that can learn from data and generalize to unseen data and perform tasks without explicit instructions. In view of the growing presence of Generative AI tools, teachers and students would likely find ML as an advanced mathematics topic relatable to their daily encounter with AI.

In this section, we create and organize a sequence of learning experiences that leverages on ML to equip students with the mathematical competencies to make the desired internal and external connections articulated in 1. Owing to the limitation of space here, we anchor our discussion on the *Gradient Descent Method* which is an important optimization method in ML. Each ensuing subsection contains a description of the learning experience that manifests as part of a lesson development or classroom task.

4.1 Steepest slope of a plane

4.1.1 Lesson tasks

Having learnt the sub-topic of the Cartesian equation of planes, the students are given a 3D-Desmos visualization of the plane $\Pi : x - 2y + 3z = 0$ (Figure 1).

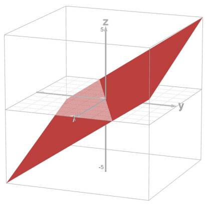

Figure 1: 3D visualization of the plane Π.

- The teacher assigns Task A to each group. As the students work on the task, they interact with the 3D visualization of Π in the Dynamic Geometry environment of 3D Desmos.

> **Task A** With respect to an origin O, the plane Π has Cartesian equation given by
>
> $$x - 2y + 3z = 0.$$
>
> (i) Verify that O lies on the plane Π and write down a vector normal to Π.
>
> (ii) Obtain a vector equation of the *line of greatest slope*, through O, lying on Π. (Recall that the line of greatest slope on a plane is one which is steepest.)

- Task B is then assigned after completion of Task A, where the students are to make connections between Tasks A and B, and based on these complete Task B.

> **Task B**
> Consider the function
>
> $$f(x, y) = -\frac{1}{3}x + \frac{2}{3}y,$$
>
> and the point $(x_0, y_0) = (0, 0)$.
> A point P moves from (x_0, y_0) in the direction of $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$ in such a way that f decreases fastest. Give an example of such a vector $\mathbf{u}$.

4.1.2 Tyler's rationale in action

Task A(ii) is a non-routine problem. To solve this problem, students can rely on Dynamic Geometry Software to visualize the relationships between (1) the normal vector $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, (2) the line of intersection of the plane $z = 0$ and Π, and (3) the line of greatest slope through O. Figure 2 (left) depicts the line of intersection of the plane $z = 0$ and Π, while Figure 2 (right) shows that the normal vector $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ is perpendicular to the line of intersection and the line of greatest slope through O.

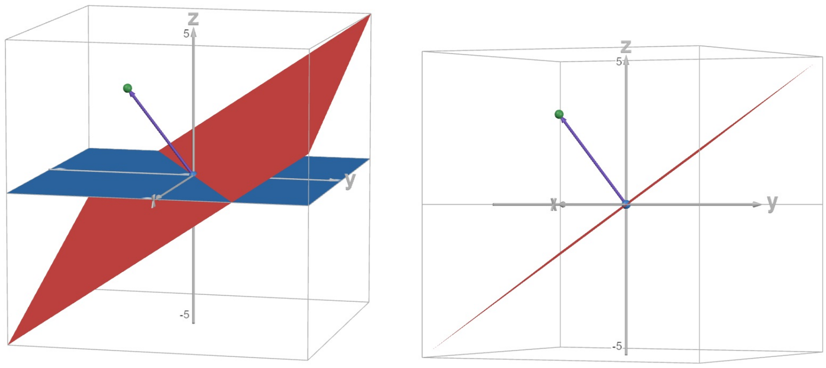

Figure 2: Two different 3D views

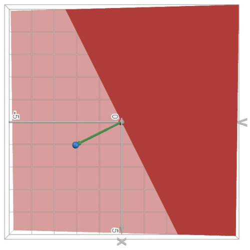

Figure 3: Top view

The learning experience explicitly requires students to relate Task A to Task B, which is an instance of making internal connections within topics in H2 Math through Big Ideas. In particular, the Big Idea of Equivalence is activated when the students realize that moving downwards along the line of greatest slope is the same as moving in the direction of greatest descent parallel to the plane $z = f(x, y)$. Based on this connection, the students can now view the projection of the normal vector $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ onto the horizontal plane, and determine that the required directional vector $\mathbf{u}$ is just this projection, i.e., $\mathbf{i} - 2\mathbf{j}$ (see Figure 3).

Remark 1 *The plane* Π *intersects* $x = 0$ *(respectively,* $y = 0$*)along the line through* O *parallel to* $-3\mathbf{i} + \mathbf{k}$ *(respectively,* $3\mathbf{j} + 2\mathbf{k}$*). Since these two vectors are the spanning vectors for* Π*, they are both perpendicular to the normal vector* $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ *of the plane* Π *(see Figure 4).*

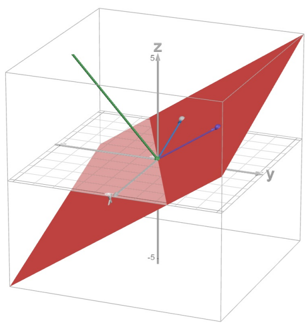

Figure 4: An alternative way of obtaining the normal vector

4.2 Greatest descent along a surface

4.2.1 Guided discovery

The teacher introduces Task C and adopts a problem solving approach. This lesson is an example of guided discovery leading the discovery of the *gradient of a function* f, i.e., ∇f.

> **Task C**
> Consider the function $f(x, y) = x^2 + y^2$ and the point $(x_0, y_0) = (1, 1)$.
> A point P moves from (x_0, y_0) in the direction of $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$ in such a way that f decreases fastest. Determine an instance of $\mathbf{u}$.

Adopting the conception of lesson play in the classroom from [18], the teacher can anticipate a dialogue with the students given in Figure 5.

> T: How would you want to approach this problem?
> S1: It looks like Task B. So we can find the normal vector to the plane...
> S2: But this is not a plane. 3D Desmos shows a curved surface when we plot $z = x^2 + y^2$.
> T: Maybe what S1 refers to the tangential plane to the surface at $(1, 1, 2)$?
> S1: You mean find the normal vector this tangential plane?
> S2: Then we need two vectors parallel to this plane, which can be crossed to get the normal vector. Right?
> T: From Task 2, can you recall the alternative method to find the normal vector?
> S3: Intersect the plane $x = 0$ with the curved surface. But you don't get a line...
> S1: This time they intersect along a curve.
> T: The equation of the curve is $z = 1 + y^2$ because $x = 1$. Now the gradient at $y = 1$ is?
> S2: We calculate $z'(y) = 2y$ and plug in $y = 1$.
> S3: It looks like you are differentiating $f(x, y) = x^2 + y^2$ but treating x as constant because x is always 1, correct?
> T: This is formally called the partial derivative of f with respect to y. So what S3 calculated is denoted by f_y.
> S1: So a vector tangential to this curve of intersection is $\mathbf{j} + f_y\mathbf{k}$. This vector will also be tangential to the surface $z = f(x, y) = x^2 + y^2$.
> S2: But we need another vector parallel to the surface.
> T: How do you think you may get that?
> S3: Can we intersect the surface $z = x^2 + y^2$ with the plane $y = 0$? Then get another curve of intersection, $z = x^2$.
> T: That's a good idea.
> S1: So another vector tangential to the surface must be $\mathbf{i} + f_x\mathbf{k}$.
> S3: Now we have to get a normal vector to this tangential plane. But this time it is straightforward. We just cross those two tangential vectors to get the normal vector.
> T: Why not you try to write down the working of what we discussed so far?

Figure 5: A sample dialogue between teacher and students.

4.2.2 Tyler's rationale in action

The above dialogue helps the teacher plan out a possible discursive road-map as he/she walks through the solution of Task C with the students. Through this discourse, the teacher has led the students to calculate a normal vector to the surface given by

$$\begin{pmatrix} 1 \\ 0 \\ f_x \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ f_y \end{pmatrix} = \begin{pmatrix} -f_x \\ -f_y \\ 1 \end{pmatrix}.$$

Again, taking the top view the students deduce that moving along $-(f_x\mathbf{i} + f_y\mathbf{j})$ gives the direction of fastest descent. The teacher may take this time to introduce the more advanced mathematical concept below:

Definition 2 (Gradient of a function) *Let $f = f(x, y)$ be a function of two real variables x and y. The* gradient *of f, denoted by ∇f, is defined to be the vector*

$$\nabla f := f_x\mathbf{i} + f_y\mathbf{j}.$$

This learning experience helps the students learn new mathematics from the H2 Math they already knew. This concept of gradient of a function is an essential ingredient of the Gradient Descent Method, which is the subject of the next learning experience.

4.3 Introducing gradient descent

4.3.1 Class discussion and activity

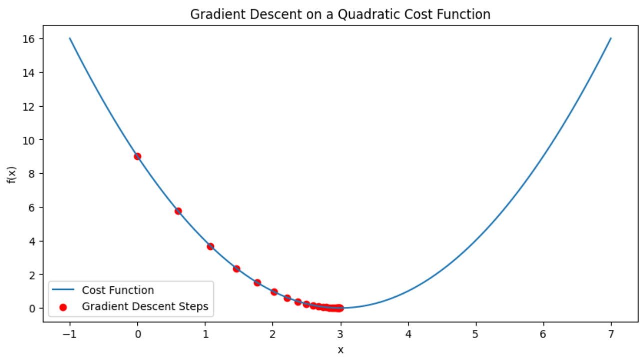

Figure 6: Gradient Descent on a Quadratic Cost Function

The teacher begins with the following scenario. Suppose you are at the point $(0, 9)$ on the curve $f(x) = (x - 3)^2$. You wish to inch your way to the lowest point on the curve by adopting the following strategy. When you are located at a point x_k you make a horizontal displacement that is proportional to the gradient $f'(x_k)$ (by a constant factor of α) which results in a drop in vertical distance (Figure 6). That is,

$$x_{k+1} = x_k - \alpha f'(x_k). \tag{1}$$

A possible discussion can emerge based on this scenario.

> T: Why is this strategy a reasonable one?
> S1: If the gradient is steep, the point is still quite far from the lowest point.
> T: What do you mean?
> S1: I mean as you approach the lowest point, the slope gets gentler ... the gradient gets near to zero, and so you inch forward with a smaller step not to go past the lowest point.
> S2: Yes, the last time my friends and I were trekking to look for low-lying water bodies we do the same thing. We went down along the steepest part of the slope to get to the lowest point as quickly as we could.
> T: I see ... so you can relate the math here with your own outdoor experience. Good.

The teacher then raises the students' attention on Equation 1. This equation is one of the many of its kind called *Gradient Descent Update Rule*. In this example, the gradient descent update rule associated to the function $f(x) = (x-3)^2$ is given by $x_{k+1} = x_k - \alpha \cdot \underbrace{2(x_k - 3)}_{f'(x_k)}$, and $x_1 = 0$. The constant of proportionality α is called the *learning rate*.

Given that students have some experience in using electronic spreadsheets or graphing calculators to generate recurrence sequences, the teacher then task them to engage in a hands-on computer-aided activity.

> **In-class activity**
> Use an electronic spreadsheet (e.g., Excel or Graphing Calculator) to generate the sequence defined by the recurrence equation $x_{k+1} = x_k - 2\alpha(x_k - 3),\ x_1 = 0$ for various learning rates of $\alpha = 0.1, 0.2, 0.5, 1.0$. Which learning rate, α, is the best?

Remark 3 *Gradient descent is crucial because it is the foundation of many machine learning algorithms, enabling models to learn from data by improving their predictions over several iterations. It is especially important for training complex models like neural networks, where analytical solutions are not feasible. The efficiency and effectiveness of gradient descent make it a cornerstone in the field of optimization and machine learning.*

4.3.2 Tyler's rationale in action

It is important that the teacher helps students relate their interests and daily experience with the learning objectives. The hands-on activity involves using computer technology which makes the application of mathematics to real world situations authentic. Firstly, the mathematics – Gradient Descent – used to solve the minimization problem is closer to what people use in the real world. Secondly, the students get to appreciate that computers are essential in this activity to help them compute and visualize the convergence.

5 Conclusion

The integration of machine learning concepts, specifically Gradient Descent, into the H2 Math syllabus provides an excellent opportunity to bridge the identified gap between pre-university mathematics in H2 Math syllabus and the mathematics applied in real-world situations. Each learning experience presented in Section 4 has been specially crafted based on Tyler's rationale, i.e., we offer students meaningful in-class interactions with authentic mathematics actually used in real-life within the context of the H2 Math topics they learn in school. These carefully designed lessons not only align with the current H2 Math syllabus but also enrich it by demonstrating practical applications, thereby enhancing students' engagement and motivation. The hands-on activities and the Python implementation offer students a chance to develop valuable 21st century competencies in mathematics and other related disciplines, preparing them for the interdisciplinary demands of higher education and modern careers.

Ultimately, the proposed approach in this paper fosters a deeper appreciation for mathematics and its relevance in today's technological landscape, helping both teachers and students make *internal* connections within H2 Math topics and *externally* with real-life applications of mathematics. By making advanced topics accessible and relatable, we hope to contributes towards "*Innovations in Mathematics and Mathematics Education in Technology*".

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A Survey questionnaire

This is a short survey that seeks to find out how you plan and enact lessons that achieve one or both of the H2 Math syllabus aims: (i) connect ideas within mathematics and apply mathematics in the contexts of sciences, engineering and other related disciplines; (ii) experience and appreciate the nature and beauty of mathematics and its value in life and other disciplines.

Internal connection: learning experiences, themes, big ideas

1. How often do you teach towards big ideas?
   Once a week/Once a month/As it appears in Scheme of Work/Incidental/Never

2. Give one example to illustrate how you help students see the connections between topics. Be specific which theme or big idea your example targets.

3. Comment about the efficacy of this particular example, e.g., do students gain an appreciation, a deeper understanding of mathematics, etc?

4. If your answer to Q1 is “Never”, state briefly why.

5. Write down briefly two challenges you face in planning and enacting lessons when you think of or attempt to make connections *internally* within the topics taught in H2 Math.

External connection: mathematics applied to problems in real world contexts

1. How often do you make use of application of mathematics in real world contexts?
   Once a week/Once a month/As it appears in Scheme of Work/Incidental/Never

2. Give one example of how you incorporate mathematics in real world contexts in your teaching (e.g., lesson example, tasks and activities, assessment).

3. Comment about the efficacy of this particular example, e.g., are students convinced that mathematics is useful in real-life problems?

4. If your answer to Q1 is "Never", state briefly why.

5. Write down briefly two challenges you face in planning and enacting lessons when you think of or attempt to use mathematics applied to real world contexts to *externally connect* H2 Math topics with the real world.

B H2 Related Topics

Topic/sub-topic	**Content**
Sequences and Series	**Sequences and series** 1. Sequence given by a formula for nth term 2. Sequence generated by the relation $u_{n+1} = f(u_n)$, using graphing calculator or computer to generate the sequence 3. Convergence of a series and the sum to infinity 4. Formula for the nth term and sum of a finite arithmetic series 5. Formula for the nth term and sum of a finite geometric series 6. Condition for convergence of an infinite geometric series 7. Formula for sum to infinity of a convergent geometric series
Vectors	**Three-dimensional vector** 1. Vector and Cartesian equations of lines and planes
Calculus	**Differentiation** 1. Problems involving tangents and normals to curves, including cases where the curve is defined implicitly or parametrically 2. Local maxima and minima problems
Probability and Statistics	**Correlation and Linear regression** 1. Use of scatter diagram to judge if there is a plausible linear relationship between two variables 2. Concepts of linear regression and method of least squares to find the equation of the regression line

Table 1: Extracted items from H2 Math Syllabus [10].